Profiting from Mean-Reverting Yield Curve Trading Strategies

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ABSTRACT

This paper studies a set of yield curve trading strategies that are based on the view that the yield curve mean-reverts to an unconditional curve. These mean-reverting trading strategies exploit deviations in the level, slope and curvature of the yield curve from historical norms. Some mean-reverting strategies were found to have significant positive profits. Furthermore, the profitability of one of these strategies significantly outperforms, on a risk-adjusted basis, alternative strategies of an investment in a bond or equity index.
Trading in fixed income assets is a profitable business in global investment banks. Besides providing market liquidity through market-making activities, investment banks also devote significant amounts of proprietary capital to trade a wide variety of fixed income instruments, such as Treasury bills to 30-year government bonds, corporate bonds and mortgage-backed securities, etc. Besides investment banks, hedge funds and dedicated bond funds also actively pursue trading opportunities in fixed income assets.

The strategies deployed range from simple arbitrage-trading, to complex trades based on technical or market views on the term structures of interest rates and credit risks. These yield curve trading strategies are essentially bets on changes in the term structure. These trading strategies can be broadly classified as directional and relative-value plays. Directional trading, as the name implies, are bets on changes in the interest rates in specific directions. Relative-value trading, by contrast, focuses on the market view that the unconditional yield curve is upward sloping, and that the current yield curve would mean-revert to an unconditional yield curve. A wide variety of trading techniques are used to construct relative-value trades based on this market view. However, there have been few efforts to examine the performance of these trading strategies or to compare them with equity investment strategies. Litterman and Scheinkman [1991], Mann and Ramanlal [1997] and Drakos [2001] are recent studies on the subject.

In this paper, we analyze the performance of a specific class of such relative-value trading techniques that are directly implied by the notion that mean-reversion of the yield curve occurs. We consciously avoid “data-snooping” by not searching through a large number of possible strategies to find a few that are profitable. Instead, we start
from the market view that the yield curve mean-reverts and derive trading strategies that follows most naturally from such a view—if the level, spread or curvature is higher (lower) than the historical average, bet that the level, spread or curvature, respectively, will decrease (increase) towards the historical average. We shall refer to this class of technical trading strategies as “mean-reverting” trading strategies. Following Litterman and Scheinkman [1991], we consider the three aspects of the yield curve – namely, the interest rate level, the slope (i.e. yield spread) and the curvature – and construct a portfolio of yield-curve trading strategies centering on each aspect. To facilitate a consistent comparison of their performance, we impose cash neutrality and consider one-month holding period for each category of strategies, and adjust the payoff for risk, as measured by the standard deviation of the payoffs. Our study abstracts from credit risk --in particular, default risk – and chooses as our dataset, the U.S. Treasury interest rates, from the period 1964 to 2004 for our study. For each aspect of the yield curve, we consider strategies that trade on the whole yield curve, as well as strategies that trade on individual portions of the yield curve.

Our analysis shows that there exists a set of mean-reverting trades that appear to offer, on average, superior payoffs, even after accounting for transaction costs, over the period considered in our study. We compare these payoffs to two benchmarks. The first benchmark is a cash-free investment in the Lehman Brothers Government Intermediate Index. This involves essentially buying the index, which consists of a portfolio of bonds with maturities ranging from 1 year to 10 years, and selling short 1-month U.S. Treasury Bills, thereby earning the term premium (see Stigum and Fabozzi [1987], pp 271). The second benchmark involves a risk-adjusted strategy of investing in the S&P index, and funding the trade also by shorting one-month U.S Treasury bills. In this
comparison, we found that some yield curve strategies outperform the S&P strategy by up to 5.7 times, and the Lehman Brothers Bond index strategy by up to 4.8 times, based on a comparison of the risk-adjusted average gross payoffs.

While factoring in trading costs may appear to diminish the profits from some of the mean-reverting yield curve trades (one of the strategies still return profits that were significantly higher than the benchmarks, even after accounting for transaction costs), we must add that the implied transaction cost we calculated is based on the assumption of actually trading the whole principal value of the Treasury securities. The transaction costs can be significantly reduced by structuring derivative trades on a notional basis, mirroring the economic cash flows of the underlying yield curve trades but without actually funding and holding the bonds. These derivative trades are commonly carried out in the fixed income market. Hence, the potential remains for more mean-reverting yield curve strategies to yield significant positive returns.

MEAN-REVERTING YIELD CURVE STRATEGIES

There is a wide variety of yield curve trading strategies. The literature on yield curve trading dates back to the late 1960s; a sample of the earlier literature includes De Leonardis [1966], Freund [1970], Darst [1975], Weberman [1976], Dyl and Joehnk [1981] and Stigum and Fabozzi [1987]. More recent analysis of the subject are found in Jones [1991], Mann and Ramanlal [1997], Grieves and Marchus [1992], Willner [1996] and Palaez [1997].

Our focus in this paper is on yield curve trading strategies that are based on the conventional view that the yield curve mean-reverts to some historical norm. This
The market view is consistent with historical experience. For instance, U.S. Treasury bill rates, spreads and curvature all trade within tight, finite bounds. The interest rate term structures in other countries also exhibit similar patterns. This suggests that some form of mean-reversion mechanism is at work that prevents the yield curve from drifting to extreme levels or shapes over time.

The market view of yield curve mean-reversion is also represented in theoretical models of the interest rate term structure – as discussed in Vasicek [1977], Cox, Ingersoll and Ross [1981, 1985], and Campbell and Shiller [1991], for example – which incorporate some form of mean-reversion mechanisms and are based on some form of the expectations hypothesis.\(^1\) In essence, the pure expectations hypothesis of the term structure is the theory that the long-term interest rate is the average of the current and expected short-term rates, so that the yield spread is mean-reverting.\(^2\) Interest rates along the yield curve adjust to equalize the expected returns on short- and long-term investment strategies.\(^3\) Furthermore, by incorporating rational expectations, the pure expectations hypothesis implies that excess returns on long bonds over short bonds are un-forecastable, with a zero mean in the case of the pure expectations hypothesis. Any arbitrage opportunity should be captured and realized by investors immediately. Therefore, by the pure expectations hypothesis, yield curve trading strategies

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\(^1\) Shiller [1990], Campbell [1995] and Fisher [2001] provide surveys of the literature on interest rate term structure.

\(^2\) This was first propounded by Fisher [1986] and refined by Lutz [1940] and Meiselman [1962].

\(^3\) A weaker version, referred to as the expectations hypothesis, states that the difference between the expected returns on short- and long-term fixed income investment strategies is constant, although it need not be zero as required under the pure expectations hypothesis.
attempting to exploit anomalies or mis-pricings in the term structure would not yield consistently positive payoffs.

The expectations hypothesis of the term structure, therefore, stands in contrast to the practitioner’s view that it is possible to construct mean-reverting yield curve trading strategies to generate consistent positive payoffs. Broadly speaking, mean-reverting yield curve strategies attempt to take advantage of deviations in the current yield curve relative to an unconditional yield curve. Three commonly-used trades are: (a) bullet strategy, which is constructed so that maturities of bonds are concentrated at a particular part of the yield; (b) ladder strategy, which involve investments across a range of maturities; and (c) barbell strategy, which are constructed, for example, by investing in two ends of the yield curve, and shorting the middle portion, or vice versa (see Fabozzi [1996]). It is easy to see that bullet strategies are essentially bets on the level of the interest rates, while ladder strategies and barbell strategies are bets on the yield spreads and curvatures, respectively.

There have not been systematic efforts to examine the performance of these trading strategies, and relate them to the predictions of the expectations hypothesis. An exception is Culbertson [1957] who computed and graphed holding period returns, between one week and three weeks, for short and various long term Treasury securities. He found that the holding period returns were very different from observed spot interest rates, and concluded that the pure expectations hypothesis, as propounded by Lutz [1940] did not hold.

The predictability of the spot yield curve and the forward interest rates, as implied by the expectations hypothesis, has also not found unambiguous empirical support (see Hamburger and Platt [1975]). Shiller, Campbell and Schoenholtz [1983]
showed that the term structure does not provide information on the future change in the short-term rates. Moreover, as Cox, Ingersoll and Ross [1985] first showed, the different versions of the expectations hypothesis are not theoretically consistent. Mankiw and Miron [1986] also found that the predictability of the term structure disappears after the founding of the Federal Reserve. Subsequent work by Rudebusch [1995] and Balduzzi, Bertola and Foresi [1997] also found that changes in the interest rate were due to unexpected changes in the Fed targeting.

Data

The dataset we use for our study is the Fama-Bliss dataset obtained from CRSP (Centre for Research in Securities Prices, 2004). The data set contains monthly data on zero coupon yields derived from a yield curve of U.S. government Treasury bills and bonds from 30 June 1964 to 31 December 2004. We acknowledge that zero coupon yield data derived from the US Treasury markets prior to the mid 1980’s might contain some systematic biases.

We first note that many of the bonds in that period are callable bonds; thus, the price of the bond includes the value of the call option. These bonds are also likely to possess tax effects that are different from pure zero-coupon bonds (since STRIPS only start to trade actively from the mid-1980’s onwards). The liquidity of these bonds was also relatively low and could result in a liquidity premium being priced in. However, all these factors contributed to systematic bias in the price and yields; therefore, they should have only a tangential effect on our results. This is the case since the strategies we considered are always long-short strategies: for every bond that we go long in, we short another bond with the same value to maintain zero initial cash-flow. Moreover,
the decisions to long or short bonds at specific tenors are based on comparisons with historical averages. If the historical averages are similarly biased, then the results should be unaffected by the bias. Also, any bond of any tenor is, unconditionally, equally likely to be shorted as it is to be longed at any point of time. Therefore, systematic biases in the relative valuation between 2 bonds (for instance, if short maturity bonds were to be consistently undervalued relative to longer maturity bonds) should not bias the overall direction of the result when summed over the time series. In aggregate, due to the large number of both long and short trades made across different bonds and time, we have good reasons to believe that the data quality issue before the mid 1980s would not present a bias in our results in any particular direction. In fact, the noise created by this data quality issue may have caused the true level of significance to be understated.

For the purpose of this study, we express all zero coupon yields in the form of continuously compounded yields. These zero-coupon yields are of maturities that are approximately 1-month, 2-month, … , 12-month, 24-month, 36-month, 48-month and 60-month. The observed maturities are approximate in the sense that some bonds may be of 0.9 month, 3.3 month or 11.8 month in maturities. Moreover, the observation interval for each yield curve is only approximately one-month apart (e.g. 28 days or 33 days). The total number of time-series yield curve observations in our dataset is 487.

For the purpose of our study, we regularize the dataset. This is carried out in two steps. First, we perform a cross-sectional linear interpolation to each zero yield curve in order to obtain the yields at exact monthly tenors from 1-month to 60-month. For instance, if the observed yields are 9.8 months, 11.3 months and 12.3 months, we interpolate linearly to obtain the yields for the 10-month, 11 month and 12-month
tenors. Also, we linearly interpolate to obtain yields for maturities of 13-months, ..., 23-month, 25-month, ..., 35-months, 37-months, ..., 47-months and 49-months, ..., 59-months based on observed yields for months 12, 24, 36, 48 and 60. For our analysis, we shall refer to bonds with yields that are observed in the market as ‘primary’ bonds, and bonds with maturities that are not observed in the market as ‘hypothetical’ bonds. Hypothetical bonds therefore have maturities greater than 12 months, but are not divisible by 12. The distinction is made to facilitate a comparison of alternative yield curve strategies in our analysis.

The second step that we took to regularize the dataset is a temporal linear interpolation procedure. The following example explains the procedure. Suppose the interpolated 13-month yield are observed at three dates, 7% (date 0), 7.5% (28 days later) and 6% (another 33 days later). Since we focus on a holding period of one month, we require the yield curves to be at exactly one-month intervals, in order to calculate the payoff at the end of each holding period. For our purpose, we define this to be 365.25 days divided by 12, i.e. 30.4375 days. Hence, the temporally interpolated 13-month yield in this example are 7% (date 0), 7.3892% (30.4375 days) and 6.0057% (another 30.4375 days later).

Since the holding period of each trade is one-month, the relevant forward yield curve with which to compare against the unconditional yield curve is the one-month forward yield curve. The one-month forward interest rate at a maturity of \( X \) months is calculated as follows. Let \( r_{X,0} \) denote the current interest rate while \( r_{X,1} \) denote the one-month forward interest rate. We have

\[
e^{\frac{r_{x,0}}{12} \cdot \frac{1}{12}} = e^{r_{x+1,0} \cdot \frac{1}{12}}
\]

(1)
Finally, the unconditional yield at each maturity (for primary and hypothetical bonds) at any date is calculated as the simple average of all the yields observed for that maturity since June 1964 till the preceding month. We define the unconditional yield curve at any date as the set of unconditional yields over all the maturities. Exhibit 1 below illustrates the unconditional yield curve for various dates.

Strategies

We consider three classes of mean-reverting yield curve strategies, focusing on the three aspect of the yield curve: level, slope (i.e. yield spread) and curvature. For each strategy, the holding period of a trade is fixed at one month, after which a new trade is initiated. We impose cash neutrality, so that any excess cash is deposited at the 1-month tenor. Similarly, if additional funding is required, this is carried out at the 1-month tenor. Since the holding period is 1 month, a bond of maturity $X$ months has duration $(X-1)$ months. Consequently, the deposits and borrowings at the one-month tenor have no impact on the duration of each trade—interest rate movements have no effect on deposits and borrowings at the one-month tenor. We recognize that borrowing at Treasury bill rates is usually impossible; however the cash-neutrality design of our study makes actual shorting of Treasury bills unnecessary. Whenever a Treasury bill needs to be shorted, we will correspondingly need to go long on some other Treasury security. The combined effect of these two transactions can be achieved via a derivative
such as Treasury forwards and futures. Stringing together a series of Eurodollar futures can also produce a good approximation to the required cash-flows. For the S&P Index strategy, a position in the S&P futures will generate the required cash-neutral investment without actually shorting any Treasury Bills.

We allow for a 102-month training period in the construction of the unconditional yield curve, so that our calculation of the average payoff of each yield-curve strategy starts from January 1973 to December 2004. The reason for the selection of this particular training period is the fact that the Lehman Brothers U.S. Government Intermediate Bond Index, which is one of our benchmarks, starts in January 1973.

Class 1: Mean-reversion of yield levels

This class of yield-curve trading strategies is based on the view that the level of the yield curve mean-reverts to the unconditional level. We consider two strategies.

Strategy 1-A: Mean-reversion of average yield to the unconditional average

This strategy takes the view that the average level of the yield curve mean-reverts to that of the unconditional yield curve. In this trade, we compare the average of all the one-month forward yields at a particular date against the corresponding average for the unconditional yield curve. If the average interest rate level for the one-month forward yield curve is higher (lower) than the average for the unconditional yield curve, the expectation is that one-month forward yield curve would shift down (up). The implied strategy is to go long (short) all the bonds with maturities longer than one month. We consider two versions of the trade, one for maturities of only primary bonds, and another for all maturities, including all the interpolated maturities of the hypothetical bonds.
The trade is constructed as follows. If \( k/59 \) dollars are invested in the 60-month bond, with a duration of 59 months over the one-month holding period, the amount of cash invested in a bond of maturity of \( X \) months, with duration of \( (X-1) \) months, will be \( k/(X-1) \) dollar. The funds required to invest in all the bonds are borrowed at the month tenor. Similarly, if the trade is to go short all the bonds, then the cash is deposited in the one-month tenor. Therefore, the strategy is a duration-weighted, cash neutral trade. In this strategy, a parallel shift in the yield curve generates approximately equal contribution to the payoff at each maturity.

**Strategy 1-B: Mean-reversion of yield at each maturity to its unconditional level**

This strategy is based on the view that the yield at each maturity mean-reverts to its unconditional level. In this trade, if the one-month forward yield is higher (lower) than the corresponding level on the unconditional yield curve, the expectation is that one-month forward yield curve would fall (rise). Except for the one-month maturity, the implied strategy is to go long (short) the bond. The trade is constructed so that a parallel shift in the yield curve generates approximately equal contribution to the payoff at each maturity. If we go long or short \( k/59 \) dollars in the 60-month bond, the amount to long or short a bond of maturity of \( X \) months, with duration of \( (X-1) \) months will be \( k/(X-1) \) dollar. Again, the one-month sector is where deposits and borrowings are made to achieve cash neutrality. We consider two versions of the trade, one for maturities of only primary bonds, and another for maturities, including all the interpolated maturities of the hypothetical bonds.

**Class 2: Mean-reversion of yield spreads**
In this strategy, the focus is on the mean reversion of the slope of the yield curve. Two versions of the trade are carried out.

**Strategy 2-A: Mean-reversion of yield spread for the whole yield curve**

The trade is constructed as follows. Consider the spread between the 59-month and 1-month maturities on the one-month forward yield curve, and compare it with that of the unconditional yield curve. If the one-month forward yield spread is larger (smaller) than the historical average, the expectation is that the slope of the yield curve would fall (rise). The implied strategy is to go long (short) the 60-month bond and go short (long) the 2-month bond.

The trade is constructed as follows. Suppose \(k/59\) dollars are invested in the 60-month bond, we need to short the 2-month bond by \(k\) dollars, to achieve duration-matching. The excess cash of \(58k/59\) dollars is deposited in the one-month tenor. This strategy is a cash neutral trade and has a zero net duration. A parallel shift in the yield curve has negligible impact on the payoff.

**Strategy 2-B: Mean reversion of the yield spreads between 2 adjacent bonds.**

This trade is based on the view that the yield spread between two adjacent bonds of maturities \((X-1)\) months and \((Y-1)\) months, with \(Y > X\), on the one-month forward yield curve would mean-revert to the corresponding spread on the unconditional yield curve. We compare the yield spread of adjacent pairs of bonds on the one-month forward yield curve against the historical average on the unconditional yield curve. If the one-month forward spread is larger (smaller) than that for the unconditional curve, go long the bond, with maturity of \(Y\) months, and short the bond with maturity of \(X\) months. We duration-weight each leg of the trade so that changes in the yield spread
with equal magnitude across different trades would generate approximately equal payoff contribution to the portfolio. For any bond with maturity of \( Z \) months, the cash to go long or short the bond is \( k/(Z-1) \) dollars. We again impose cash neutrality.

This trade essentially focuses on the slope of the yield curve for adjacent bonds on the one-month forward yield curve. We consider two versions of the trade, for both yield curves with only primary bonds and another set with maturities one month apart from one month to 60 months.

**Class 3: Mean reversion of curvature**

We define curvature as follows. Take three zero coupon bonds, with maturities of \( X, Y \) and \( Z \) months and corresponding one-month forward yields of \( r_X, r_Y \) and \( r_Z \). The curvature of the curve yield curve, as defined by the three bonds, is the measure:

\[
c(X,Y,Z) = \frac{r_Y - r_X}{Y - X} - \frac{r_Z - r_Y}{Z - Y} \tag{2}
\]

If the curvature is smaller (larger) relative to the corresponding measure for the unconditional yield curve over the same set of maturities, the expectation is that the curvature of the one-month forward yield curve would increase (decrease). We consider two strategies.

**Strategy 3-A: Mean reversion of the curvature of the yield curve**

This strategy focuses on the entire yield curve. Specifically, we consider the maturities of 1-month, 29-month (a hypothetical bond, and the mid-point) and the 59-month bond, on the one-month forward yield curve. If the curvature is expected to increase (decrease), the implied trade is to go long (short) the 2-month and 60-month bond and short (long) the 30-month bond, on the current yield curve. We match the
durations of the various portions of the trade as follows. For every $k/59$ dollars invested in the 60-month bond (with a duration of 59 months), the amount invested in the 2-month bond is $k$ dollars. Next, for the 30-month bond (with a duration of 29 months), the amount to short is $2k/29$ dollars. The excess funding needs is met by borrowing $k(1/59 + 1 - 2/29)$ dollars at the one-month tenor. The trade is cash-neutral and has zero duration, so that a parallel shift in the yield curve or a change in the slope of the yield curve without a change in curvature has negligible impacts on the payoff. The curvature trading strategy we just described is often referred to as a barbell strategy.

**Strategy 3-B: Mean reversion of the curvature of 3 adjacent bonds to the unconditional curvature**

In this trade, we compare the curvature of any three adjacent bonds, say with maturities of $(X−1)$, $(Y−1)$ and $(Z−1)$ months on the 1-month forward yield curve, as measured by $c(X−1,Y−1,Z−1)$ described in (2), with the corresponding curvature by the unconditional yield curve. If the curvature is smaller (larger) relative to that for the unconditional yield curve, the expectation is that the curvature of the current yield curve over the three maturities would increase (decrease). The implied trade is go long (short) the $X$-month and $Z$-month bond and short (long) the $Y$-month bond.

Again, we match the durations of the various portions of the trade so that the trade is immune to shifts in the yield curve. The amount of cash to be invested in the $X$-month and $Z$-month bonds are, respectively, $k/(X−1)$ dollars and $k/(Z−1)$ dollars. As for the bond with $Y$-month maturity, the cash amount is given by $2k/(Y−1)$ dollars. The funding need or excess cash for this trade is $k/(X−1) + k/(Z−1) - 2k/(Y−1)$ dollars. The strategy is essentially a portfolio of curvature trades, using all the primary bonds.
Since the hypothetical bonds are linearly interpolated from the primary bonds, the curvatures of the hypothetical bonds are zero. Hence, the trade does not work with hypothetical bonds.

**Benchmarks**

In order to be able to compare the performance of the mean-reverting trades described in the preceding subsection, we construct two benchmarks. The first is a fixed income strategy benchmark while the second is an equity investment benchmark.

*Benchmark 1 – Investment in the Lehman Brothers U.S. Government Intermediate Bond Index*  

This benchmark is constructed by assuming that we go long on the Lehman Brothers U.S. Government Intermediate Bond Index. The trade is funded by shorting 1-month Treasury bills. This is a standard benchmark in the fixed income market, essentially deriving its returns from the term premium of interest rates (see Stigum and Fabozzi [1987]). This trade, like all the other strategies that we are testing, is cash neutral. When used as a benchmark, we will match the volatility of this strategy to the other strategies, and then compare the means.

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4 There is a similar, though less common, benchmark that we can use. Profiting from the term premium involves buying a long-dated bond, and holding it for a period of time. Therefore, a logical benchmark is to simply buy a 60-month bond every month and holding it to maturity, all the while repeatedly funding the long positions with corresponding short positions in the 1-month Treasury Bills. A new 60-month bond is bought each month. Hence, at any one time, there is portfolio of bonds of maturities ranging from one month to 60 months. The payoff of the portfolio is calculated as the marked-to-market profits each month. As expected, this benchmark is almost identical to an investment in the Lehman Brothers U.S. Government Intermediate Bond Index.
Finally, we construct an equity benchmark to compare the performance of mean-reverting trades against an alternative investment strategy in equity assets. Most studies on fixed income investment strategies do not compare the performance against the alternative strategy of investing in equity instruments. Any attempt at doing so often runs into problems of comparability, in terms of risk adjustments, holding period and credit risks etc. The equity benchmark we construct addresses these issues.

We use the S&P index, starting from January 1973. Invest a dollar in the S&P index, and borrow a dollar for one-month by shorting 1-month Treasury bills. The trade is cash-neutral, with a one-month holding period. We found that the average profit is $4.91 for every $1000 invested in the S&P, funded by 1-month borrowings – the average monthly excess returns of the S&P index over our sample period is 0.491% per month.

RESULTS AND ANALYSIS

By adjusting the cash amounts, we can derive comparable volatilities (standard deviation) in payoffs for the S&P investment against a particular mean-reverting yield curve strategy. Let the standard deviation of payoffs for the cash-neutral investment in the S&P index from January 1973 to December 2004 be denoted by $\sigma^E$ (the standard deviation of the monthly excess returns of the S&P index). Similarly, let $\sigma^\#$ denote the standard deviation of payoffs from a $1$ nominal position for a yield curve strategy numbered $\#$, from January 1973 to December 2004. Hence, to yield identical volatility in payoffs, the cash amount of $k$ dollars for a particular yield curve strategy is given by
for each dollar invested in the S&P trade. Note that the matching of volatilities across different strategies is done after all the payoffs are realized. This is to ensure that the volatilities of the 2 competing strategies will be matched exactly. This procedure does not, in any way, compromise the fact that all investment decisions are made “out-of-sample”. It merely seeks to evaluate any two competing strategies on a fair and comparable basis by scaling the size of the monthly payoffs to match the standard deviations of the 2 strategies.

Exhibit 2 below presents performance of the various strategies and benchmarks before accounting for trading costs (We defer the discussion of transaction costs to a later section). From Exhibit 2, we note that, on a comparable risk-adjusted basis, only strategies 2-B, 3-A and 3-B yield higher payoffs compared with the two benchmarks. In particular, not all mean-reverting yield curve strategies beat the simple buy-and-hold bonds strategy (Benchmark 1). In the following subsections, we analyze in detail the set of profitable mean-reverting yield-curve strategies.

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**Performance against the Benchmarks**

Against the two benchmarks, strategies 2-B and 3-B have performed remarkably well. On a comparable basis, Exhibit 2 shows that the monthly payoff of
strategy 2-B is about 5.7 times that of the monthly payoff of the equity benchmark (benchmark 2). This means that while investing $1000 in S&P (and funding the investment by shorting 1-month Treasury Bills) generates an average profit of $4.91 per month, strategy 2-B generates $27.78 per month, after adjusting the volatility of payoffs for strategy 2-B to exactly match the volatility of payoffs from the S&P strategy. For strategy 3-B, the corresponding ratio is about 3.9 times against the equity benchmark. Hence, yield-spread mean-reverting and curvature mean-reverting strategies can outperform an equity investment strategy, on a risk-adjusted basis.

Moreover, Strategies 2-B and 3-B also outperformed the bond benchmark. In the case of strategy 2-B, the average monthly payoff is about 4.8 times that of Benchmark 1, while for strategy 3-B, the average monthly payoff is about 3.3 times that of Benchmark 1.

The next subsection will test whether these superior performance of (gross) payoffs relative to the benchmarks are statistically significant.

**Test of Significance of Excess Payoffs against Benchmarks**

To test whether strategies 2-B, 3-A and 3-B significantly outperform the benchmarks, we conduct two statistical tests of significance; these are: the paired t-test and the z-test using the Newey-West estimator (Hereafter, N-W test, Newey and West [1987]. Also see Diebold and Mariano [1995] for another possible test of significance for auto-correlated series).

The paired t-test requires that the time-series of payoff differences be uncorrelated. Positive auto-correlations will incorrectly overstate the power of the test. Exhibits 3, 4 and 5 respectively plot the first 60 auto-correlation of the payoff
differences between the strategies and the benchmarks. The autocorrelations are small in absolute values and are also distributed across positive and negative values. This means that the paired t-test, while not perfect, is still reasonable for our purpose.

The Newey-West estimator can be used to ascertain whether the mean of an autocorrelated and heteroskedastic series is significantly different from zero. It is less powerful than the t-test, but it requires weaker assumptions by accounting for autocorrelation. We also allow autocorrelations of up to 60 lags. The Newey-West generates a variance estimate that can then be used to compute the z-score for a particular series. Therefore, a statistic higher than 1.96 will imply that the difference between the two means being tested is statistically significant.

Exhibit 6 shows that while strategy 3-A does not significantly outperform the benchmarks, strategies 2-B and 3-B do. In particular, the p-value of the t-tests for strategies 2-B and 3-B are negligible. For the Newey-West test, strategy 2-B managed a p-value of 0.002 and 0.001 against benchmarks 1 and 2 respectively; while strategy 3-B
obtained a p-value of 0.013 and 0.008 against benchmarks 1 and 2 respectively. These p-values of these tests for strategy 2B are so low that our results are still highly significant even after making simple bonferroni adjustments to account for the fact that we tested 6 strategies in this study.\(^5\)

Having a profitability that is not significantly more than the benchmarks, but that is significantly more than zero could still mean that the strategy is useful as a positive-mean diversification tool if the correlations with the benchmarks are low. This is indeed the case for the strategies in this study. For all 3 strategies, the profitability is significantly more than zero for both the t-test as well as the N-W test. The correlations between the profits of the strategies and both the benchmarks are also extremely low. For strategy 2-B, the correlations of profits with benchmarks 1 and 2 are 0.0258 and 0.0567 respectively. For strategy 3-A, the correlations of profits with benchmarks 1 and 2 are -0.2188 and 0.0453 respectively. For strategy 3-B, the correlations of profits with benchmarks 1 and 2 are 0.0670 and -0.0028 respectively.

**Transaction Costs**

Thus far, all our analyses are done in terms of the gross payoffs of the different mean-reverting yield curve strategies. An obvious question to ask is whether the set of profitable trades, specifically strategies 2-B and 3-B, would continue to outperform the indices (or even yield positive returns) when the appropriate transaction costs are taken into account. Transaction costs in bond trading are embedded in the form of the spread

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\(^5\) The simple bonferroni correction adjusts the required p-value for rejection to account for multiple tests by dividing the alpha-level by the number of tests conducted. Therefore, in the case of our study where 6 tests are conducted, the p-value required for a rejection at the 5% level is 0.008333. The p-value from Strategy 2-B is still smaller than 0.00833.
between the ‘bid’ and ‘ask’ yields. The 5-year average spreads are approximately 1 basis point for Treasury bills that mature in 1 year or less, 0.8 basis points for 2-year bonds and 0.35 basis points for 5-year bonds. A reasonable assumption would be that the effective transaction cost for each trade is half the quoted spread. For the purpose of this paper, we assume a spread of 3 basis points for all the bonds traded from Jan 1973 to Dec 1980, 2 basis points for all bonds traded from Jan 1981 to Dec 1990 and 1 basis point for all bonds traded from Jan 1991 to Dec 2004 (and therefore pay a transaction cost of 1.5, 1 and 0.5 basis points respectively). Assuming a cost of 1 basis point, the cost expressed in dollars is a function of the maturity of the bond and the value of the bond, and can be approximated as follows:

\[
\text{(Transaction Cost)} \approx 0.0001 \times (\text{Maturity in Years}) \times (\text{Value of Bond})
\]  

(4)

As an illustration, buying or selling $100,000,000 worth of 6-month Treasury Bills will attract a transaction cost of $5,000.

The profitability of strategies 2-B, 3-A and 3-B after accounting for transaction costs are reported in Exhibit 7. We assume that the benchmarks are traded without any

---


7 Assume a yield of \( r \) for the bond with \( T \) years to maturity. If we were to buy the bond, based on a 1 basis point transaction cost, we obtain a yield of \( (r-0.0001) \). Thus, the transaction cost, in dollar terms would be:

\[
e^{-rT} - e^{0.0001T} = e^{rT} (e^{0.0001T} - 1) \approx e^{rT} (1 + 0.0001T - 1) = 0.0001 \times T \times e^{rT}
\]
transaction costs. Strategy 2-B is still significantly more profitable than both the benchmarks under all measures (both the t-tests and the N-W tests). Both strategies 3-A and 3-B no longer perform better than the benchmarks. However, the average profits are still positive; and in the case of strategy 3-B, significantly so.

It is important to note that the transaction costs we calculated are based on the assumption that the mean-reverting yield curve strategies are executed on a physical basis, i.e. the actual bonds are bought and sold and funds are borrowed (if required) to construct the trades on a monthly basis. The transaction costs can be diminished by reducing the frequency of the entering and exiting trades. For instance, instead of executing the trades on a monthly basis, the trades could be executed on a quarterly basis, or when the relevant deviations on forward yield curves for spreads and curvatures exceed certain thresholds.

More importantly, the transaction costs can be reduced substantially if the yield curve strategies are structured as derivative trades (on a notional basis) to mirror the economic cash flows of the underlying strategies, without actually funding and holding the bonds. These derivative trades are commonly carried out in the fixed income market. Therefore, while factoring in transaction costs may appear to diminish the profits from some the mean-reverting yield curve trades, there are different ways to lower the transaction costs. Nevertheless, Strategy 2-B still returns a significantly better profit than all the benchmarks even after accounting for these costs.

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8 Of course, the pricing of the derivative trades may involve other costs as well, as investment banks take a cut from the potential profits. Fortunately, there are some standard derivatives that can be traded at extremely low cost and can substitute for a pair of long-short trade in bonds. For instance, the highly liquid Eurodollar futures gives identical payoff as shorting a bond of a certain maturity, and at the same time going long a another bond of maturity 90 days longer than the shorted bond.
Value-Add of Mean-Reverting Strategy to Investment in the S&P Index

In the preceding sections, we have shown that a number of mean-reverting yield-curve strategies can be highly profitable. Another way to demonstrate the attractiveness of mean-reverting yield curve strategies is to consider the incremental value-add of including such strategies to an existing investment strategy. In this regard, Foster and Stine [2003] introduce a convenient test to ascertain whether a particular strategy can add value to a buy-and-hold investment in the S&P index. The Foster-Stine test is essentially a test on Jensen’s alpha (see Jensen [1968]), where the benchmark is the S&P index-- it involves regressing the excess returns of the selected strategy against the excess returns from the buy-and-hold investment in the S&P index. Based on this regression, we can obtain the t-statistic as well as the p-value of the intercept that allows us to test if adding a new strategy leads to a significant improvement in the performance of the portfolio. Again, the critical p-value needs to be adjusted using the bonferonni correction when multiple strategies are tested. If the regression intercept is statistically significant, then we can infer that the particular strategy does in fact add value to the original strategy of buy-and-hold the S&P index.

The basic premise behind this test is that a strategy that gives a positive mean return and is not too highly correlated to the S&P index can be linearly combined with the S&P index to obtain a better mean-variance return profile. In other words, a strategy that serves as a good addition to diversify holdings in the S&P index can therefore add value.

In the case of the mean-reverting yield-curve strategies we examined in this paper, Strategies 2-B and 3-B are found to have significant value-add even after accounting for transaction costs and the bonferonni correction. In particular, Strategies
2-B and 3-B have t-statistics of 6.816 and 2.685 respectively, with significant corresponding p-values. The results of the Foster-Stine test are reported in Exhibit 8 below.

Breakdown of the Payoffs

Since strategy 2-B seems to be highly profitable, we find it necessary to investigate the robustness of its profitability. As an initial check, we further analyzed the breakdown of payoffs. Exhibit 9 shows the contribution of the payoffs from each trade-segment in the portfolio for Strategies 2-B.

The results show that no single trade dominates the entire portfolio, and almost all the trade segments generate positive payoffs. Interestingly, trading the yield spread between 10-month and 11-month maturities on the one-month forward yield curve, generate substantial positive profits while trading the yield spreads between 11- and 23-months, as well as 23- and 35-months generate mild negative profits.
We also show the scatter-plot of the monthly payoffs against the absolute deviations of the relevant parameter from the unconditional yield curve for trade-segment 10—trading the 10-11 month spread. The scatter-plot is shown in Exhibit 10 below.

Exhibit 10 shows that, for this trade segment, the monthly payoffs have a high positive correlation with the absolute deviations from the unconditional yield curve (correlation = 0.819). In other words, the positive payoffs from this particular trade are not random payoffs: the larger the deviation from the unconditional yield curve, the larger the resulting profit from that particular trade. This result strongly supports the view that the spread of these portions of the yield-curve do in fact mean-revert and the reversion can be profitably exploited.

The presence of a very profitable trade segments in the 10-11 month portion of the one-month forward yield curve followed by unprofitable trade segments from 11- to 35-months provides some support for the “market-segmentation” view of the interest rate term structure in the fixed income market. This is the market view that many participants in the fixed income market have preferred habitats that are dictated by the nature of liabilities and investments, so that a major factor influencing the shape of the yield curve is the asset-liability management constraints that are either regulatory or self-imposed. Specifically, the yield curve is viewed as comprising a “short-end” — up
to the 12-month maturity – and a “long-end” – from 12-month onwards. Asset-liability management constraints, when they exist, restrict lenders and borrowers to the short-end or the long-end of the yield curve, or even certain specific maturity sectors, and, as a result, investors and borrowers do not shift from one maturity sector to another to take advantage of opportunities arising from differences between market expectations and the forward interest rates. Arbitrage trades in the fixed income market are frequently constructed in the transition between the short-end and the long-end of the yield curves.

Time Series Analysis

To investigate the profitability of strategy 2-B over time, we plot the 10-year moving average of the payoffs of strategy 2-B as well as the two benchmarks. These are shown in Exhibit 11 below.

From Exhibit 11, it can be seen that while the average monthly payoffs for strategy 2-B stays significantly positive throughout the sample, the increasing profitability of the benchmarks towards the end of the sample gradually eroded the out-performance of the strategy over time. Nevertheless, the results remain significant, and the absolute profitability (relative to zero) of the strategy does not seem to be sensitive to the time period.
CONCLUSION

The objective of this paper is to examine the profitability of a range of yield-curve trading strategies that are based on the view that the yield curve mean-reverts to an unconditional yield curve. Our study has shown that a small number of these yield-curve trading strategies can be highly profitable. In particular, trading strategies focusing on the mean-reversion of the yield spreads significantly outperformed two commonly-used benchmarks of investing in the Lehman Brothers U.S. Government Intermediate Bond Index and investing in the S&P, on a risk-adjusted basis. Although factoring in transaction costs lower the profitability of these trades against the benchmarks, the significant result still remains for this strategy. Furthermore, transaction costs can be reduced substantially, for instance, through structured derivative trades that mirror the underlying cash flows or by reducing the frequency of the trades.

We also investigate the profitability of these mean-reverting yield curve trades over time. A time series analysis of the performance of the various yield-curve trading strategies show that while the scope for excess returns over the benchmarks has diminished over time, the absolute level of profitability has not suffered. Therefore, profitable trading opportunities still exist (up to December 2004) in yield-spread mean-reversion strategies. Moreover, these strategies as well as strategies that exploit the mean-reversion of the curvature of the yield curve are found to have significant value-add to a strategy of buy-and-hold the S&P index.
REFERENCES


Exhibit 1

Unconditional Zero-Coupon Yield Curve

This figure plots the estimated unconditional zero-coupon curve across time. At each point in time, the unconditional curve is estimated by averaging all past yield curves up to that date.
Exhibit 2

Risk-adjusted Average Gross Payoff of Mean-Reverting Yield-Curve Strategies

For each of the 6 strategies, we calculate the average monthly payoff from Jan 1973 to Dec 2004 scaled such that the volatility of the payoff exactly matches that of the benchmarks. We then compare the average profitability of the strategies vis-à-vis the benchmarks.

<table>
<thead>
<tr>
<th>Class</th>
<th>Strategy</th>
<th>Bonds</th>
<th>Mean Payoff</th>
<th>Against Benchmark 1</th>
<th>Against Benchmark 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield Level</td>
<td>1-A</td>
<td>P</td>
<td>0.00102</td>
<td>0.174</td>
<td>0.208</td>
</tr>
<tr>
<td></td>
<td></td>
<td>H</td>
<td>0.00022</td>
<td>0.037</td>
<td>0.045</td>
</tr>
<tr>
<td></td>
<td>1-B</td>
<td>P</td>
<td>0.00127</td>
<td>0.216</td>
<td>0.259</td>
</tr>
<tr>
<td></td>
<td></td>
<td>H</td>
<td>-0.00003</td>
<td>-0.005</td>
<td>-0.006</td>
</tr>
<tr>
<td>Yield Spread</td>
<td>2-A</td>
<td>P</td>
<td>0.00274</td>
<td>0.467</td>
<td>0.558</td>
</tr>
<tr>
<td></td>
<td>2-B</td>
<td>P</td>
<td>0.02695</td>
<td>4.591</td>
<td>5.489</td>
</tr>
<tr>
<td></td>
<td></td>
<td>H</td>
<td>0.02778</td>
<td>4.773</td>
<td>5.658</td>
</tr>
<tr>
<td>Curvature</td>
<td>3-A</td>
<td>H</td>
<td>0.00968</td>
<td>1.649</td>
<td>1.971</td>
</tr>
<tr>
<td></td>
<td>3-B</td>
<td>P</td>
<td>0.01919</td>
<td>3.269</td>
<td>3.908</td>
</tr>
<tr>
<td>Benchmark 1</td>
<td></td>
<td></td>
<td>0.00587</td>
<td>1.000</td>
<td>1.196</td>
</tr>
<tr>
<td>Benchmark 2</td>
<td></td>
<td></td>
<td>0.00491</td>
<td>0.836</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Notes:
1. P – the trade is structured for primary bonds only; H – the trade is structured for both primary and hypothetical bonds.
Exhibit 3

Autocorrelations of Strategy 2-B Against Benchmarks

This figure plots the auto-correlations of the monthly difference of the payoffs between strategy 2-B and the two benchmarks. A low auto-correlation across all lags would imply that a t-test is valid for testing the significance of out-performance of the strategy vis-à-vis the benchmarks.
Exhibit 4

Autocorrelations of Strategy 3-A Against Benchmarks

This figure plots the auto-correlations of the monthly difference of the payoffs between strategy 3-A and the two benchmarks. A low auto-correlation across all lags would imply that a t-test is valid for testing the significance of out-performance of the strategy vis-à-vis the benchmarks.
Exhibit 5

Autocorrelations of Strategy 3-B Against Benchmarks

This figure plots the auto-correlations of the monthly difference of the payoffs between strategy 3-B and the two benchmarks. A low auto-correlation across all lags would imply that a t-test is valid for testing the significance of out-performance of the strategy vis-à-vis the benchmarks.
Exhibit 6

Significance Tests of Excess Payoffs of Strategies with respect to Benchmarks

For the 3 most profitable strategies (2-B, 3-A and 3-B), we perform significance test on their profitability. We test whether the average profits are significantly more than zero, as well as whether the profits are significantly more than the benchmarks. We use two types of test—the more powerful (but potentially less valid) t-test, as well as the Newey-West test.

<table>
<thead>
<tr>
<th></th>
<th>2-B</th>
<th>3-A</th>
<th>3-B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Payoff</td>
<td>0.02695</td>
<td>0.00968</td>
<td>0.01919</td>
</tr>
<tr>
<td>vs zero profits</td>
<td>11.885</td>
<td>4.270</td>
<td>8.464</td>
</tr>
<tr>
<td>vs Benchmark 1</td>
<td>6.659</td>
<td>1.076</td>
<td>4.300</td>
</tr>
<tr>
<td>vs Benchmark 2</td>
<td>7.076</td>
<td>1.523</td>
<td>4.447</td>
</tr>
<tr>
<td>vs Benchmark 1</td>
<td>3.116</td>
<td>0.526</td>
<td>2.479</td>
</tr>
<tr>
<td>vs Benchmark 2</td>
<td>3.432</td>
<td>0.357</td>
<td>2.654</td>
</tr>
</tbody>
</table>

Note:

1. We report the significance tests for the trade structured with primary bonds only. The results of the significance tests for strategy 2-B using the trade structured for both primary and hypothetical bonds are essentially identical.

2. We assume that the benchmarks are traded with zero transaction cost.
Exhibit 7

Significance Tests of Excess Payoffs of Strategies, net of transaction costs with respect to Benchmarks

For the 3 most profitable strategies (2-B, 3-A and 3-B), we perform significance test on their profitability, after accounting for transaction costs in executing the strategies. We test whether the average profits are significantly more than zero, as well as whether the profits are significantly more than the benchmarks. We use two types of test—the more powerful (but potentially less valid) t-test, as well as the Newey-West test.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>2-B(^1)</th>
<th>3-A</th>
<th>3-B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Payoff</td>
<td>0.01584</td>
<td>0.00355</td>
<td>0.00613</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Test</th>
<th>Statistic</th>
<th>p-value</th>
<th>Statistic</th>
<th>p-value</th>
<th>Statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>t-test</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>vs zero profits</td>
<td>6.985</td>
<td>0.000</td>
<td>1.566</td>
<td>0.118</td>
<td>2.702</td>
<td>0.007</td>
</tr>
<tr>
<td>vs Benchmark 1</td>
<td>3.159</td>
<td>0.002</td>
<td>-0.656</td>
<td>0.512</td>
<td>0.083</td>
<td>0.934</td>
</tr>
<tr>
<td>vs Benchmark 2</td>
<td>3.517</td>
<td>0.000</td>
<td>-0.435</td>
<td>0.664</td>
<td>0.379</td>
<td>0.705</td>
</tr>
<tr>
<td><strong>N-W test</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>vs zero profits</td>
<td>3.870</td>
<td>0.000</td>
<td>0.354</td>
<td>0.927</td>
<td>1.624</td>
<td>0.104</td>
</tr>
<tr>
<td>vs Benchmark 1</td>
<td>1.875</td>
<td>0.061</td>
<td>-1.321</td>
<td>0.186</td>
<td>0.060</td>
<td>0.952</td>
</tr>
<tr>
<td>vs Benchmark 2</td>
<td>2.160</td>
<td>0.031</td>
<td>-1.225</td>
<td>0.221</td>
<td>0.277</td>
<td>0.782</td>
</tr>
</tbody>
</table>

Notes:

1. We report the significance tests for the trade structured with primary bonds only. The results of the significance tests for strategy 2-B using the trade structured for both primary and hypothetical bonds are essentially identical.

2. We assume that the benchmarks are traded with zero transaction cost.
Exhibit 8

Test of Value-Added of Mean-Reverting Strategies (net of transaction costs) to a Buy-and-Hold Investment in the S&P Index (Jensen’s alpha)

This table lists the usefulness of each strategy when it is added to one that buys-and-holds the S&P index (Foster and Stine [2003]). Excess returns of the strategy (Y) are regressed on the excess return of the S&P index (X). If the t-stat of the intercept is significantly positive, this will imply that the particular strategy can add value to a simple buy-and-hold S&P strategy.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>2-B</th>
<th>3-A</th>
<th>3-B</th>
</tr>
</thead>
<tbody>
<tr>
<td>t-stat</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>p-value</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>alpha</td>
<td>6.816</td>
<td>0.000</td>
<td>1.449</td>
</tr>
</tbody>
</table>
Exhibit 9

Contribution of Different Trades to the Payoff of Strategy 2-B (Primary Bonds)

This figure plots the contribution of various trade segments of strategy 2-B to the overall profitability of the strategy. The overall profitability of the strategy is not dominated by any particular segment of the yield curve. Rather, almost all of the segments (with the exception of 2 segments) contribute substantially to the profitability.

Trade segments of strategy 2-B: Yield spread mean-reversion trade, on the 1-month forward curve
1. 1-2 month spread
2. 2-3 month spread
3. 3-4 month spread
4. 4-5 month spread
5. 5-6 month spread
6. 6-7 month spread
7. 7-8 month spread
8. 8-9 month spread
9. 9-10 month spread
10. 10-11 month spread
11. 11-23 month spread
12. 23-35 month spread
13. 35-47 month spread
14. 47-59 month spread
Exhibit 10

10-11 month Yield Spread Trade of Strategy 2-B, on the one-month forward curve

This figure is a scatter plot of the monthly profit of trade segment 10 (the yield spread between 10- and 11-month maturity on the zero coupon yield curve) versus the difference between the observed spread and the average historical spread. The strong relationship exhibited in the scatter plot indicate that whenever the yield spread is large relative to historical spreads, the resulting profit for the month is also large.
Exhibit 11

10-year Moving Average of Monthly Payoffs of Strategy 2-B
(for all Bonds)

This figure plots the time series of 10-year moving average of monthly profits for strategy 2-B. Throughout the entire sample period, the 10-year moving average of profits stayed around 0.015 to 0.02 (this means that if we scale strategy 2-B to have the same volatility as a $1 investment in the S&P index, the average profit per month is between $0.015 and $0.02).