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A Theory of Lowest-Price Guarantees

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A Theory of Lowest-Price Guarantees

Abstract

Retailers offer lowest-price guarantees (LPGs) to make their lowest-price claims credible. LPGs induce customers to purchase before search and even abandon post-purchase search. Moreover, firms gain or lose customer goodwill depending on the credibility of their lowest-price claims. We find that the change in search pattern raises prices while goodwill intensifies price competition. Consequently, equilibrium prices may be lower even if total search decreases with LPGs. If goodwill is exogenous, the rate of price decline in goodwill increases with search intensity. In the limit where all customers search, any positive goodwill leads to Bertrand prices. If goodwill varies linearly with the price differential, prices are at either monopoly or Bertrand levels.
1 Introduction

Retailers often offer lowest-price guarantees (LPGs) which promise to match the lowest price offered in the market. To customers, such guarantees increase the credibility of the firms’ lowest price claims. The conventional wisdom is that LPGs are adopted to pre-empt price cuts: a rival cannot steal market share by undercutting because customers who search before purchase can always invoke the LPG to obtain the lowest price at the high-price store (Salop, 1986, Doyle 1988). If the market consists of “searchers” and “non-searchers”, Png and Hirshleifer (1987) show that LPGs can be used as a discriminatory device to allow the retailers to charge higher prices to the latter. Moorthy and Winter (2002) also find that even when products are differentiated and firms have different costs, prices are monopolist if all firms offer LPGs. Consequently, LPGs lead to higher prices. This anti-competitive effect of LPGs has recently been challenged. Hviid and Shaffer (1999) show that if customers incur hassle costs of obtaining refunds, the Bertrand result can be restored since firms can steal market share through lowering price by less than the hassle cost. Moreover, in a market with heterogeneous customer segments, prices in a LPG regime can even be lower if LPGs encourage an additional segment of customers to undertake search (Chen et al, 2001).

The existing research makes three assumptions about customer shopping behavior. First, customers search before purchase and always buy at their favorite stores regardless of the stores’ posted prices. They invoke the LPGs if their favorite stores post higher prices and thus pay the lowest price. Second, the intensity of search either remains unchanged or increases with the adoption of LPGs. Third, it is assumed that there are no potential adverse reactions by customers when firms offering LPGs fail to charge the lowest price.

Our paper differs from the literature in three significant ways in that it captures the following elements of an oligopolistic market with LPGs:

1. LPGs induce customers to search after purchase. This is consistent with the observation that retailers often afford customers a period of up to 45 days after purchase to claim price-matching refunds. The fact that refunds are being issued in the mar-
ket also supports the existence of post-purchase search. Hence, a model of LPG should capture post-purchase search behavior.

2. LPGs may reduce the level of search by customers. Srivastava and Lurie (2001) reported an experiment which shows both search intentions and actual search behavior declined significantly with LPGs. In addition, Jain and Srivastava (2000) found that customers view LPGs as signals of lowest prices. This is unsurprising since LPGs are always offered in concert with lowest-price claims. More importantly, since customers believe firms offering LPGs do have the lowest price, it implies that they have less incentive to search. Hence, it is important to allow for a lower level of search in a model with LPGs.

3. Firms which offer LPGs and subsequently charge higher prices may lose customer goodwill because customers will doubt the credibility of future lowest-price claims. This ex post loss of goodwill can reduce future purchases at the firms and hence their payoffs. The concern over this loss of goodwill may explain why some firms offer refunds that are greater than the actual price differentials. On the other hand, firms which offer the lowest price enhance their low-price reputation and enjoy a gain in goodwill. Consequently, firms should set their prices accordingly to reflect these ramifications.

In this paper, we develop a model of price competition with LPGs in which customers undertake only post-purchase search; moreover, compared to price competition without LPGs, customers may decrease their search intensity. Firms gain or lose customer goodwill depending on whether their lowest-price claims are true. We compare the results of our model to a benchmark model without LPGs and show that equilibrium prices can be higher or lower with LPGs. This finding is true even if the level of search intensity becomes lower with LPGs. While the change in search pattern raises prices, the presence of customer goodwill intensifies price competition. In the case where customer goodwill is exogenous, that is, independent of the prices, we find that prices decrease linearly in goodwill initially and then in a convex manner. Moreover, this rate of decline increases with the intensity of post-purchase search. In the limit where all customers undertake post-purchase search, any positive level of goodwill is sufficient to drive prices down to Bertrand levels. This finding provides a fresh interpretation of the Traveler’s Dilemma.
(Basu, 1994) as a special case of price matching competition with customer goodwill. Finally, if customer goodwill is endogenous and varies with the price differential, equilibrium prices become more extreme at either monopoly or Bertrand levels.

The paper proceeds as follows: Section 2 presents a model of a LPG duopoly with post-purchase search and customer goodwill. Section 3 characterizes the solution to the model in the case where customer goodwill is independent of prices and compares the equilibrium prices to a benchmark model without LPGs. We show that our model generalizes the Traveler’s Dilemma and a store-switching model where customer goodwill is synonymous with gains or losses in future market share. Section 4 analyzes the model with endogenous customer goodwill and discusses an application to price-beating competition. Section 5 concludes.

2 Model Setup

We consider a symmetric duopoly where the firms, denoted by $i$ and $j$, set prices in a non-differentiated product market\(^1\). Each firm claims that it has the lowest price and guarantees a refund of the difference if it charges a higher price. Firms have zero marginal costs and set prices $P_i, P_j \in [0, 1]$. Each firm has a customer base of 1 so that the total market size is 2.

Every customer has a willingness to pay of 1 and purchases a unit of the product before price search. We assume that a fraction $\alpha$ of customers undertake post-purchase price comparisons. Hence, if a firm’s posted price is higher, $\alpha$ of its customers will invoke the LPG and receive a refund of the price differential. Moreover, that firm suffers a loss of goodwill $L(P_i, P_j) > 0$. This loss of goodwill includes loss in credibility of future claims, negative word-of-mouth, defection of customers to the competing firm, and so on. Put differently, $L(P_i, P_j)$ represents the net present value of the potential loss in future income from the customer base. Conversely, the firm with the lower posted price gains a goodwill of $G(P_i, P_j) > 0$. We will consider cases where $L(\cdot, \cdot)$ and $G(\cdot, \cdot)$ are both exogenous and endogenous to the price differential.

\(^1\)We also analyze asymmetric competition where firms have unequal customer bases. The results are qualitatively similar to those presented below and are available from the authors upon request.
The payoff for firm $i$ can thus be written as:

$$
\Pi_i(P_i, P_j) = \begin{cases} 
  P_i + G(P_i, P_j) & \text{if } P_i < P_j \\
  P_i & \text{if } P_i = P_j \\
  \alpha P_j + (1 - \alpha) P_i - L(P_i, P_j) & \text{if } P_i > P_j
\end{cases}
$$

(2.1)

In equation (2.1) above, the first line is the sum of the profit and gain in goodwill when the firm charges a lower price. The second line denotes the profit when prices are identical. The first two terms of the last line represent the profit adjusted for LPG refund when the firm charges a higher price, while the last term denotes the loss in goodwill.

There is an alternative interpretation of the parameter $\alpha$. Let us consider a market where a fraction of customers conduct post-purchase search. However, not every customer who finds a lower price elsewhere will return to the store to claim the price-matching refund. This could occur for two reasons. First, the size of the refund might be small compared to the hassle costs of visiting the store (Hviid and Shaffer, 1999). Second, the customer might have missed the window of opportunity to claim the refund. The parameter $\alpha$ may be interpreted as the fraction of customers who search and claim the refund. This interpretation provides a natural explanation for the development of adverse goodwill: it originates from those unhappy customers who search and do not claim the refund. We shall discuss more about this in Section 3.4.

In order to study whether LPGs lead to higher or lower prices, we compare our model to a benchmark model without LPGs. This benchmark model is a standard model of price competition (see for example Varian, 1981 and Narasimhan, 1988) which assumes that a fraction of $\hat{\alpha}$ consumers search prices before purchase. These customers always purchase from the firm with the lower price. Also, since there are no LPGs, there are no concomitant ex post gains or losses in customer goodwill. The benchmark model is specified as follows:

$$
\Pi_i^b = \begin{cases} 
  (1 + \hat{\alpha}) \cdot P_i & \text{if } P_i < P_j \\
  P_i & \text{if } P_i = P_j \\
  (1 - \hat{\alpha}) \cdot P_i & \text{if } P_i > P_j
\end{cases}
$$

(2.2)
Comparing equations (2.1) and (2.2), we note the following differences. First, the timing of search moves from before (in the benchmark model) to after purchase (LPG model). Second, the total intensity of search changes from $\hat{\alpha}$ to $\alpha$. We assume that $\hat{\alpha} \geq \alpha$ (Srivastava and Lurie, 2001) in our subsequent analyses. Note also that firms can have unequal market shares in the benchmark model while these market shares are always identical under LPGs. Finally, the model with LPGs considers the ramifications of not fulfilling the lowest-price claims by incorporating gains and losses in customer goodwill.

Proposition 1 characterizes the equilibrium prices in the benchmark model:

**Proposition 1**: For $\hat{\alpha} \in (0, 1)$, there exists a symmetric mixed-strategy Nash equilibrium with the following strategy profile$^2$:

$$F^b(P_i) = \begin{cases} 0 & \text{if } P_i \in [0, P_{bm}); \\ \frac{1}{2\hat{\alpha}} \left(1 + \hat{\alpha} - \frac{1-\hat{\alpha}}{P_i} \right) & \text{if } P_i \in [P_{bm}, 1); \\ 1 & \text{if } P_i = 1; \end{cases}$$

(2.3)

where $P_{bm} = \frac{1-\hat{\alpha}}{1+\hat{\alpha}}$.

The expected price is:

$$E(P_i) = \frac{1 - \hat{\alpha}}{2\hat{\alpha}} \ln \left(\frac{1 + \hat{\alpha}}{1 - \hat{\alpha}}\right)$$

(2.4)

PROOF. See Appendix.

Note that prices decline in the intensity of search $\hat{\alpha}$. The results in Proposition 1 will be used to evaluate the relative competitiveness of prices in markets with LPGs.

3 Exogenous Goodwill

In this section, we solve the LPG model assuming that customer goodwill depends solely on whether prices are lower or higher, independent of the price differential between the

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$^2$If $\hat{\alpha} = 1$, equilibrium prices are Bertrand. If $\hat{\alpha} = 0$, the Pareto-dominant equilibrium is the monopolist pair $(1, 1)$. 
two firms. In general, the magnitude of gains and losses in customer goodwill need not be identical; however, since the results remain qualitatively unchanged, we present only the case where \( G(P_i, P_j) = L(P_i, P_j) = \gamma > 0 \).

**Proposition 2** If \( \alpha, \gamma \in (0, 1) \), there is a symmetric mixed-strategy Nash equilibrium that can be characterized as follows\(^3\):

**Case 1.** If \( \gamma < \frac{-\alpha}{2\ln(1-\alpha)} \), the equilibrium strategy profile is:

\[
F(P_i) = \begin{cases} 
0 & \text{if } P_i \in [0, P_m); \\
\frac{1}{\alpha} + \left(1 - \frac{1}{\alpha}\right) e^{\frac{\alpha(1-P_i)}{2\gamma}} & \text{if } P_i \in [P_m, 1); \\
1 & \text{if } P_i = 1;
\end{cases}
\]

(3.1)

where \( P_m = 1 + \frac{2\alpha}{\alpha} \ln(1 - \alpha) \).

The expected price is:

\[
E(P_i) = 1 + \frac{2\gamma}{\alpha} \left(1 + \frac{1}{\alpha} \ln(1 - \alpha)\right)
\]

(3.2)

\(^3\)When \( \alpha \in (0, 1) \) and \( \gamma \in [1 - \alpha, 1) \), there are two equilibria. There is a mixed-strategy equilibrium which is characterized in the proposition and an equilibrium at Bertrand prices. If \( \gamma \geq 1 \), there is an unique equilibrium at Bertrand prices for any value of \( \alpha \).

If \( \alpha = 1 \), prices are Bertrand for any positive \( \gamma \). If \( \alpha = 0 \) and \( \gamma \in \left(0, \frac{1}{2}\right) \), there is a symmetric mixed-strategy Nash equilibrium with the following strategy profile:

\[
F(P_i) = \begin{cases} 
0 & \text{if } P_i \in [0, P_m); \\
1 - \frac{1 - P_i}{2\gamma} & \text{if } P_i \in [P_m, 1); \\
1 & \text{if } P_i = 1;
\end{cases}
\]

where \( P_m = 1 - 2\gamma \). The expected price is \( 1 - \gamma \).

If \( \alpha = 0 \) and \( \gamma \in \left[\frac{1}{2}, 1\right) \), then there is a mass point at \( P_i = 0 \), and the mixed-strategy equilibrium is:

\[
F(P_i) = \begin{cases} 
1 - \frac{1 - P_i}{2\gamma} & \text{if } P_i = 0; \\
1 & \text{if } P_i \in (0, 1); \\
1 - \frac{1 - P_i}{2\gamma} & \text{if } P_i = 1;
\end{cases}
\]

with an expected price of \( \frac{1}{4\gamma} \).
Case 2. If $\gamma \geq \frac{-\alpha}{2 \ln(1-\alpha)}$, the equilibrium strategy profile is:

$$F(P_i) = \begin{cases} \frac{1}{\alpha} + (1 - \frac{1}{\alpha})e^{\frac{\alpha}{2\gamma}} & \text{if } P_i = 0; \\ \frac{1}{\alpha} + (1 - \frac{1}{\alpha})e^{\frac{\alpha(1-P_i)}{2\gamma}} & \text{if } P_i \in (0, 1); \\ 1 & \text{if } P_i = 1; \end{cases}$$

The expected price is:

$$E(P_i) = \frac{1 - \alpha}{\alpha} \left( \frac{2\gamma}{\alpha} \left( e^{\frac{\alpha}{2\gamma}} - 1 \right) - 1 \right)$$

PROOF. See Appendix.

In Case 2, there is a mass point at the competitive price $P_i = 0$. Note that $\frac{\partial F(0)}{\partial \alpha} > 0$, $\frac{\partial F(0)}{\partial \gamma} > 0$, and $\lim_{\alpha \to 1} F(0) = 1$. Equations (3.2) and (3.4) suggest that expected prices are continuous when $P_m = 0$ or $\gamma = \frac{-\alpha}{2 \ln(1-\alpha)}$.

3.1 Comparison with Benchmark Model

The LPG model differs from the benchmark model in three significant ways: 1) customers conduct post-purchase instead of pre-purchase search, 2) the intensity of search can be lower in the LPG regime, and 3) customers develop positive or negative goodwill depending on the credibility of lowest-price claims. It is interesting to examine the relative competitiveness of LPG model by analyzing the incremental effect of each of these changes.

Figure 1 shows the effect of a change from pre-purchase to post-purchase search behavior in the absence of customer goodwill. The change in search pattern raises equilibrium prices to monopoly levels for all values of search intensity $\alpha$. Note that in the benchmark model, prices decline with search intensity $\hat{\alpha}$. In the LPG model, for any level of search, a price cut by a firm will not attract customers from the rival firm but will reduce revenue from its own customers. Consequently, LPGs reduce price competition when there is no customer goodwill.

With customer goodwill, equilibrium prices decline with the level of post-purchase search intensity ($\frac{\partial E(P_i)}{\partial \alpha} < 0$). Figure 2 describes this relationship for $\gamma = 0.25$ while
keeping the level of search intensity in both the benchmark and LPG regimes identical (i.e., \( \alpha = \hat{\alpha} \)). As more customers undertake post-purchase search and demand refunds, the benefit of increasing prices diminishes relative to the benefit of gaining customer goodwill. Figure 2 also shows that equilibrium prices can be higher or lower with LPGs. The following proposition states this result formally:

**Proposition 3**: Assume the total search intensity remains unchanged, i.e., \( \alpha = \hat{\alpha} \in (0, 1) \) and \( \gamma \in (0, 1) \). Let \( \bar{\alpha} \approx 0.655 \) be the solution to the following implicit equation:

\[
\frac{1}{\ln(1 - \alpha)} + \frac{1 - \alpha}{\alpha} \left( 1 + \frac{1}{2} \ln \left( \frac{1 + \alpha}{1 - \alpha} \right) \right) = 0 \tag{3.5}
\]

**Case 1**: \( \alpha < \bar{\alpha} \). LPG regime is more competitive than the benchmark regime if \( \gamma > \gamma^* \), where \( \gamma^* \) is characterized in (3.6). Otherwise, LPG regime is less competitive.

\[
\gamma^* = \frac{1 - \alpha \ln(1 + \alpha) - \alpha}{2(1 + \frac{1}{\alpha} \ln(1 - \alpha))} \tag{3.6}
\]

**Case 2**: \( \alpha \geq \bar{\alpha} \). LPG regime is more competitive than the benchmark regime if \( \gamma > \gamma^* \), where \( \gamma^* \) solves the implicit equation (3.7). Otherwise, LPG regime is less competitive.

\[
2\gamma^* \left( e^{\frac{\alpha}{\gamma^*}} - 1 \right) = \alpha \left( 1 + \frac{1}{2} \ln \left( \frac{1 + \alpha}{1 - \alpha} \right) \right) \tag{3.7}
\]

**PROOF.** See Appendix.

The expression \( \gamma^* \) traces the iso-competitive curve for the two regimes, which is defined as the level of goodwill where expected prices in both regimes are equal (see Figure 3). In the region above the iso-competitive curve, the LPG regime is more competitive, whereas it is less competitive in the region below the curve. This result is interesting as it shows that LPG regime can lead to more competitive prices even if total search intensity remains unchanged.
The LPG regime may still be more competitive even if customers search less with LPGs. Figure 4 illustrates such an example, where \( \hat{\alpha} = 0.3 \) and \( \gamma = 0.25 \), and \( \frac{\alpha}{\alpha} \) varies from 0 to 1 (i.e., \( \alpha \) varies from 0 to 0.3). Note that as \( \frac{\alpha}{\alpha} \) increases, expected prices decrease and become lower than those of the benchmark model when \( \frac{\alpha}{\alpha} \) exceeds 0.53.

Since goodwill is central to our model, it is interesting to see how it influences equilibrium prices. Figure 5 shows the effect of customer goodwill \( \gamma \) on equilibrium prices for two levels of \( \alpha \). Equilibrium prices decline linearly in customer goodwill for \( \gamma < \frac{-\alpha}{2\ln(1-\alpha)} \) and in a convex manner for \( \gamma \geq \frac{-\alpha}{2\ln(1-\alpha)} \) (corresponding to Cases 1 and 2 respectively in Proposition 2). Also, the rate of decline accelerates with an increase in the level of search intensity \( \alpha \).

### 3.2 Traveler’s Dilemma

Our model can be used to re-interpret the Traveler’s Dilemma game (Basu, 1994). The game tells of a story where two travellers visited an exotic island and bought the same piece of antique. They took the same flight back and the airline lost their baggage. The following game was devised to compensate the travellers for their lost antique. The travellers were to reveal independently the true value of the antique \( (P_i, P_j) \). The rule is such that the person who submitted a lower value will receive that value plus a bonus \( R > 0 \) for her honesty. The person who submitted a higher value will receive the lower value minus \( R \). Formally, the payoff structure for the Traveller’s Dilemma is as follows:

\[
\Pi_i(P_i, P_j) = \begin{cases} 
P_i + R & \text{if } P_i < P_j \\
P_i & \text{if } P_i = P_j \\
P_j - R & \text{if } P_i > P_j 
\end{cases}
\]  

(3.8)

Note that the above payoff structure corresponds to our model for \( \alpha = 1 \) and \( \gamma = R \). Thus, the Traveler’s Dilemma is a special case of a price-matching duopoly where all customers undertake post-purchase search and develop positive or negative goodwill depending on whether the firms meet their lowest-price claims. Under these limiting conditions, the theoretical prediction is that equilibrium prices are uniquely Bertrand, independent of the level of customer goodwill. Thus, the Traveler’s Dilemma game demonstrates the fragility of the collusive equilibrium in models of price-matching where all customers
search - any level of customer goodwill is sufficient to restore the Bertrand outcome. Capra et. al (1999) and Goeree and Holt (2001)) studied Traveller’s Dilemma game experimentally and showed that prices are seldom at Bertrand levels. More significantly, there appears to be an inverse relationship between prices and the level of customer goodwill. We note that both experimental “anomalies” are consistent with our theoretical results as long as the level of search intensity \( \alpha \) is slightly perturbed to be less than one.

3.3 Two-Period Store-Switching Model

The LPG model presented above can easily be extended to a two-period model of store-switching where customer goodwill is explicitly defined as gains or losses in future market share. The setup of this model is identical to the LPG model with the following differences. In the store-switching model, firms set prices in each period and customers purchase one unit of the product in every period. In the first period, firms sell to their initial customer base, each of size 1.

A fraction \( \alpha + \beta \) of each firm’s customers make price comparisons after purchase. If the firm’s posted price is higher, \( \alpha \) customers will invoke the LPG and receive a price-matching refund. A fraction of \( \beta \) customers do not claim the refund but switch to the other firm in the second period in order to “penalize” the firm. Thus, the firm that posts the lower first-period price not only retains its initial share of share of customers but plus attract \( \beta \) of the rival’s customers in the second period. Assuming no discounting, the total payoffs for firm \( i \) across two periods can be expressed as:

\[
\Pi_i = \begin{cases} 
P_i + 1 + \beta & \text{if } P_i < P_j \\
P_i + 1 & \text{if } P_i = P_j \\
\alpha P_j + (1 - \alpha)P_i - (1 - \beta) & \text{if } P_i > P_j 
\end{cases}
\] (3.9)

We know by backward induction that since the game ends in the second period, firms will charge will the monopoly price of 1 in that period.\(^4\) If the firm post a lower first-period price, its total payoffs is the sum of its first-period profit \( P_i \) and the profit from

\(^4\)Customers are intemporally rational even if firms charge identical prices in the second period as long as switching is costless and they want to penalize the firm with the higher first-period price.
the second period \((1 + \beta)\). If prices are identical, the firm sells only to its own customer base of 1 in each period. If the firm’s price is higher, it gives out post-purchase refunds to \(\alpha\) of its customers and its second-period profit is the product of the resultant market \((1 - \beta)\) multiplied by the monopoly price. It can be easily seen that the solution to the model of store-switching is equivalent to:

\[
\Pi_i = \begin{cases} 
P_i + \beta & \text{if } P_i < P_j \\
P_i & \text{if } P_i = P_j \\
\alpha P_j + (1 - \alpha)P_i - \beta & \text{if } P_i > P_j 
\end{cases} \tag{3.10}
\]

This is the LPG model with exogenous goodwill if we set \(G(P_i, P_j) = L(P_i, P_j) = \gamma = \beta\).

### 3.4 Automatic Price Protection

Our model can be used to understand an interesting and unique pricing policy of a Boston-based electronic retailer, Tweeter etc. (Gourville and Wu, 1996). The firm offers "Automatic Price Protection" (APP) to customers who shop at its stores. Under APP, Tweeter etc. undertakes post-purchase search on behalf of its customers by tracking prices of other firms that are published in 8 major local newspapers. If Tweeter etc. charges a higher price, it will mail out rebates to customers automatically.

Consider a market that consists of two firms offering APP. APP is equivalent to a market where all customers search and receive a refund whenever a firm charges a higher price (i.e., \(\alpha = 1\)). Since all customers receive refunds and there are no hassle costs, there are no customers who search and do not claim the refunds. Consequently, no adverse goodwill would be developed against the firm. Under our model setup, this implies that \(\gamma = 0\). Consequently, APP leads to monopoly prices.

### 4 Endogenous Goodwill

In Section 3, we assume customer goodwill to be exogenous to the price differential between the two firms. In reality, it is reasonable to posit that firms not only gain or lose customer goodwill depending on whether their prices are lower or higher, but that
the magnitude of goodwill increases with the difference in prices. A straightforward way to operationalize this idea is to assume that customer goodwill varies linearly with the price differential. Specifically, for firm $i$, we can define $G(P_i, P_j) = \omega_G \cdot (P_j - P_i)$ and $L(P_i, P_j) = \omega_L \cdot (P_i - P_j)$, where $\omega_G$ and $\omega_L$ represent the constant marginal gain and loss in customer goodwill. If customers care more about losses relative to gains, we would expect $\omega_L > \omega_G$. (Kahneman and Tversky, 1979). The payoffs for firm $i$ can be written as:

$$\Pi_i = \begin{cases} 
P_i + \omega_G \cdot (P_j - P_i) & \text{if } P_i < P_j \\ 
P_i & \text{if } P_i = P_j \\ 
\alpha P_j + (1-\alpha)P_i - \omega_L \cdot (P_i - P_j) & \text{if } P_i > P_j 
\end{cases} \quad (4.1)$$

The above game can delineated into four cases depending on the magnitudes of $\omega_G$ and $\omega_L$. In equation (4.1), if $\omega_G > 1$, the marginal gain in customer goodwill outweighs the loss in revenue from the price cut. Consequently, the firm always has an incentive to undercut its rival’s price. The firm does not have the same incentive when $\omega_G \leq 1$. If $\omega_L < 1-\alpha$, the benefit of increased revenue from raising prices outweighs the concomitant loss in goodwill. Thus, the firm is better off raising prices to the monopoly levels. When $\omega_L > 1-\alpha$, the same incentive disappears. The following proposition describes the equilibrium under the four cases formally (see Figure 6):

**Proposition 4** The symmetric Nash equilibria in each of the four cases are:

**Case I.** If $\omega_G > 1$ and $\omega_L \geq 1 - \alpha$, the equilibrium prices are Bertrand.

**Case II.** When $\omega_G > 1$ and $\omega_L < 1 - \alpha$, there are multiple equilibria. The two pure-strategy equilibria are $(0, 1)$ and $(1, 0)$ and there is a symmetric mixed-strategy equilibrium where firms randomize between 1 and 0. The probability that a firm chooses 1 is $\frac{1 -(\alpha + \omega_L)}{\omega_G - (\alpha + \omega_L)}$, which is also the expected price for this equilibrium.

**Case III.** If $\omega_G \leq 1$ and $\omega_L \geq 1 - \alpha$, all symmetric price pairs constitute Nash equilibria. The Pareto-dominant equilibrium is thus the monopolist price pair$^5$.

$^5$In the special case of $\omega_G = 1$ and $\omega_L = 1 - \alpha$, any price is a Nash equilibrium.
Case IV. If $\omega_G \leq 1$ and $\omega_L < 1 - \alpha$, the equilibrium prices are monopolist.

PROOF. See Appendix.

Proposition 4 suggests that if customer goodwill varies linearly with the price differential, equilibrium prices become more extreme at either the Bertrand or monopoly levels (except for quadrant II where there is a symmetric mixed-strategy equilibrium). Consequently, these expected price do not vary with search intensity $\alpha$ and marginal gain $\omega_G$ and loss $\omega_L$ in goodwill in Quadrants I, III, and IV of Figure 6.

Figure 7 shows the expected prices for the benchmark model and LPG model under four different sets of parametric combinations of $\omega_L$ and $\omega_G$ (where each of them take values of either 0.5 or 1.5). The benchmark model is less competitive in Quadrants I and II and more competitive in Quadrants III and IV. In Quadrants III and IV, LPGs lead to monopoly prices. In Quadrant I, LPGs yield Bertrand prices. In Quadrant II, the expected prices decrease with search but it can be easily shown that they are lower than the expected prices in the benchmark model for all level of search intensity. Consequently, the LPG model with endogenous goodwill can be more or less competitive than the benchmark model.

4.1 Price-Beating Competition

A model of price-beating competition where firms offer to compensate customers an additional percentage of the price differential in addition to the price-matching refund is a special case of the above model. This compensatory amount is commonly fixed at five or ten percent of the price differential. For example, Circuit City currently offers a 10% compensation for charging a higher price.

Relative to price-matching, price-beating guarantee may send a stronger signal to customers so that their lowest-price claims become even more credible. Consequently, more customers may be persuaded to abandon post-purchase search (i.e., a lower $\alpha$). If

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6The LPG model with exogenous goodwill can also represent a form of price-beating competition where firms offer a lump-sum payment to customers on top of the refund. However, the policy of compensating an additional fraction of the price differential is much more prevalent in practice.
we assume that there is no gain in goodwill due to lower prices (i.e., $\omega_G = 0$) and the loss in goodwill is captured entirely by the compensation amount (i.e., $\omega_L = 0.1$ for a 10% price beating guarantee), then our model predicts that price-beating competition leads to monopoly prices. Hence, adopting a price-beating policy may increase expected prices relative to price-matching.

5 Conclusion

This paper presents a model of LPGs that incorporates three plausible effects of LPGs on customer shopping behavior. First, customers in our model purchase before search. Second, consistent with behavioral evidence, the level of post-purchase search intensity can be lower with LPGs. Third, customers may develop positive or adverse goodwill towards the firms depending on whether their lowest-price claims are credible. The loss of goodwill can also be attributed to defection by those customers who search prices but do not claim refund because of hassle costs.

We contribute to the literature by providing a more realistic model of LPG competition and by integrating several well-known special cases into our model framework. We show that the Traveler’s Dilemma is a LPG duopoly where all customers search and goodwill is exogenous to the price differential. In this case, LPG leads to competitive prices. Our model also helps to analyze the implications of certain pricing practices such as automatic price protection and price-beating. We show that both practices lead to monopoly prices.

Our results show that expected prices can be higher or lower with LPGs even if total search decreases. This is interesting because it shows that search intensity and hassle costs are not the only drivers of price competitiveness. Under our model, expected prices can be higher because the pattern of search changes from pre to post purchase and total level of search intensity can decrease. They can be lower because customers develop positive and negative goodwill towards the firms. Firms which charge higher prices are

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7Although our model of LPGs considers the case where customers only undertake post-purchase search, the model can be readily extended to incorporate pre-purchase search by a segment of customers through combining the payoffs of the benchmark and LPG models.
penalized while those which charge lower prices are rewarded. Finally, our results suggest that prices are more extreme when goodwill is endogenous to the price differential.

6 Appendix

6.1 Proof of Proposition 1

We are interested in the symmetric Nash equilibrium of the benchmark model. Define \( f^b(.) \) and \( F^b(.) \) to be the mixing density and cumulative density function (c.d.f.) respectively for either firm over the support \([P_m^b, 1]\) where \( P_m^b \) is the lowest price both firm charge in equilibrium. Denoted firm \( i \)'s expected profit by \( E(\Pi_i^b) \).

From equation (2.2), if firm \( i \) charges \( P_i \in [P_m^b, 1] \),

\[
E(\Pi_i^b) = \int_{P_i}^{1} ((1 + \hat{\alpha}) P_i) \cdot f^b(P_j) dP_j + \int_{P_m^b}^{P_i} ((1 - \hat{\alpha}) P_i) \cdot f^b(P_j) dP_j 
\]

(6.1)

This yields:

\[
E(\Pi_i^b) = P_i (1 + \hat{\alpha}) - 2 \hat{\alpha} P_i F^b(P_i) 
\]

(6.2)

Equating the above with the expected profit when firm \( i \) charges 1 (the maximum price) and simplifying, we have

\[
F^b(P_i) = \frac{1}{2\hat{\alpha}} \left( 1 + \hat{\alpha} - \frac{1 - \hat{\alpha}}{P_i} \right) 
\]

(6.3)

Narasimhan (1988) shows that there is no mass point at \( P_m^b \) so that we have \( F^b(P_m^b) = 0 \). Solving, we obtain

\[
P_m^b = \frac{1 - \hat{\alpha}}{1 + \hat{\alpha}} 
\]

(6.4)

The expected price for firm \( i \) under the mixed-strategy Nash equilibrium is

\[
E(P_i) = \int_{P_m^b}^{1} P_i \cdot f^b(P_i) dP_i = 1 - \int_{P_m^b}^{1} F^b(P_i) dP_i 
\]

(6.5)

This yields

\[
E(P_i) = \frac{1 - \hat{\alpha}}{2\hat{\alpha}} \ln \left( \frac{1 + \hat{\alpha}}{1 - \hat{\alpha}} \right) 
\]

(6.6)

Of course, when \( \hat{\alpha} = 1 \), prices are Bertrand. When \( \hat{\alpha} = 0 \), prices are monopolist.
6.2 Proof of Proposition 2

The symmetric mixed-strategy equilibrium depends on whether $\gamma$ is greater or less than $\frac{\alpha}{2 \ln(1-\alpha)}$. We shall consider each case in turn. In both cases, $\alpha \in (0, 1)$.

Case 1. $\gamma < \frac{\alpha}{2 \ln(1-\alpha)}$.

Define $f(.)$ and $F(.)$ to be the probability and cumulative densities of each firm’s mixing distribution over the support $[P_m, 1]$, where $P_m$ is the lowest price charged by the firms. The expected profit of firm $i$, denoted by $E(\Pi_i)$ when the firm charges $P_i \in [P_m, 1]$ is:

$$E(\Pi_i) = \int_{P_i}^{1} (P_i + \gamma) \cdot f(P_j) \, dP_j + \int_{P_m}^{P_i} (\alpha P_j + (1 - \alpha)P_i - \gamma) \cdot f(P_j) \, dP_j$$

(6.7)

This reduces to:

$$E(\Pi_i) = P_i + \gamma - 2\gamma F(P_i) - \alpha \int_{P_m}^{P_i} F(P_j) \, dP_j$$

(6.8)

Equating this with the expected profit when firm $i$ charges 1, we have:

$$P_i + \gamma - 2\gamma F(P_i) + \alpha \int_{P_i}^{1} F(P_j) \, dP_j = 1 - \gamma$$

(6.9)

Differentiating with respect to $P_i$, we obtain the following first-order differential equation:

$$1 - 2\gamma f(P_i) - \alpha F(P_i) = 0$$

(6.10)

Solving, we have

$$F(P_i) = \frac{1}{\alpha} + A \cdot e^{\frac{-\alpha P_i}{2\gamma}}$$

(6.11)

Using $F(1) = 1$, we have $A = (1 - \frac{1}{\alpha})e^{\frac{\gamma}{2\gamma}}$. Hence

$$F(P_i) = \frac{1}{\alpha} + (1 - \frac{1}{\alpha})e^{\frac{-\alpha(1-P_i)}{2\gamma}}$$

(6.12)

$P_m$ can be solved by setting $F(P_m) = 0$, we obtain

$$P_m = 1 + \frac{2\gamma}{\alpha} \ln(1 - \alpha)$$

(6.13)
\( P_m > 0 \) as long as \( \gamma < \frac{-\alpha}{2\ln(1-\alpha)} \). The expected price under the mixed strategy Nash equilibrium can be obtained from applying integration by parts:

\[
E(P_i) = \int_{P_m}^{1} P_i \cdot f(P_i) \, dP_i = 1 - \int_{P_m}^{1} F(P_i) \, dP_i
\]

This yields

\[
E(P_i) = 1 + \frac{2\gamma}{\alpha} \left( 1 + \frac{1}{\alpha} \ln(1 - \alpha) \right)
\]

**Case 2:** \( \gamma \geq \frac{-\alpha}{2\ln(1-\alpha)} \).

There is a mass point at \( P_i = 0 \), denote by \( F(0) > 0 \). If firm \( i \) charges \( P_i > 0 \), firm \( i \)'s expected profit is:

\[
E(\Pi_i) = \int_{P_i}^{1} (P_i + \gamma) \cdot f(P_j) \, dP_j + \int_{0}^{P_i} \left( \alpha P_j + (1 - \alpha)P_i - \gamma \right) \cdot f(P_j) \, dP_j
\]

\[+ F(0)((1 - \alpha)P_i - \gamma) \]

This yields

\[
E(\Pi_i) = P_i + \gamma - 2\gamma F(P_i) - \alpha \int_{0}^{P_i} F(P_j) \, dP_j
\]

If firm \( i \) charges 0, the expected profit is

\[
E(\Pi_i) = (1 - F(0)) \cdot \gamma
\]

Solving for \( F(.) \) by equating the two equations and differentiating with respect to \( P_i \), we get

\[
F(P_i) = \frac{1}{\alpha} + (1 - \frac{1}{\alpha})e^{\frac{\alpha(1-P_i)}{\alpha \gamma}}
\]

Hence, \( F(0) = \frac{1}{\alpha} + (1 - \frac{1}{\alpha})e^{\frac{\alpha}{\gamma}} \). Note that \( \frac{\partial F(0)}{\partial \alpha} > 0 \), \( \frac{\partial F(0)}{\partial \gamma} > 0 \), and \( \lim_{\alpha \to 1} F(0) = 1 \). The expected price is

\[
E(P_i) = F(0) \cdot 0 + \int_{0}^{1} P_i \cdot f_i(P_i) \, dP_i = 1 - \int_{0}^{1} F_i(P_i) \, dP_i
\]

This yields

\[
E(P_i) = \frac{1 - \alpha}{\alpha} \left( \frac{2\gamma}{\alpha} \left( e^{\frac{\alpha}{\gamma}} - 1 \right) - 1 \right)
\]

Note that the expected prices in both cases are identical when \( \gamma = \frac{-\alpha}{2\ln(1-\alpha)} \), giving a value of \( \frac{\alpha-1}{\alpha} - \frac{1}{\ln(1-\alpha)} \).
6.3 Proof of Proposition 3

We compare the expected price for the benchmark model in equation (2.4) and those of LPGs in equations (3.2) and (3.4) to determine whether LPG is more or less competitive than the benchmark model. Note that when \( \gamma = \frac{-\alpha}{2 \ln(1 - \alpha)} \), (3.2) and (3.4) give the same expected price. Define \( \alpha^* \) to be level of search where expected prices in the benchmark and LPG models are identical when \( \gamma = \frac{-\alpha}{2 \ln(1 - \alpha)} \). \( \alpha^* \) is solved by equation (3.5), which is obtained by equating expression (2.4) and (3.2) where we set \( \gamma = \frac{-\alpha}{2 \ln(1 - \alpha)} \). Solving (3.5), we obtain \( \tilde{\alpha} = 0.6551 \). So when \( \tilde{\alpha} = 0.6551 \) and \( \tilde{\gamma} = 0.3077 \), expected prices are identical in the benchmark and LPG models.

When \( \gamma < \tilde{\gamma} \), expected price in LPG model is given by (3.2), otherwise it is given by (3.4). For any \( \alpha \), we determine the level of \( \gamma \) that make the two regimes equally competitive. Call this level of goodwill \( \gamma^* \). For \( \alpha < \tilde{\alpha} \), \( \gamma^* = \frac{-\alpha}{2 \ln(1 + \frac{\alpha}{1 - \alpha})} \), which is (3.6). Similarly, for \( \alpha \geq \alpha^* \), \( \gamma^* \) is solved by the implicit equation \( 2\gamma^*(e^{\frac{\alpha}{2\gamma^*}} - 1) = \alpha \left(1 + \frac{1}{2} \ln \left(\frac{1 + \alpha}{1 - \alpha}\right)\right) \), which is (3.7). This iso-competitive curve traces the values of \( \gamma^* \) for all \( \alpha \). Figure 3 shows this iso-competitive curve and the point \((\tilde{\alpha}, \tilde{\gamma})\). Hence, if \( \gamma > \gamma^* \), the LPG regime is more competitive. If \( \gamma \leq \gamma^* \), the LPG regime is less competitive.

6.4 Proof of Proposition 4

The proofs of Cases I, III and IV are straightforward. We simply check whether the firm has an incentive to deviate by inspecting \( \frac{\partial \Pi_i}{\partial P_j} \) for each parametric combination.

For Case II, note that for any \( P_j \), both undercutting and raising prices are profitable deviations. The only price pairs in which there are no profitable deviations are (1, 0) and (0, 1). For the mixed strategy equilibrium, we solve for \( p \), the probability that \( i \) charges 1, by equating the following two equations:

\[
E(\Pi_i \mid P_i = 1) = p \cdot 1 + (1 - p) \cdot (1 - \alpha - \omega_L) \tag{6.22}
\]

\[
E(\Pi_i \mid P_i = 0) = p \cdot \omega_G + (1 - p) \cdot 0 \tag{6.23}
\]
References


Figure 1: Effect of Post-Purchase Search (No Goodwill)

Figure 2: Effect of Search Intensity with Customer Goodwill
\( \gamma = 0.25 \)
Figure 3: Iso-Competitive Curve $\gamma^*$ for LPG model

$\gamma^*$ for LPG model

LPG More Competitive

LPG Less Competitive

$\bar{\alpha} = 0.655$

Figure 4: Effect of Reduced Search in LPG Regime

for $\bar{\alpha} = 0.3, \gamma = 0.25$

$E(Price)$

$\alpha, \bar{\alpha}$
Figure 5: Effect of Goodwill for $\alpha = 0.3$ and $\alpha = 0.6$

E(Price)

$\gamma$

$\alpha=0.3$

$\alpha=0.6$
Figure 6: Solution to Model with Endogenous Goodwill

<table>
<thead>
<tr>
<th>$\omega^L$</th>
<th>$\geq 1 - \alpha$</th>
<th>$&lt; 1 - \alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\geq 1$</td>
<td>I $\omega_G$ Bertrand</td>
<td>II ${(1,0), (0,1)}$; Mixed Strategy</td>
</tr>
<tr>
<td>$\leq 1$</td>
<td>III $\omega_G$ Monopoly (Pareto Criterion)</td>
<td>IV $\omega_G$ Monopoly</td>
</tr>
</tbody>
</table>
Figure 7: Competitiveness of LPG with Endogenous Goodwill
$\omega_G = 0.5, 1.5$ and $\omega_L = 0.5, 1.5$