Stochastic Capacity Investment and Flexible versus Dedicated Technology Choice in Imperfect Capital Markets

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Abstract
This paper analyzes the impact of endogenous credit terms under capital market imperfections in a capacity investment setting. We model a monopolist firm that decides on its technology choice (flexible vs dedicated) and capacity level under demand uncertainty. Differing from the majority of the stochastic capacity investment literature, we assume that the firm is budget-constrained and can relax its budget constraint by borrowing from a creditor. The creditor offers technology-specific loan contracts to the firm, after which the firm makes its technology choice and subsequent decisions. Capital market imperfections impose financing frictions on the firm. We derive the unit financing costs and the firm’s decisions in equilibrium. Our analysis contributes to the capacity investment literature by analyzing the impact of the internal budget level and the demand characteristics (variability and correlation) on the operational decisions and the performance of the firm in an imperfect market equilibrium; and by delineating the differences between perfect and imperfect markets as a function of firm and capital market characteristics. We demonstrate that the endogenous nature of credit terms in imperfect capital markets may modify or reverse conclusions concerning capacity investment and technology choice obtained under the perfect market assumption. For example, the value of dedicated technology may decrease with an increase in the demand correlation. This is because the equilibrium financing cost increases as the default risk increases due to a lower value of financial pooling (diversification benefit of operating in two markets). The value of flexible technology may decrease with an increase in the demand variability. This is because the equilibrium financing cost increases due to a higher default risk; and this may outweigh the increasing value of capacity pooling.

Key Words: Capacity, Flexibility, Financing, Newsvendor, Limited Liability, Collateral, Market Imperfection.

July 2007, Revised 24 June 2010
1 Introduction and Literature Review

Capacity investment is subject to internal or external financing frictions, especially in capital-intensive industries. If the internal capital of the firm is not sufficient to finance the desired investment level, then the firm may decide to raise external capital. External capital is more expensive because there exist capital market imperfections such as bankruptcy costs, taxes, financial distress cost, underwriter fees or agency costs due to asymmetric information etc. (Froot et al. 1993) that create frictions in the borrowing process of the firm. However, as highlighted by Van Mieghem (2003, p. 275) “stochastic capacity models assume (often implicitly) [...] perfect capital markets, so that frictionless borrowing is possible [...].” In imperfect capital markets, the investment decision and the cost of external capital are interdependent. The objective of this paper is to increase our understanding of how capital market imperfections affect stochastic capacity investment and technology choice. A key feature of our paper is that we endogenize the cost of borrowing in a creditor-firm equilibrium.

To this end, we model a firm who produces and sells two products under demand uncertainty. The firm chooses between flexible and dedicated technologies that incur variable investment costs, and determines the capacity level and the production quantities with the chosen technology. Differing from the majority of the stochastic capacity investment literature, we assume that the firm is budget-constrained and can relax its budget constraint by borrowing from a creditor. The creditor offers technology-specific loan contracts to the firm, after which the firm makes its technology choice and subsequent decisions. We assume that the creditor incurs a fixed cost of bankruptcy if the firm defaults on the loan. In the basic model, we assume that the credit market is perfectly competitive. We allow for a positive return requirement of the creditor to capture financing frictions other than bankruptcy cost that may arise in this market. At the other end of the spectrum, we analyze a credit market with a monopolist creditor. In summary, the fixed bankruptcy cost, the positive return requirement of the creditor, and the monopolistic nature of the credit market constitute the capital market imperfections considered in this paper.

We derive the technology choice and external borrowing, capacity, and production level decisions of the firm and the creditor’s loan terms in equilibrium. First, we analyze the single-product version of our model. In particular, we investigate how the internal budget level and the demand variability affect the capacity investment level and the performance of the firm in equilibrium. Second, in the two-product firm, we answer the following questions: How do demand variability and demand correlation affect the capacity investment level and
the performance of the firm in equilibrium for a given technology? What is the equilibrium technology choice and what are the main drivers of this choice?

In answering these questions, our analysis is focused on determining the differences between perfect and imperfect capital markets; and understanding the impact of firm characteristics and different capital market conditions. We note that the objective of this paper is not to solve for the optimal capital structure of the firm (equity financing, debt financing with debt contracts of different rates, maturities, covenants, etc); rather we focus on technology-specific loan contracts characterized by their unit financing cost, and analyze the creditor-firm strategic interaction in that setting. Our results contribute to several streams of research, as detailed below.

The stochastic capacity investment literature analyzes the capacity-pooling value of flexible technology over dedicated technology in a variety of models. We refer readers to Van Mieghem (2003) for an excellent review. As highlighted in this review paper, the operations management literature (often implicitly) assumes that capital markets are perfect, in which case operational and financial decisions decouple (Modigliani and Miller 1958). In practice, capital market imperfections exist (Harris and Raviv 1991) and impose deadweight costs of external financing, leading operational and financial decisions to interact with each other. This is because these financing costs are affected by the firm’s operational decisions, and are endogenously determined in equilibrium. There is a growing body of work in operations and finance that analyzes these interactions. Our paper’s overall contribution to this literature is to analyze the effect of different capital market imperfections on capacity investment and flexible versus dedicated technology choice. We demonstrate that the endogenous nature of credit terms in imperfect capital markets may modify or reverse conclusions obtained under the perfect market assumption. For example, with the dedicated technology, the capacity investment level and the performance of the firm in equilibrium may decrease in demand variability and correlation; whereas with the flexible technology, they may decrease with an increase in demand variability and a decrease in demand correlation. These results are driven by the impact of demand uncertainty on the financing cost in equilibrium; and are observed under all the imperfect capital market models analyzed in this paper.

In the Operations Management literature, a recent stream of papers (Lederer and Singhal 1988 and 1994, Buzacott and Zhang 2004, Xu and Birge 2004, Babich and Sobel 2004, Babich et al. 2006, Xu and Zhang 2006, Dada and Hu 2008 and Caldentey and Haugh 2009) analyze the joint financing and operating decisions of the firm and demonstrate the value of integrated decision making. All these papers focus on a single-product setting
where technology choice is irrelevant. Besides our two-product firm analysis, which is new, our single-product firm analysis contributes to this literature by providing new comparative statics results under different capital market conditions. We compare our results to three of these papers in particular.

Xu and Birge (2004) analyze the effect of capital market imperfections (in particular, taxes and bankruptcy costs) on the firm’s joint financing and operating decisions in a perfectly competitive credit market (with a zero expected return requirement). Our single-product analysis complements theirs. They numerically show that an increase in demand variability decreases the capacity investment level of the firm. We provide conditions under which this observation holds analytically and demonstrate that the opposite can be true if the expected return requirement in a perfectly competitive credit market is sufficiently large or the creditor is a monopolist.

Dada and Hu (2008) analyze the interaction between a monopolist creditor and a newsvendor in a single-period setting. They provide comparative statics results on how the capacity investment level and the financing cost in equilibrium are affected by changes in cost parameters. Different from our work, they neither consider the effect on the equilibrium firm performance nor the effect of demand variability. Similar to our work, they analyze the effect of an internal budget constraint. They demonstrate that an increase in the internal budget increases the financing cost in equilibrium, but also increases the capacity investment level for a given financing cost; hence the resulting effect on the capacity investment is ambiguous. Our analytical results provide insights into the impact of internal budget level on the capacity investment level and the performance of the firm in equilibrium for different firm characteristics and capital market conditions.

Lederer and Singhal (1994) study the joint financing (optimal mix of debt and equity) and capacity investment problem in a single-product, multi-period setting under the assumption of a perfectly competitive credit market (with a zero expected return requirement). In a numerical example, they analyze the capacity-pooling benefit of flexible technology in a multi-product firm. They show that the value of flexible technology decreases in demand variability, and argue that this is because the default risk of the firm decreases, which allows the firm to secure a lower financing cost in equilibrium. In our model, we demonstrate that this result is only valid at high demand correlations. At low demand correlations, the default risk of the firm is not affected by the change in demand variability because the diversification benefit of operating in two markets (which we call “financial pooling”) is sufficiently large. It follows that at low demand correlations, the value of flexible technol-
ogy increases in demand variability. This is because i) the value of flexible technology at a
given financing cost increases in demand variability (due to capacity pooling), and ii) the
equilibrium level of financing cost is insensitive to changes in demand variability (due to
financial pooling).

Several finance papers also investigate the interaction of financing and operational de-
cisions. Dotan and Ravid (1985) and Dammon and Senbet (1988) are examples of early
research that demonstrates the effect of operational investments on the financing policy
of the firm in a single-period setting. We refer the reader to Childs et al. (2005) for a
recent review of papers in this stream. The main focus of these papers is on financial issues;
and therefore they have strong modeling assumptions concerning the firm’s operations. We
demonstrate that new trade-offs arise and new conclusions are obtained with a more detailed
formalization of the firm’s operations (the sequential nature of technology choice, capacity
investment and production decisions and the impact of demand uncertainty). For example,
we show that firms with higher internal capital may perform worse than firms with lower
capital due to higher financing costs in equilibrium.

More recently, a number of papers in the finance literature (Mauer and Triantis 1994,
Mello et al. 1995, and Mello and Parsons 2000) analyze the effect of various forms of
operational flexibility (e.g. shutting down the production plant) on the joint operational
and financing decisions of firm in the contingent claims framework. As highlighted in
MacKay (2003), without agency cost concerns, operational flexibility is associated with a
lower default risk in equilibrium: Operational flexibility decreases the firm’s default risk by
generating higher returns due to its option value. We demonstrate that this argument may
not hold in general with a more detailed formalization of the firm’s operations. Anticipating
the option value of operational flexibility (flexible technology in our case), the firm optimally
adjusts its capacity investment decision. As a result, operational flexibility may increase the
default risk of the firm. Even if operational flexibility is costless (in our model, this means
the flexible technology has the same cost structure as the dedicated technology), the firm
may be worse off with operational flexibility due to a higher financing cost in equilibrium.

The remainder of this paper is organized as follows: In §2, we describe the model and
discuss the basis for our assumptions. §3 focuses on the single-product version of our model
and analyzes the impact of the demand variability and the internal budget level on the
firm’s operational decisions and performance in equilibrium. We provide the analysis for
the two-product version of our model in §4 and investigate the effect of demand variability
and correlation on the firm’s operational decisions and performance for a given technology,
as well as the technology choice in equilibrium. In §5, we present some extensions to our model and discuss the robustness of our results to some of our assumption. We conclude in §6 with a discussion of future research directions.

2 Model Description and Assumptions

We consider a creditor-firm interaction where borrowing terms are determined before the firm makes any decisions. The firm is a budget-constrained monopolist that makes its technology choice and capacity investment decision (potentially after borrowing from the creditor) under demand uncertainty; and produces and sells two products after the resolution of this uncertainty. The firm chooses the technology (dedicated $D$ versus flexible $F$), and the borrowing, capacity investment, and production levels so as to maximize the expected equity value. We model the firm’s decisions as a two-stage stochastic recourse problem. We focus on a stylized firm that lives for a single period and is liquidated at the end of the period. After operating profits are realized, the firm pays back its debt (if any), and default occurs if it is unable to do so. Before discussing the timeline in detail, we introduce our assumptions about the firm and the credit market.

The firm’s objective is to maximize the expected shareholder wealth by maximizing the expected value of equity. Shareholders have limited liability. The risk-free rate $r_f$ is normalized to 0.

We assume that the creditor offers a technology-specific unit financing cost $a_T$ for $T \in \{D, F\}$ to the firm. The creditor has perfect information about the firm. We assume that the firm has physical assets of value $P$ (e.g., real estate) that are pledged to the creditor as collateral. The physical assets are illiquid; they can only be liquidated with a lead time. Therefore, default can occur even if the loan is secured. We discuss how the results would change should there not be a liquidity problem in Section §5.1.

As discussed in Froot et al. (1993), outside capital is more expensive than internally generated funds. This is because there exist transaction costs of external financing that give rise to capital market imperfections. We assume that the creditor incurs a fixed bankruptcy cost $BC$ if the firm defaults on its loan; this cost is incurred as an out-of-pocket fee. $BC$ represents the direct cost of bankruptcy to the creditor, which includes the administrative and legal fees of the bankruptcy process (Altman 1980), and is often used in the literature to represent default-related capital market imperfections (e.g., Smith and Stulz 1985). Thus, the existence of bankruptcy cost introduces a capital market imperfection in our model.

In our basic model, the credit market is perfectly competitive. In the financial economics literature, the common assumption is to have a perfectly competitive credit market such
that the creditor makes zero expected return (e.g., Melnik and Plaut 1986). We generalize
this by implicitly assuming that the competition in the credit market forces the equilibrium
expected return for the creditor to be $U \geq 0$. The strictly positive level of $U$ can be a
consequence of frictions (e.g. fixed administrative costs for loan arrangements including
due diligence costs) in the perfectly competitive credit market. In our framework, this is
analyzed by determining the equilibrium unit financing cost that results in an expected
return of $U \geq 0$. We investigate the $U = 0$ and $U > 0$ cases separately. At the other
end of the spectrum, we analyze a monopolist creditor who maximizes his expected return
from lending. In summary, the fixed bankruptcy cost, the positive return requirement of
the creditor, and the monopolistic nature of the credit market constitute the capital market
imperfections considered in this paper.

Returning to the timeline, before the firm makes any decisions, the creditor offers its
borrowing terms $a_T \geq 0 (= r_f), T \in \{D,F\}$. In stage 1, the firm determines its technology
choice $T \in \{D,F\}$ (if this dominates the doing nothing option in expectation), capacity
investment level and borrowing level under the corresponding financing contract $a_T$ with
respect to the internal budget constraint $B$. The flexible technology ($F$) has a single resource
that is capable of producing two products and the dedicated technology ($D$) consists of two
resources that can each produce a single product. Thus, the flexible technology has a
capacity pooling benefit over the dedicated technology. Technology $T$ incurs unit capacity
investment cost $c_T$. Since flexible technology is more costly, we assume $c_F \geq c_D$. Capacity
investment can be salvaged at a rate of $0 \leq \gamma_T < 1$. Since flexible capacity is typically more
marketable than dedicated capacity, we assume $\gamma_F \geq \gamma_D$.

In stage 2, demand uncertainty is resolved. The firm then chooses the production
quantities (equivalently, prices) to satisfy demand optimally. Price-dependent demand for
each product is represented by the iso-elastic inverse-demand function $p_i(q_i; \xi_i) = \xi_i q_i^{1/b}$ for
$i = 1, 2$. Here, $b \in (-\infty, -1)$ is the constant elasticity of demand, and $p$ and $q$ denote price
and quantity, respectively. $\xi_i$ represents the idiosyncratic risk component. We make specific
assumptions about the distribution of $(\xi_1, \xi_2)$ throughout the text whenever necessary. For
tractability, we assume that the marginal production costs of each product are 0. This
is an assumption that is widely used in the literature (see Goyal and Netessine 2007 and
references therein). We discuss the implications of relaxing this assumption in Section 6.

At termination, the firm salvages its capacity investment. If the firm is able to repay its
debt from its final cash position (that consists of operating revenues and the salvage value
of capacity), it does so and, since the firm lives for a single-period, terminates by liquidating
its physical assets. Otherwise, default occurs and the firm goes into bankruptcy. The cash on hand and the ownership of the collateralized physical assets \( P \) are transferred to the creditor. The creditor may or may not be able to retrieve the face value of the loan from the seized assets of the firm depending on whether the firm is solvent or not when the value of \( P \) is taken into account. In the former case, the firm collects the remaining cash.

We use the following mathematical representation throughout the text: A realization of the random variable \( \xi \) is denoted by \( \tilde{\xi} \) and its expectation is denoted by \( \bar{\xi} \). Bold face letters represent vectors of the required size. Vectors are column vectors and ' denotes the transpose operator. \( \mathbf{x}^a \) denotes the componentwise exponent \( a \) of the vector \( \mathbf{x} \). \( \mathbf{x} \mathbf{y} \) denotes the componentwise product of vectors \( \mathbf{x} \) and \( \mathbf{y} \) with identical dimensions. \( \Pr \) denotes probability, \( \mathbb{E} \) denotes the expectation operator and \((x)^+ \equiv \max(x,0)\). Monotonic relations (increasing, decreasing) are used in the weak sense unless otherwise stated. Tables 4 and 5 in the Technical Appendix summarize all the notation.

3 The Single-Product Firm

In the single product setting, the firm uses a single resource and technology choice is not relevant so we eliminate the \( D \) and \( F \) subscripts. We also have a uni-dimensional product market uncertainty \( \xi \). Let \( F(.) \) denote the cdf of \( \xi \) with \( \bar{F}(.) = 1 - F(.) \), \( f(.) \) denote the pdf. All the proofs for this section are provided in \( \S A \) of the Technical Appendix.

3.1 Analysis of the Firm’s Problem for a Given Financing Cost

In this section, we describe the optimal solution for the firm’s capacity investment, external borrowing and production decisions. We solve the firm’s problem by using backward induction starting from stage 2.

Stage 2, Production Decision: In stage 1, the firm with budget \( B \) borrowed \( e \), invested in capacity level \( K \) and placed \( B + e - cK \) into the cash account (at the risk-free rate). In this stage, the firm observes the demand realization \( \tilde{\xi} \) and determines the production quantity \( q \) within the existing capacity limit \( K \) to maximize the stage-two equity value \( \Pi(q; K, e, B, \tilde{\xi}) \).

To derive \( \Pi \), note that two outcomes are possible: If the firm’s final cash position (consisting of stage-two operating profits, cash account holdings and the salvage value of capacity) is sufficient to cover the face value of the loan, i.e. if \( q \bar{p}(q; \tilde{\xi}) + (B + e - cK) + \gamma cK \geq e(1 + a) \), then the firm does not default. Otherwise, it defaults and its assets (including the ownership of physical assets \( P \)) are transferred to the creditor. The bankruptcy cost \( BC \) is borne by the creditor as an out-of-pocket expenditure. The firm receives the remaining cash (if any) after the face value of the loan is deducted from its seized assets. With the limited liability
assumption, we can therefore write

$$\Pi(q; K, e, B, \tilde{\xi}) = [qp(q; \tilde{\xi}) + (B + e - cK) + \gamma cK + P - e(1 + a)]^+.$$  \hspace{1cm} (1)$$

Maximizing the stage-two equity value is equivalent to maximizing the operating profit. We find

$$q^* = \arg\max_{0 \leq q \leq K} qp(q; \tilde{\xi}) = K,$$

in other words, the firm optimally utilizes all of its available capacity. Then

$$\Pi^*(K, e, B, \tilde{\xi}) = [\tilde{\xi}K^{(1 + \frac{1}{b})} + (B + e - cK) + \gamma cK + P - e(1 + a)]^+.$$$$

**Stage 1, Capacity Choice and External Financing:** In this stage, the firm has an internal budget $B \geq 0$ and determines the optimal capacity investment level $K^*$ and the optimal external borrowing level $e^*$ so as to maximize its expected equity value,

$$\pi(K, e; B) = E[\Pi^*(K, e, B, \tilde{\xi})].$$$$

It is easy to show that at optimality, $e = (cK - B)^+$ is satisfied, that is, the firm exactly borrows what it needs to cover its capacity investment. This is because production is costless, so that the firm does not incur any further costs after investing in capacity, and therefore only borrows to finance its capacity investment. Thus, the optimal expected equity value of the firm, $\pi^*(B)$, can be found as follows:

$$\pi^*(B) = \max_{K \geq 0} E[\tilde{\xi}K^{(1 + \frac{1}{b})} + (B - cK)^+ + \gamma cK + P - (cK - B)^+(1 + a)]^+.$$  \hspace{1cm} (2)$$

For a given capacity investment level $K$, if the firm has borrowed, it does not default when demand is such that $\tilde{\xi} \geq b(K) = \left((1 + a - \gamma)cK^{-\frac{1}{b}} - \frac{B(1+a)}{K^{(1+\frac{1}{b})}}\right)$, while it defaults, but is able to pay back the loan after its collateralized assets are liquidated when $\tilde{\xi} \geq l(K) = b(K) - \frac{P}{K^{(1+\frac{1}{b})}}$. We call $b(K)$ and $l(K)$ the **bankruptcy threshold** and the **limited liability threshold**, respectively, for investment level $K$. If the firm does not have any physical assets to collateralize ($P = 0$), then the bankruptcy threshold equals the limited liability threshold.

It is easy to establish that $l(K)$ is strictly increasing in $K$. We define $K^l$ as the (unique) solution to $l(K^l) = \xi_l$, where $\xi_l$ is the lower bound on demand. For $K \leq K^l$, the bracketed expression in (2) is non-negative for any demand realization and the limited liability of the shareholders is not useful, i.e. the firm uses a secured loan if borrowing takes place. For $K > K^l$, for some demand realizations, the bracketed expression is negative but the stage-two equity value is zero as limited liability is utilized, i.e. the firm uses an unsecured loan. The objective function in (2) is strictly concave for $K \in [0, K^l]$, but is not necessarily globally concave as the firm uses an unsecured loan when $K > K^l$. As the bracketed expression in (2) becomes more negative, not being liable for negative cash flows becomes more valuable. In this case, we say that the value of the limited liability option of the firm increases.
Proposition 1 For the firm with an internal budget $B \geq B^h \equiv c\hat{K} \left[1 - \frac{\xi^t}{\xi^u(1+\frac{1}{b})} \right] \left[1 - \frac{\gamma}{1+a} \right] - \frac{P}{1+a}$, where $\hat{K} \equiv \left(\frac{\xi^u(1+\frac{1}{b})}{(1+a-\gamma)c}\right)^{-b}$, the unique $K^*$ is given by

$$K^* = \begin{cases} 
K^0 = \left(\frac{\xi(1+\frac{1}{b})}{(1-\gamma)c}\right)^{-b} & \text{if } B \geq cK^0 \\
\frac{B}{c} & \text{if } cK^1 \leq B < cK^0 \\
K^1 = \left(\frac{\xi(1+\frac{1}{b})}{(1+a-\gamma)c}\right)^{-b} & \text{if } B < cK^1 
\end{cases}$$

(3)

The condition $B \geq B^h$ ensures that the objective function is globally concave and the optimal investment level can be found in closed form. Here, $K^0$ is the budget-unconstrained capacity level and $K^1$ is the capacity investment level with borrowing if the loan needed to make this investment is secured. If the budget realization is high enough to cover the corresponding cost $cK^0$, then $K^* = K^0$ with no borrowing. Otherwise, for each budget level $B < cK^0$, the firm determines to borrow or not by comparing the marginal revenue from investing in an additional unit of capacity over its available budget with the marginal cost of that investment including the external financing cost and the salvage value. For $cK^1 \leq B < cK^0$, the budget is insufficient to cover the budget-unconstrained capacity level $K^0$, and the marginal revenue of capacity is lower than its marginal cost. Therefore, the firm optimally does not borrow, and only purchases the capacity level $B/c$ that fully utilizes its budget. For $B < cK^1$, the marginal revenue of capacity is higher than its marginal cost. Therefore, the firm optimally borrows from the creditor to invest in capacity. The condition $B \geq B^h$ ensures that the marginal profit decreases for $K > K^t$, i.e. the firm uses a secured loan at $K^t$ ($K^1 \leq K^t$). Note that for sufficiently large $P$, we have $B^h < 0$. In other words, Proposition 1 characterizes the optimal solution for any $B \geq 0$ when $P$ is large enough.

Proposition 2 For the firm with an internal budget $B < B^l \equiv cK^l \left[1 - \frac{\xi^t}{\xi^u(1+\frac{1}{b})} \right] \left[1 - \frac{\gamma}{1+a} \right] - \frac{P}{1+a}$, $K^*$ is a solution to $MP(K^*) = 0$, where $MP(K) = \int_{l(K)}^{u(K)} \left[\frac{(1+\frac{1}{b})\xi}{K - \frac{1}{b}} - (1 + a - \gamma)c\right] f(\xi) d\xi$. $K^* \in (K^1, \hat{K})$.

If the internal budget is sufficiently low ($B < B^l$), the objective function in (2) increases in $K$ for $K \in [0, K^l]$. The marginal profit $MP(K)$ is negative for $K \geq \hat{K}$, hence there exists at least one $K^l < K < \hat{K}$ that makes the marginal profit equal to zero. The optimal solution is not necessarily unique, as neither global concavity nor unimodality can be guaranteed. It can be shown that in this budget range, $K^1 > K^t$, that is the firm would use an unsecured loan to invest in $K^1$. Therefore, the firm optimally takes more investment risk with borrowing, and increases $K^*$ beyond $K^1$. 

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For the firm with $B \in [B^l, B^h)$, we cannot explicitly characterize the optimal capacity investment level for a general distribution of $\xi$. To guarantee the unimodality of the objective function and thus, the uniqueness of the solution, we impose the following assumption:

**Assumption 1** Let $b \geq -2$ and $F(\xi)$ be such that $\frac{\xi F'(\xi) d\xi}{F(l(K))}$ decreases in $K$ for $K > K^l$.

The distributional assumption is satisfied for the uniform, exponential and triangular distributions, for example. We note here that, in the literature, the commonly used assumption to guarantee unimodality is to focus on distributions with increasing generalized failure rates (see for example, Buzacott and Zhang 2004). This assumption is relevant for the price-taker newsvendor problem but is not useful for our price-setting newsvendor model.

**Proposition 3** If Assumption 1 is satisfied, then the unique $K^*$ is given by

$$K^*$ = \begin{cases} 
K^0 & \text{if } B \geq cK^0 \\
\frac{B}{c} & \text{if } cK^1 \leq B < cK^0 \\
K^1 & \text{if } cK^1 \left(1 - \frac{\xi}{\xi(1+\frac{1}{1+a})}\right) \left[1 - \frac{\gamma}{1+a}\right] - \frac{P}{1+a} \leq B < cK^1 \\
\bar{K} & \text{if } 0 \leq B < cK^1 \left(1 - \frac{\xi}{\xi(1+\frac{1}{1+a})}\right) \left[1 - \frac{\gamma}{1+a}\right] - \frac{P}{1+a}
\end{cases}$$

(4)

where $\bar{K} \in (K^1, \hat{K})$ is the unique solution to $MP(\bar{K}) = 0$.

The intuition of the first two cases in (4) is similar to Proposition 1. If the budget is large enough such that the firm can invest in $K^1$ using a secured loan, then the optimal capacity level with borrowing is $K^1$. If the budget is sufficiently low such that the firm would use an unsecured loan to invest in $K^1$, then the firm optimally takes more investment risk and the optimal capacity investment level with borrowing is $\bar{K} > K^1$.

Using (4), the optimal expected equity value of the firm, $\pi^*$, can be obtained, and is given in the proof of Proposition 3. Throughout the analysis in §3, we will assume that Assumption 1 holds and focus on the characterization given in Proposition 3.

### 3.2 Characterization of the Creditor’s Expected Return

Let $\Lambda(a)$ denote the creditor’s expected return for a given rate $a$. As shown in Proposition 3, if $B \geq cK^1$, or equivalently, if $a \geq a^{max} = \left(\frac{cK^0}{B}\right)^{-\frac{1}{b}} - 1$ $(1 - \gamma)$, the firm does not borrow. For $0 \leq a < a^{max}$, $\Lambda(a)$ is given by

$$\Lambda(a) = (cK^*(a) - B) a - F(b(K^*(a))) BC - \int_{\xi^l}^{\max[l(K^*(a)), \xi]} \left[(cK^*(a) - B)(1 + a) - \xi (K^*(a))^{\left(1 + \frac{1}{1+a}\right)} - \gamma cK^*(a) - P\right] f(\xi) d\xi,$$

(5)
where $K^*(a)$ denotes the optimal capacity investment level of the firm for a given $a$. In (5), the first term is the creditor’s net gain from lending if the loan is secured by the collateralized assets of the firm, the second term denotes the expected default cost (payable when the firm goes to bankruptcy), and the third term is the expected loss due to the unsecured part of the loan. Incorporating $K^*(a)$ from Proposition 3, $\Lambda(a)$ is characterized as follows:

**Proposition 4** The creditor’s expected return $\Lambda(a)$ is characterized by

\[
\begin{align*}
\text{i)} & \quad = (cK^1(a) - B)a \\
\text{for } 0 \leq a < a^{\text{max}} & \quad \text{if } cK^0 (1 - \gamma) \frac{1}{\xi (1 + \frac{1}{\xi})} \leq B, \\
& \quad < cK^0 \\
\text{ii)} & \quad \begin{cases} 
(cK^1(a) - B)a - F(b(K^1(a)))BC \\
(cK^1(a) - B)a
\end{cases} \\
\text{for } 0 \leq a < a^d & \quad \text{if } cK^0 (1 - \gamma) \frac{1}{\xi (1 + \frac{1}{\xi})} - P \leq B, \\
& \quad < cK^0 (1 - \gamma) \frac{1}{\xi (1 + \frac{1}{\xi})} - P \\
\text{iii)} & \quad \begin{cases} 
(cK^1(a) - B)a - F(b(K^1(a)))BC - L(K(a)) \\
(cK^1(a) - B)a - F(b(K^1(a)))BC
\end{cases} \\
\text{for } a^d \leq a < a^{\text{max}} & \quad \text{if } B < cK^0 (1 - \gamma) \frac{1}{\xi (1 + \frac{1}{\xi})} - P,
\end{align*}
\]

where $a^{\text{max}} = \left[ \left( \frac{cK^0}{B} \right)^{-\frac{1}{\xi}} - 1 \right] (1 - \gamma)$, $L(K) \equiv \int_{\xi l}^{l(K)} \left[ (cK - B) (1 + a) - \xi (K) \right. (1 + \frac{1}{\xi}) - \gamma cK - P \left. \right] dF(\xi)$, $a^d$ is the unique solution to $B = cK^0 (1 - \gamma) - b \frac{1}{\xi (1 + \frac{1}{\xi})} \left( 1 + a^d - \gamma \right)^{\frac{b}{1 + a^d}} - \frac{P}{1 + a^d}$, and $a^d$ is the unique solution to $B = cK^0 (1 - \gamma) - b \frac{1}{\xi (1 + \frac{1}{\xi})} \left( 1 + a^d - \gamma \right)^{\frac{b}{1 + a^d}} - \frac{P}{1 + a^d}$.

If $B \geq cK^0$, the firm does not need to borrow, and the creditor does not have any returns (this is omitted from the statement of the proposition). Otherwise, if the internal budget level is sufficiently high (case (i)), the firm always borrows to invest in $K^1(a)$ but never defaults. This case can only occur if there is a positive lower bound on demand or a positive salvage value of capacity. If the internal budget level is moderate (case (ii)), for a small $a$, the firm borrows to invest in $K^1(a)$ and may default on the loan, but the creditor can always retrieve the face value of the loan through the collateralized assets. For large $a$, the firm borrows less to invest in $K^1(a)$ and does not default. In summary, in case (ii), the firm may default but the borrowing is always secured. If the internal budget level is sufficiently low (case (iii)), the firm uses an unsecured loan to invest in $K(a)$ for small $a$. For moderate $a$, the firm borrows less to invest in $K^1(a)$, may default but the loan is secured. For large $a$, the firm borrows even less and does not default.

### 3.3 Equilibrium Characterization

We now turn to the characterization of the equilibrium. We use the $\dot{a}$ notation to denote equilibrium quantities: $\dot{a}$ is the equilibrium unit financing cost and $K \approx K^*(\dot{a})$ is the equilibrium capacity investment level. Where there are multiple $\dot{a}$'s that satisfy
the objective of the creditor, we set $\hat{a}$ to the smallest such value. $\hat{a}$ is Pareto-optimal for the firm because the optimal expected equity value of the firm increases as the external capital becomes available at a lower unit cost. Let $a^N$ be the unique solution to

$$(1 + a - \gamma)(b-1)(1 + a - \gamma + ab) = \frac{B}{cK^0(1-\gamma)\delta}.$$ 

**Proposition 5** For the perfectly competitive credit market with $U = 0$, $\hat{a} = 0$ for case (i) and $\hat{a} \in (0, a^d)$ for cases (ii) and (iii) of Proposition 4. For $U > 0$, if the contract is offered, $\hat{a}$ can take any value in $(0, a^{\text{max}})$. In the monopolistic credit market, a contract is always offered, and $\hat{a} \in (0, a^{\text{max}})$. In particular, for cases (i) and (ii) of Proposition 4, $\hat{a} = a^N$ if $a^N \geq a^d$ and $\hat{a} \in (a^N, a^d)$ otherwise.

For the perfectly competitive credit market with $U = 0$, a loan contract is always offered in equilibrium, but this is not the case for sufficiently large $U$, in which case we use the convention that $\hat{a} = a^{\text{max}}$. For the monopolist creditor, since there exists a feasible unit financing cost level $a \in (a^d, a^{\text{max}})$ such that the creditor makes a strictly positive profit, the creditor always offers a loan contract in equilibrium. In contrast to the perfectly competitive credit market case, the monopolist creditor uses the expected marginal profit and not the expected profit from lending to determine the unit financing cost in equilibrium.

For firms that may use an unsecured loan (case (iii) of Proposition 4), Figure 1 demonstrates that the creditor’s expected return may not be unimodal in $a$. In particular, there may be two different local maxima, one in the unsecured lending region, and the other one in the secured lending region. Consequently, in the monopolist creditor case, all three equilibria ($\hat{a} < a^d$, $a^l \leq \hat{a} < a^d$, $a^d \leq \hat{a}$) may arise depending on the parameter specifications. Figure 1 shows this progression as a function of the internal budget level $B$ of the firm.

Proposition 5 and Figure 1 jointly show that depending on the credit market, firm and product market characteristics, any of the following three types of equilibria can be observed: an equilibrium where the firm uses a secured loan without default possibility, i.e. $a^d \leq \hat{a} < a^{\text{max}}$, an equilibrium where the firm uses a secured loan with default possibility, i.e. $a^l \leq \hat{a} < a^d$, and an equilibrium where the firm uses an unsecured loan, i.e. $\hat{a} < a^l$. When the firm uses a secured loan it invests in $K^1(\hat{a})$ whereas when the firm uses an unsecured loan, it invests in $\overline{K}(\hat{a})$. We will use this equilibrium classification throughout the paper.

**3.4 Comparative Static Analysis**

The goal of this paper is to analyze the impact of endogenous credit terms under capital market imperfections in a capacity investment setting. To this end, we first identify the perfect capital market equilibrium and present comparative statics results with respect to
Figure 1: The location of the equilibrium financing cost in the monopolist creditor case ($\hat{a} = \hat{a}^M$) for firms that may use an unsecured loan (case (iii) of Proposition 4) with $c = 1$, $b = -2$, $P = 350$, $BC = 75$, $B \in \{800, 1000, 1500\}$, $\xi \sim U[20, 200]$: For $B = 800$, $\hat{a}^M < a^l$ and the firm uses an unsecured loan. For $B = 1000$, $a^l \leq \hat{a}^M < a^d$ and the firm uses a secured loan with default possibility. For $B = 1500$, $\hat{a}^M \geq a^d$ and the firm uses a secured loan without default possibility.

demand variability and the internal budget level. We then carry out comparative statics analysis under the three different imperfect capital market models that we study.

In our modeling framework, if the capital markets are perfect, there is no fixed bankruptcy cost ($BC = 0$) and the creditor’s expected return is zero ($U = 0$). In this case, the firm’s capacity investment decision is independent of financing decision:

**Remark 1** *In the perfect capital market equilibrium, for any firm with $B \geq 0$, we have $\hat{K} = K^0 = \left(\frac{\xi(1 + \frac{1}{\gamma})}{(1 - \gamma)c}\right)^{-b}$ and the expected equity value of the firm is given by $\hat{\pi} = B + P + \frac{cK^0(1 - \gamma)}{(b + 1)}$. $\hat{K}$ and $\hat{\pi}$ are independent of demand variability, whereas $\hat{\pi}$ increases and $\hat{K}$ does not change with an increase in the internal budget level of the firm.*

The equilibrium investment level is the budget-unconstrained investment level $K^0$ for the firm with any internal budget level as in traditional stochastic capacity models: The firm simply chooses the optimal investment level without regard to the budget limit, and implements it by borrowing if necessary. This replicates the well-known result about the decoupling of operational and financial decisions in perfect markets (Modigliani and Miller
1958); but we do it to have the benchmark specific to our model. We now show that the impact of demand variability and the internal budget level shown in Remark 1 are modified or reversed once capital market imperfections and endogenous credit terms are taken into account, and we explain why.

Our goal is not to undertake a complete characterization of the equilibrium, but to show the existence of certain effects that arise from capital market imperfections. For this reason, we focus on a specific demand distribution and assume that $\xi$ is uniformly distributed between $[\xi^l, \xi^u]$ as the uniform distribution allows us to derive analytical results. For brevity, we also normalize the salvage rate of capacity $\gamma$ to zero. For consistency, we use the same base parameter set ($c = 1, b = -2, P = 350, BC = 75, \xi^l = 20, \xi^u \in \{120, 140\}$) for numerical examples appearing in this section.

### 3.4.1 The Impact of Demand Variability

To analyze the impact of demand variability $\sigma$, we use the mean-preserving spread of the uniform distribution to characterize an increase in $\sigma$, i.e. $\xi \sim U[\xi^l - \epsilon, \xi^u + \epsilon]$ for $\epsilon \in (0, \xi^l)$. A higher $\epsilon$ leads to a higher variance of $\xi$. As follows from Remark 1, in perfect capital markets, the firm’s equilibrium capacity decision and expected equity value do not depend on demand variability $\sigma$. This result continues to hold at equilibria where the firm uses a secured loan without default possibility. In all other cases, $\dot{K}$ and $\dot{\pi}$ depend on demand variability. For convenience, we summarize the results of this section in Table 1, where the boxed comparative statics results are proven analytically and the rest are existence results observed in numerical experiments. The detailed analysis (analytical proofs and numerical experiments) of summary results in Table 1 as well as the proofs for the supporting lemmas, denoted by Lemma A.x, are provided in §A.4.1 of the Technical Appendix.

For brevity, we only discuss two of our more interesting results.

**A higher demand variability may increase $\dot{K}$.** One may expect that a higher demand variability decreases the equilibrium capacity investment level $\dot{K}$ because it increases the default risk of the firm, which in turn, increases the equilibrium financing cost. Indeed, Xu and Birge (2004) numerically demonstrate this in a setting where the firm is a price-taker newsvendor in a perfectly competitive credit market with $U = 0$. In contrast, we find that $\dot{K}$ may increase.

For the perfectly competitive credit market with $U \geq 0$, this is observed at equilibria where the firm uses an unsecured loan. For a given $a$, an increase in demand variability increases $\overline{K}$ (Lemma A.5). This is because as the likelihood of low demand states increases, the value of the limited liability option of the firm increases, and the firm optimally takes
more investment risk and invests more in capacity. From the creditor’s perspective, increasing demand variability has three distinct effects. First, the default risk increases (Lemma A.6) as i) for a fixed $K^0$, the downside risk of the firm’s operating cash flows increases, and ii) $K$ increases and the firm borrows more. Second, the expected loss due to the unsecured part of the loan increases due to a similar reasoning (Lemma A.6). Third, the creditor’s net gain increases as the firm invests and borrows more (Lemma A.6). The first two effects work to increase $\hat{a}$, whereas the third effect works to decrease it. Our numerical results show that the first two effects can dominate such that $\hat{a}$ increases in demand variability. However, the increase in the value of the limited liability option of the firm can outweigh the negative effect of higher financing cost and $\hat{K}$ increases in demand variability.

For the monopolist creditor, this result is also observed at the same equilibria (where the firm uses an unsecured loan). Our numerical results show that similar to the perfectly competitive credit market case, with an increase in $\sigma$, even though $\hat{a}$ increases, the increase in the value of the limited liability option of the firm may outweigh this effect and $\hat{K}$ increases. The same result is also obtained when the firm uses a secured loan with default
possibility in equilibrium, as discussed next.

**Proposition 6** For the monopolist creditor, at equilibria where the firm uses a secured loan with default possibility, the expected equity value and the capacity investment level in equilibrium strictly increase in demand variability through a decrease in \( \dot{a} \).

For a given \( \dot{a} \), \( K^1(\dot{a}) \) is independent of demand variability because the loan is secured. In this case, the impact of demand variability is solely determined by the effect on \( \dot{a} \). In determining the equilibrium financing cost, the monopolist creditor equates the marginal cost (that is, the reduction in the net gain of the creditor with an increase in \( a \)) with the marginal revenue (that is, the reduction in the expected default cost with an increase in \( a \)). Only the marginal revenue term is affected by \( \sigma \). With the uniform distribution of \( \xi \), an increase in the demand variability decreases the marginal revenue term and the equilibrium financing cost decreases. This is because the density function of the uniform distribution decreases in the demand variability where it is defined. Thus, \( K \) increases.\(^1\)

**A higher demand variability may increase** \( \dot{\pi} \). As follows from Proposition 6, firms that use a secured loan with default possibility benefit from demand variability. This is because for a given \( a \), demand variability does not alter the firm’s expected equity value (as the loan is secured); whereas a lower financing cost in equilibrium increases \( \dot{\pi} \).

### 3.4.2 The Impact of the Internal Budget Level

As follows from Remark 1, in perfect capital markets, the effect of an increase in the internal budget level on the equilibrium capacity investment level is zero (because the firm is not budget-constrained), and the effect on the expected equity value is positive. In imperfect capital markets, this result only holds for the perfectly competitive credit market with \( U = 0 \) at equilibria where the firm uses a secured loan without default possibility. In all other cases, this result changes. For convenience, we summarize the results of this section in Table 2 where the boxed comparative statics results are proven analytically and the rest are existence results observed in numerical experiments. The detailed analysis (analytical proofs and numerical experiments) of the summary results in Table 2 as well as the proofs for the supporting lemmas are provided in §A.4.2 of the Technical Appendix.

\(^1\)With the normal distribution of \( \xi \), if we assume a sufficiently large \( P \) such that the optimal capacity investment level is given by Proposition 1, we can prove that this result is reversed. This is because with the normal distribution, the marginal revenue term is increasing in the demand variability as the probability density function of \( \xi \) increases in \( \sigma \) over the relevant parameter range.
<table>
<thead>
<tr>
<th>Perfect Market</th>
<th>Imperfect Market</th>
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<tr>
<td>Perfectly Competitive Credit Market</td>
<td>Monopolist Creditor</td>
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<tr>
<th>Form of $\dot{K}$</th>
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<tr>
<td>$K^0$</td>
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<td>secured w/o default possibility</td>
<td>$K^1(\hat{a})$</td>
<td>secured w/o default possibility</td>
<td>$K^1(\hat{a})$</td>
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<td></td>
<td>$\dot{K}^-, \Pi^+$</td>
<td>$K^1(\hat{a})$</td>
<td>secured w/o default possibility</td>
<td>$\dot{K}^+$, $\Pi^-$</td>
<td>$U \leq BC \frac{\xi^l}{\xi^u - \xi^l}$</td>
<td>$K^1(\hat{a})$</td>
<td>secured w/o default possibility</td>
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<td>$\dot{K}^-$, $\Pi^+$</td>
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<td>secured w/o default possibility</td>
<td>$\dot{K}^+$, $\Pi^-$</td>
<td>$U &gt; BC \frac{\xi^l}{\xi^u - \xi^l}$</td>
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<td>secured w/o default possibility</td>
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<td>unsecured</td>
<td>$\dot{K}^-, \Pi^+$</td>
<td>$K(\hat{a})$</td>
<td>unsecured</td>
<td>$\dot{K}^-, \Pi^+$</td>
<td>$K(\hat{a})$</td>
</tr>
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Table 2: Differences between perfect and imperfect markets concerning the impact of an increase in the internal budget level in single-product investments with uniform $[\xi^l, \xi^u]$ demand uncertainty: The boxed comparative statics results are proven analytically. The differences in $\dot{K}$ and $\dot{\pi}$ between perfect and imperfect markets are driven by the effect of $B$ on the creditor’s expected return, and in turn, on the equilibrium financing cost $\dot{a}$. The variety of comparative statics results in imperfect capital markets depend on the firm’s loan type in equilibrium (unsecured versus secured, with or without default possibility) and the different capital market conditions studied.

For brevity, we only discuss two of our more interesting results.

**A higher internal budget level may decrease $\dot{K}$.** Intuition may suggest that as the internal budget increases, the firm’s dependence on external borrowing decreases and hence the firm invests more in capacity. As we discuss below, this intuition is not always correct.

**Proposition 7** For the perfectly competitive credit market with $U > 0$, with a small increase in the internal budget level, $\dot{a}$ strictly increases and the capacity investment level in equilibrium strictly decreases at i) equilibria where the firm uses a secured loan without default possibility, and ii) equilibria where the firm uses a secured loan with default possibility if $U > BC \frac{\xi^l}{\xi^u - \xi^l}$.

For firms that use a secured loan without default possibility in equilibrium, the creditor’s expected return is characterized by the net gain from secured lending. From the firm’s perspective, an increase in the internal budget level does not alter the optimal capacity investment level for a given $a$. From the creditor’s perspective, an increase in $B$ decreases the borrowing level for a given $a$. Therefore, $\dot{a}$ increases to satisfy $U$ and $\dot{K}$ decreases.
For firms that use a secured loan with default possibility in equilibrium, the creditor's expected return is characterized by the net gain from secured lending minus the expected default cost. An increase in the internal budget level decreases both the net gain and the default probability (as the firm optimally borrows less). The first effect works to increase $\dot{a}$ and the second effect to decrease it. For sufficiently high $U$, the first effect dominates and $\dot{a}$ increases in the internal budget level. Therefore, $\dot{K}$ decreases.

For firms that use an unsecured loan in equilibrium, for a given $a$, an increase in the internal budget level decreases $K$ (Lemma A.7). This is because as $B$ increases, the value of the limited liability option of the firm decreases; and the firm optimally takes less investment risk and invests less in capacity. For the perfectly competitive credit market, our numerical experiments reveal that even when $\dot{a}$ decreases in the internal budget level, the reduction in the value of the limited liability option of the firm may outweigh the positive effect of the lower financing cost and $\dot{K}$ decreases in $B$. There also exist instances in which $\dot{a}$ increases in $B$. In this case, both effects work in the same direction and $\dot{K}$ decreases in $B$. For the monopolist creditor, $\dot{K}$ decreasing in the internal budget level is driven by a jump in the equilibrium financing cost: As we had shown in Figure 1 of §3.3, the expected profit of the creditor is bimodal, and the increase in $B$ may induce the creditor to switch from one local maximizer (in the unsecured lending region) to the other local maximizer (in the secured lending region). This leads to a discontinuous increase in $\dot{a}$ and decreases $\dot{K}$.

A higher internal budget level may decrease $\dot{\pi}$. Our numerical experiments reveal that a higher internal budget level may hurt the firm in equilibrium. This counterintuitive result is observed when there is a discontinuous increase in the equilibrium financing cost with an increase in $B$. For the perfectly competitive credit market with $U > 0$, this result is observed at equilibria where the firm uses a secured loan without default possibility. It is driven by the unavailability of the loan contract in equilibrium: With an increase in $B$, since the borrowing level decreases, the expected return requirement $U$ may not be satisfied and the contract is not offered. Going from an equilibrium with a loan contract to another equilibrium without a loan contract leads to a sharp decline in $\dot{\pi}$. With the monopolist creditor, this result is observed at equilibria where the firm uses an unsecured loan and is driven by the jump in the equilibrium financing cost, as discussed above.

In summary, the impact of the demand variability and the internal budget level on the firm’s capacity investment and expected equity value in an imperfect market equilibrium are, in general, different from the case in perfect capital markets. The overall impact depends on the firm characteristics (in particular, type of the loan the firm is using) as well as the
capital market conditions.

4 The Two-Product Firm

This section extends the analysis of the impact of endogenous credit terms under capital market imperfections to the two-product firm. In the two-product setting, the firm determines its technology choice \( T \in \{D,F\} \) in stage 1. The analysis of the equilibrium with each technology (without the technology choice) is similar to the single product analysis of §3 with minor modifications. For brevity, we only provide a synopsis of the analysis in §4.1. The complete analysis is relegated to §B.1 of the Technical Appendix. In §4.2, we analyze the effect of demand variability and demand correlation on each technology in equilibrium. §4.3 analyzes the equilibrium technology choice.

4.1 Synopsis of The Analysis For A Given Technology

In this section, since some of the parameters are technology specific, we reintroduce subscripts \( D \) and \( F \). We assume that \( \xi' = (\xi_1, \xi_2) \) has a symmetric bivariate normal distribution (with \( \bar{\xi}_1 = \bar{\xi}_2 = \bar{\xi} \) and covariance matrix \( \Sigma \), where \( \Sigma_{ii} = \sigma^2 \) and \( \Sigma_{ij} = \rho \sigma^2 \) for \( i \neq j \)) because this is the natural setting to study the impact of demand correlation \( \rho \). Recall that we cannot guarantee the uniqueness of the optimal capacity investment level for the normal distribution as it does not satisfy Assumption 1. Therefore, we assume that the value of the physical assets \( P \) of the firm is sufficiently large\(^2\) such that for a given financing cost, it is never optimal for the firm to invest in capacity levels that necessitate an unsecured loan. In this case, we can use an analogue of Proposition 1 to characterize the optimal capacity investment level with each technology. This assumption is made for analytical convenience, as we cannot guarantee the unimodality of the firm’s problem with bivariate normal demand uncertainty. We discuss the implications of relaxing the sufficiently large \( P \) assumption and, in turn, the impact of obtaining an unsecured loan in §5.2.

We first analyze the firm’s decision problem for a given financing cost \( a_T, T \in \{D,F\} \). With the dedicated technology, since \( \xi \) has a symmetric distribution, the firm optimally invests in identical capacity levels for both resources. Therefore, we can use a single resource level \( K_D \) to characterize the firm’s optimal capacity investment portfolio. The analysis of the two-product firm decouples into an analysis of two single-product firms. Similar to Proposition 1, the firm invests in the budget-unconstrained capacity level \( K_D^0 \doteq \left( \frac{\bar{\xi}(1+\frac{1}{T})}{(1-\gamma_T)\sigma_D} \right)^{-\frac{b}{\alpha}} \) for each resource if the internal budget is sufficiently high \((B \geq 2\sigma_D K_D^0)\). Otherwise, the firm

\[ P \geq 2^{\frac{b+1}{\alpha}} \left[ \frac{\bar{\xi}}{\xi} \left( 1 + \frac{1}{T} \right) \right]^{-\frac{b}{\alpha}} \left[ 1 - \frac{\bar{\xi}^b}{\xi^b(1+\frac{1}{T})} \right] \left[ 1 - \gamma_T \right]^{b+1} \] for \( T \in \{D,F\} \).

\(^2\)More precisely, we assume \( P \geq 2^{\frac{b+1}{\alpha}} \left[ \frac{\bar{\xi}}{\xi} \left( 1 + \frac{1}{T} \right) \right]^{-\frac{b}{\alpha}} \left[ 1 - \frac{\bar{\xi}^b}{\xi^b(1+\frac{1}{T})} \right] \left[ 1 - \gamma_T \right]^{b+1} \) for \( T \in \{D,F\} \).
either fully utilizes the available budget without borrowing and invests \( \frac{B}{c_D} \) for each resource or invests in the optimal capacity investment level with borrowing \( K_D^1 = \left( \frac{\bar{\xi}(1+\frac{1}{D})}{(1+a_{D} - \gamma_{D})c_D} \right)^{-\frac{1}{b}} \) if the internal budget cannot finance this investment level \( (B < 2c_D K_D^1) \).

With the flexible technology, there is a single resource investment, and the optimal capacity investment level is also similar to Proposition 1 except for one modification: In \( K_F^0 \) (and \( K_F^1 \)), the term \( \bar{\xi} \) is replaced by \( \mathbb{E}\left[\left(\xi_1^{-b} + \xi_2^{-b}\right)^{-\frac{1}{b}}\right] \). This new term captures the capacity-pooling benefit of the flexible technology.

For the creditor’s problem, since the physical assets of the firm are sufficiently large, the creditor always retrieves the face value of the loan even if the firm defaults. The creditor’s expected return can be characterized in a similar fashion to Proposition 4 where only cases (i) and (ii) are relevant.

In the perfect market equilibrium, similar to Remark 1, the firm invests in the budget-unconstrained capacity level \( K_T^0 \) with each technology \( T \in \{D, F\} \), and the decoupling of the firm’s operational and financial decisions continue to hold as expected. In the imperfect market equilibrium, the creditor offers a unique Pareto-optimal \( \hat{a}_T \) for each technology; and the firm invests in \( K_T^1(\hat{a}_T) \) with the chosen technology. The equilibrium characterization under the different capital market conditions is similar to the single-product case. For example, for the perfectly competitive credit market with \( U > 0 \), there exist two different equilibria with each technology \( T \in \{D, F\} \): Firms that use a secured loan without default possibility in equilibrium \( (\hat{a}_T \geq a_T^d) \) and firms that use a secured loan with default possibility in equilibrium \( (\hat{a}_T \in (0, a_T^d)) \).

4.2 The Effect of Demand Uncertainty on Each Technology

We now investigate the effect of demand variability (\( \sigma \)) and correlation (\( \rho \)) on the equilibrium capacity investment level and the expected equity value of the firm with each technology. We use the same parameter set \( c_F = c_D = 1, \gamma_F = \gamma_D = 0.1, P = 650, b = -2, \bar{\xi}_1 = \bar{\xi}_2 = 20, B \in \{5, 20, 240\}, \sigma \in [3, 6], \rho \in [-0.995, 0.995], BC \in \{100, 200\}, U \in \{0, 20\} \) for numerical examples throughout this section. Our main results for the perfectly competitive credit market are summarized in Table 3 where the boxed comparative statics results are proven analytically and the rest are existence results observed in numerical experiments. The table for the monopolist creditor is also identical except the impact of \( \rho \) and \( \sigma \) at equilibria where the firm uses a secured loan without default possibility are not proven analytically. The detailed analysis (analytical proofs and numerical experiments) of the summary results in Table 3 as well as the results for the monopolist creditor are provided.
in §B.2 of the Technical Appendix.

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<td>Form of $K_T$</td>
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<tr>
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<td>$\dot{K}_D^-, \dot{\Pi}_D^-$</td>
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<td></td>
<td>$K_F^0$</td>
<td>$\dot{K}_F^+, \dot{\Pi}_F^+$</td>
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</table>

| Demand Correlation ($\rho$) | $K_D^0$        | $\dot{K}_D^-, \dot{\Pi}_D^-$ | $K_{PD}^0(\hat{a}_D)$ | w/o default | $\dot{K}_D^-, \dot{\Pi}_D^-$ | $K_{PD}^0(\hat{a}_F)$ | w/o default |
|                          | $K_F^0$        | $\dot{K}_F^-\downarrow, \dot{\Pi}_F^-\downarrow$ | $K_{PF}^0(\hat{a}_D)$ | w default    | $\dot{K}_F^-, \dot{\Pi}_F^-\downarrow$ | $K_{PF}^0(\hat{a}_F)$ | w default |

Table 3: Differences between perfect and imperfect markets in two-product investments with bivariate normal demand uncertainty for a perfectly competitive credit market with $U \geq 0$. The boxed results are proven analytically. The two differences between the perfect and the imperfect capital markets are the following: With the dedicated technology, $\dot{K}_D$ and $\dot{\Pi}_D$ may decrease in $\sigma$ and $\rho$ while they do not change in perfect markets. With the flexible technology, $\dot{K}_F$ and $\dot{\Pi}_F$ may increase with a decrease (increase) in $\sigma$ ($\rho$) while they decrease in perfect markets.

**Dedicated Technology:** In perfect capital markets, the firm’s equilibrium capacity decision and the expected equity value with the dedicated technology depend only on the mean demand and not on the covariance matrix. In imperfect capital markets, the elements of the covariance matrix ($\rho$ and $\sigma$) can matter:

A higher demand variability or demand correlation may decrease $\dot{K}_D$ and $\dot{\Pi}_D$.

**Proposition 8** For the perfectly competitive credit market with $U \geq 0$, at equilibria where the firm uses a secured loan with default possibility, the expected equity value and the capacity investment level with the dedicated technology strictly decrease in the demand variability $\sigma$ and the correlation $\rho$ through an increase in $\hat{a}_D$. For the monopolist creditor, the same result holds locally.

From the firm’s perspective, for an arbitrary financing cost $a_D$, the demand variability or the correlation do not alter the optimal capacity investment level or the equity value (as
the firm uses a secured loan). Therefore, the impact of $\sigma$ or $\rho$ is determined by their effect on $\dot{a}_D$.

With our assumption on $P$, the creditor’s expected return with the dedicated technology ($\Lambda_D(a_D)$) is given by

$$\left(2c_DK_D^1 - B\right) a_D - BC \Pr \left\{ \frac{(\xi_1 + \xi_2)}{2(1+\frac{1}{b})} < \frac{(1 + a_D)(2c_DK_D^1 - B) - \gamma_D 2c_DK_D^1}{(2K_D^1)^{(1+\frac{1}{b})}} \right\},$$

where the first term is the net gain from secured lending and the second term is the expected default cost. For the monopolist creditor, $\dot{a}_D$ is determined by equating the marginal cost (that is, the reduction in the net gain from secured lending with an increase in $a_D$) with the marginal revenue (that is, the reduction in the expected default cost with an increase in $a_D$). Only the marginal revenue term is affected by $\sigma$ or $\rho$. A small increase in $\sigma$ or $\rho$ increases the marginal revenue and $\dot{a}_D$ locally increases. Therefore, $\dot{K}_D$ and $\dot{\pi}_D$ locally decrease. Since the creditor’s expected return function may not be unimodal, we can only prove this locally, but in our numerical study, this observation is not restricted to small changes in these two parameters.

For the perfectly competitive credit market, a higher $\sigma$ increases the downside risk and, in turn, the default risk of the firm. This reduces the expected return of the creditor for an arbitrary financing cost $a_D$. The rate $\dot{a}_D$ increases to compensate for this reduction. The result with respect to $\rho$ follows from a financial-pooling argument. The firm’s default probability for a given capacity level depends on the variability in operating revenues. Operating in two markets creates a diversification benefit for the firm: When the demands are negatively correlated, the revenue variability and hence the default risk are low. With a high correlation, the firm generates similar revenues from both markets, and operates under a high default risk. As correlation increases, the diversification benefit decreases and the expected default cost increases, therefore $\dot{a}_D$ increases.

These results are demonstrated in Figure 2. In addition, in our numerical experiments, we observe that the effect of demand variability on $\dot{a}_D$ is insignificant at low correlations. This can also be observed in Figure 2: At low correlations, the diversification benefit is very high and the default risk is very low, therefore a change in $\sigma$ has negligible impact on the default risk of the firm. Hence, the financing cost offered by the creditor is insensitive to an increase in $\sigma$ (Panel A). In this case, the equilibrium capacity level (Panel B) and equity value (Panel C) are also insensitive to a change in $\sigma$.

The financial-pooling effect discussed in this section is different from the capacity-pooling benefit of flexible technology that comes from the ability to switch capacity between products. Within our modeling framework, the former effect only matters in imperfect capital
Figure 2: The effect of the demand correlation $\rho$ and the demand variability $\sigma$ on the dedicated technology investment in the imperfect market equilibrium (in a perfectly competitive credit market with $U = 0$) with $\sigma \in [3, 6]$ with 1-unit increments: A higher $\rho$ or $\sigma$ lead to a higher financing cost in equilibrium (Panel A), and this decreases $\hat{K}_D$ (Panel B) and $\hat{\pi}_D$ (Panel C). Both quantities are insensitive to changes in demand variability at low correlations: The value of financial pooling (diversification benefit of operating in two markets) is high in this range and the equilibrium financing cost is insensitive to changes in demand variability.

Flexible Technology: In perfect capital markets, the firm’s equilibrium capacity decision and the expected equity value with the flexible technology depend on the covariance matrix $\Sigma$ of $\xi$ through the term $M_F = E \left[ \left( \xi_1^{-b} + \xi_2^{-b} \right)^{-\frac{1}{b}} \right]$. This term captures the capacity-pooling feature of flexible technology. Unfortunately, it is not possible to derive analytically the effect of $\rho$ and $\sigma$ on $M_F$ for bivariate normal $\xi$. In our numerical experiments, at a sufficiently low coefficient of variation $\frac{\sigma}{\xi}$ to avoid negative demand realizations, we observe that $M_F$ increases in $\sigma$ and decreases in $\rho$. Therefore, $\hat{K}_F$ and $\hat{\pi}_F$ increase. This is in line with the traditional argument on flexible technology investment: Its value increases in demand variability and decreases in demand correlation. Therefore, in deriving the analytical results for flexible capacity in Table 3, we make the following assumption:

**Assumption 2** $M_F = E \left[ \left( \xi_1^{-b} + \xi_2^{-b} \right)^{-\frac{1}{b}} \right]$ is decreasing in $\rho$ and increasing in $\sigma$

In imperfect capital markets, the impact of $\rho$ and $\sigma$ on $\hat{K}_F$ and $\hat{\pi}_F$ may be reversed relative
to the perfect market benchmark, as discussed next.

**A higher demand variability or a lower demand correlation may decrease** $\hat{K}_F$ and $\hat{\pi}_F$. With our assumption on $P$, the creditor’s expected return with the flexible technology $(\Lambda_F(a_F))$ is given by

$$
(c_F K_1^F - B) a_F - BC Pr \left\{ \left( \xi_1^{-b} + \xi_2^{-b} \right)^{-\frac{1}{b}} < \frac{(1 + a_F)(c_F K_1^F - B) - \gamma_F c_F K_1^F}{(K_1^F)^{(1+\frac{1}{b})}} \right\},
$$

where the first term is the net gain from secured lending and the second term is the expected default cost. From the firm’s perspective, for a given $a_F$, under Assumption 2, an increase in $\rho$ or a decrease in $\sigma$ decreases $K_1^F(a_F)$ as the value of capacity pooling decreases.

In a perfectly competitive credit market, from the creditor’s perspective, an increase in $\rho$ or a decrease in $\sigma$ has three distinct effects: First, for a fixed $K_1^F(a_F)$, the default risk increases. This is because the firm is able to generate lower returns as the value of capacity pooling as well as the value of financial pooling (diversification benefit) decrease. This effect is inherent in the change of the distribution of the random variable $H_F(\xi) = \left( \xi_1^{-b} + \xi_2^{-b} \right)^{-\frac{1}{b}}$ that defines the default probability. Second, the net gain from secured lending decreases as the firm invests less and, in turn, borrows less. Third, as the firm borrows less, the default risk decreases. The first two effects work to increase $\hat{a}_F$, whereas the third effect works to decrease it.

With a decrease in $\sigma$, in our numerical experiments, the third effect dominates and $\hat{a}_F$ decreases (as depicted in Panel A of Figure 3). For the effect on $\hat{K}_F$, we have two drivers that work in opposite directions: A lower $\sigma$ induces the firm to invest less for a given $\hat{a}_F$ as capacity pooling becomes less valuable, but a lower $\hat{a}_F$ induces the firm to invest more. For low correlations, the former outweighs the latter and $\hat{K}_F$ decreases with a decrease in $\sigma$ (Panel B). This is because the value of financial pooling is sufficiently high and $\hat{a}_F$ is insensitive to changes in $\sigma$ (Panel A). For high correlations, the latter outweighs the former and $\hat{K}_F$ increases with a decrease in $\sigma$ (Panel B). This is because the value of financial pooling is very low and $\hat{a}_F$ decreases significantly with a decrease in $\sigma$ (Panel B). The effect on $\hat{\pi}_F$ is identical (Panel C) and follows from the same reasoning.

With an increase in $\rho$, in our numerical experiments, for a larger portion of the $\rho$ range, the first two effects dominate and $\hat{a}_F$ increases (Panel A). This reinforces the declining value of capacity pooling: $\hat{K}_F$ (Panel B) and $\hat{\pi}_F$ (Panel C) decrease. Our numerical experiments also reveal that at very high correlation levels (for $\rho = 0.980$ for this figure), the third effect may outweigh the first two and $\hat{a}_F$ may decrease. With an increase in $\rho$, although the capacity-pooling value decreases, the decreasing cost of financing may dominate, and $\hat{K}_F$
Figure 3: Effect of demand correlation ($\rho$) and demand variability ($\sigma$) on the flexible technology investment in the imperfect market equilibrium (in a perfectly competitive credit market with $U = 0$) with $\sigma \in [3, 4.5]$ with 0.25-unit increments: A higher $\sigma$ leads to a higher financing cost $\hat{a}_F$ (Panel A), and for high correlation levels this overcomes the increasing value of capacity pooling: $\hat{K}_F$ (Panel B) and $\hat{\pi}_F$ (Panel C) decrease. For the most part (except a few instances which are not visible here in which $\rho$ is close to 1) an increase in $\rho$ increases $\hat{a}_F$ (Panel A) and this reinforces the declining value of capacity pooling: $\hat{K}_F$ (Panel B) and $\hat{\pi}_F$ (Panel C) decrease.

and $\hat{\pi}_F$ may increase.

For the monopolist creditor, the impact of $\sigma$ and $\rho$ on $\hat{K}_F$ and $\hat{\pi}_F$ are identical to the perfectly competitive credit market case. The only difference is the impact of $\rho$ on the equilibrium financing cost. In our numerical experiments, we observe that with an increase in $\rho$, in contrast to the perfectly competitive credit market case, $\hat{a}_F$ decreases at low correlation levels. Since the capacity pooling is of high value at these low correlation levels; with an increase in $\rho$, the decline in the capacity-pooling benefit may outweigh the decline in the financing cost. Thus, $\hat{K}_F$ and $\hat{\pi}_F$ decrease.

It is important to note that with the monopolist creditor although the possible impact of $\sigma$ and $\rho$ on each technology is identical to the perfectly competitive credit market case, the underlying intuition is different and depends on the impact of these parameters on the marginal profit from lending (and not on the impact on the expected profit from lending).
4.3 Technology Choice

In §4.2, we performed comparative statics analysis for a given technology. In this section, we turn to the equilibrium technology choice. The detailed analysis (analytical proofs and numerical experiments) of the results in this section as well as the proofs for the supporting technical statements, denoted by B.x, are provided in §B.3 of the Technical Appendix. Since there is no fixed cost and the firm always invests in a positive level of capacity with each technology, investing in either technology dominates not making any technology investment. We first characterize the equilibrium technology choice in perfect capital markets. The choice \( \hat{T} \) is determined by a unit cost threshold.

**Remark 2** If the capital markets are perfect, there exists a unique variable cost threshold \( \bar{c}_F(c_D) \) such that when \( c_F \leq \bar{c}_F(c_D) \) (\( c_F > \bar{c}_F(c_D) \)), it is optimal to invest in flexible (dedicated) technology. This threshold is given by

\[
\bar{c}_F(c_D) = c_D \left( \frac{1 - \gamma_D}{1 - \gamma_F} \right) \left[ \frac{E \left( \left( \xi_1^{-b} + \xi_2^{-b} \right)^{-\frac{1}{b}} \right)}{2^{-\frac{1}{b}} \xi} \right]^{\frac{b}{b+1}} \geq c_D,
\]

where the last inequality holds at equality only if i) the salvage values are symmetric (\( \gamma_F = \gamma_D \)) and ii) the product markets are deterministic (\( \sigma = 0 \)) or the product markets are perfectly positively correlated (\( \rho = 1 \)).

The threshold \( \bar{c}_F(c_D) \) is a variant of the mix flexibility threshold of Chod et al. (2006). Since flexible capacity has a higher salvage value and has a capacity-pooling benefit, the firm can sustain a higher unit capacity investment cost with the flexible technology, and we have \( \bar{c}_F(c_D) \geq c_D \). As we discussed in §4.2, the term \( M_F = E \left( \left( \xi_1^{-b} + \xi_2^{-b} \right)^{-\frac{1}{b}} \right) \) captures the capacity-pooling benefit of the flexible technology. Under Assumption 2, \( M_F \) and this threshold increase with an increase in demand variability (\( \sigma \)) and a decrease in demand correlation (\( \rho \)) due to the increasing capacity-pooling benefit. With symmetric salvage rates, Remark 2 shows that there is no capacity-pooling benefit (\( \bar{c}_F(c_D) = c_D \)) only if the product market demands are deterministic or are random but perfectly positively correlated.

In imperfect capital markets, the equilibrium financing costs depend on the technology, and affect the equilibrium technology choice. In the rest of the analysis, we focus on symmetric salvage rates (\( \gamma_F = \gamma_D \)).\(^3\) We further restrict our analysis to the equilibria where

\(^3\)We discuss the implications of a higher salvage value of flexible technology on equilibrium technology choice in §5.3.
In imperfect capital markets, the technology choice in equilibrium depends on two factors: the capacity-pooling value of the flexible technology and the relative magnitude of \( \dot{a}_D(c_D) \) and \( \dot{a}_F(c_F) \). To understand the intuition behind Panel A, we note that if the financing cost under the two technology choices were identical (\( \dot{a}_D(c_D) = \dot{a}_F(c_F) \)), then the firm would be indifferent in imperfect capital markets as well for \( c_F = \bar{c}_F(c_D) \) (Lemma B.2). Therefore, the ordering of the financing cost in equilibrium determines the technology choice in imperfect capital markets. To analyze the ordering of the financing costs, we write the expected return of the creditor with both

\[ c_F' = \frac{c_D + \bar{c}_F(c_D; \rho, \sigma)}{2} \]

(Panel B) at equilibria where the firm uses a secured loan with default possibility in imperfect capital markets (in a perfectly competitive credit market with \( U = 0 \)) with \( B = 20, \ BC = 200, \ \gamma_D = \gamma_F = 0.1, \ c_D = 2, \ P = 650, \ b = -2, \ \bar{\xi}_1 = \bar{\xi}_2 = 20, \rho \in \{-0.9995, -0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75, 0.9995\} \), and \( \sigma \in [2, 5] \) with 0.1-unit increments.

Figure 4: The effect of the demand variability (\( \sigma \)) and the demand correlation (\( \rho \)) on the equilibrium technology choice with the technology cost pairs \( (c_D, \bar{c}_F(c_D; \rho, \sigma)) \) (Panel A) and \( (c_D, c_F') \) where \( c_F' = \frac{c_D + \bar{c}_F(c_D; \rho, \sigma)}{2} \) (Panel B) at equilibria where the firm uses a secured loan with default possibility with each technology in imperfect capital markets (in a perfectly competitive credit market with \( U = 0 \)) with \( B = 20, \ BC = 200, \gamma_D = \gamma_F = 0.1, \ c_D = 2, \ P = 650, \ b = -2, \ \bar{\xi}_1 = \bar{\xi}_2 = 20, \rho \in \{-0.9995, -0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75, 0.9995\} \), and \( \sigma \in [2, 5] \) with 0.1-unit increments.

\footnote{We discuss the technology choice at equilibria where the firm uses a secured loan without default possibility in \S 5.1.}
technologies:

$$\Lambda_F(a_F) = (c_F K^1_F - B) a_F - BC Pr \left\{ \left( \xi_1^{-b} + \xi_2^{-b} \right)^{-\frac{1}{b}} < \frac{(1 + a_F)(c_F K^1_F - B) - \gamma_F c_F K^1_F}{(K^1_F)^{(1 + \frac{1}{b})}} \right\},$$

$$\Lambda_D(a_D) = (2c_D K^1_D - B) a_D - BC Pr \left\{ \left( \frac{\xi_1 + \xi_2}{2(1 + \frac{1}{b})} \right)^{-\frac{1}{b}} < \frac{(1 + a_D)(2c_D K^1_D - B) - \gamma_D 2c_D K^1_D}{(2K^1_D)^{(1 + \frac{1}{b})}} \right\}.$$ 

Here, the first term in each expression is the net gain from secured lending, and the second term is the expected default cost. Suppose that the creditor offers identical financing costs for each technology. We can show that the total investment cost is identical, so the firm borrows the same amount and the net gain from secured lending is identical. It follows that the creditor would charge a lower financing cost when the default probability is lower. We now analyze the default probability. Since the firm borrows the same amount, the face value of the loan and the salvage value of the capacity are also identical with each technology (numerator of the right hand side of the default probability). Therefore, the default risk is determined by the total capacity investment available to generate revenue (denominator of the right-hand side of the probability) and the effectiveness of the firm in generating revenue for each unit of total capacity at any $\bar{\xi}$ (left-hand side of the default probability). The latter term captures the financial-pooling (diversification) benefit of the dedicated technology and the financial- and the capacity-pooling benefits of the flexible technology.

We can show that the firm can always generate higher returns per unit capacity level with the flexible technology due to the capacity-pooling benefit, i.e. $\left( \xi_1^{-b} + \xi_2^{-b} \right)^{-\frac{1}{b}} \geq \left( \frac{\xi_1 + \xi_2}{2(1 + \frac{1}{b})} \right)^{-\frac{1}{b}}$ for $\xi > 0$ with equality holding only for $\rho = 1$. All else being equal, this would lead to a lower financing cost with the flexible technology. We can also show that the total capacity investment is higher with the dedicated technology, i.e. $2K^1_D \geq K^1_F$ with equality holding only for $\rho = 1$. All else being equal, this would lead to a lower financing cost with the dedicated technology. In summary, the ordering of the financing cost in equilibrium is determined by the trade-off between the capacity-pooling benefit of the flexible technology and the higher total capacity investment made under the dedicated technology.

At very high correlation levels ($\rho \approx 1$), we can prove that the second effect dominates the first and the dedicated technology is strictly preferred in equilibrium (Proposition B.7). When demand is highly negatively correlated, the financial-pooling benefit is very high for each technology. When this results in the firm not defaulting with either technology we have $\dot{a}_F = \dot{a}_D$ and the firm is indifferent between them as seen in Figure 4. In between these two extremes, the first effect dominates and flexible technology is preferred. Note that the threshold value of $\sigma$ below which the firm is indifferent decreases in $\rho$. This is because
the financial-pooling benefit decreases as $\rho$ increases, so the variability in demand needs to be lower to avoid default. As bankruptcy cost decreases, the effect of the expected default cost term diminishes, and the equilibrium financing costs equalize in the limit such that the firm is indifferent between the two technologies.

In the finance literature, it is argued that operational flexibility decreases the firm’s default risk by generating higher returns due to its option value (see, for example, MacKay 2003). Our analysis shows that this argument may not hold in general with a stronger formalization of the firm’s operations. Anticipating the option value of operational flexibility (flexible technology in our case), the firm optimally adjusts other operational decisions (capacity investment level). As a result, operational flexibility may increase the default risk of the firm. While we prove this at high correlation levels under the normal distribution assumption, this is not a necessary condition in general.

In Panel B, $c_F \in [c_D, \tilde{c}_F(c_D)]$ and the firm prefers the flexible technology in perfect capital markets. Here, if the financing cost under the two technology choices were identical ($\hat{a}_F(c_D) = \hat{a}_F(c_F)$), then the firm would prefer the flexible technology in imperfect capital markets as well (Lemma B.2). If $\hat{a}_F(c_F) < \hat{a}_D(c_D)$, then the flexible technology is even more desirable. The dedicated technology can only be preferred if $\hat{a}_D(c_D)$ is sufficiently small relative to $\hat{a}_F(c_F)$, i.e., the relative magnitudes (and not just the ordering) of the financing costs matter. To understand what drives technology choice for $c_F < \tilde{c}_F(c_D)$, we reason based on $\Lambda_T(a_T)$ again as follows: The first term in $\Lambda_T(a_T)$ is the net gain from secured lending. We can show that if the financing costs were identical, the total investment cost would be strictly higher with the flexible technology, the firm would borrow more, and the net gain from secured lending would be higher. Thus, all else being equal, a lower financing cost would be offered for the flexible technology. The second term in $\Lambda_T(a_T)$ is the expected default cost. The relative default risk is determined by three effects: $i)$ the face value of the loan net of the salvage value of the capacity is higher under the flexible technology since the firm borrows more; $ii)$ there exists a capacity-pooling benefit hence the firm can generate higher revenues for each unit of capacity investment; $iii)$ the total capacity available to generate revenues is lower (higher) with sufficiently high (low) $c_F$. For sufficiently high $c_F$, all else being equal, $(i)$ and $(iii)$ would result in a higher financing cost with $F$, whereas $(ii)$ would result in a lower cost. In contrast, for a sufficiently low $c_F$, $(i)$ would result in a higher financing cost with $F$ whereas $(ii)$ and $(iii)$ would result in a lower cost. The combined effect of the four factors we discussed is indeterminate. In our numerical examples, we observe both $\hat{a}_D < \hat{a}_F$ and $\hat{a}_D > \hat{a}_F$, but the net effect is that flexible technology is preferred for
a larger set of \((\rho, \sigma)\) values relative to the \(c_F = c_D^P\) case; the indifference region shrinks. The dedicated technology can be preferred, but only at very high correlation levels, because here \(\dot{a}_D\) does not need to be much lower than \(\dot{a}_F\) to overcome the low level of capacity-pooling benefit of the flexible technology. This can happen even when the costs of the two technologies are identical \((c_F = c_D)\), which maximizes the advantage of flexible technology.

MacKay (2003) empirically demonstrates that the production flexibility of the firm may induce the creditor to tighten the credit terms. This is because the creditor may believe that shareholders will use production flexibility to increase the riskiness of the firm’s cash flows. In our model, we observe that \(\dot{a}_F\) may be higher than \(\dot{a}_D\). This happens entirely due to the expected default cost portion of the creditor’s return as follows from the above discussion, which is consistent with the empirical findings of MacKay (2003).

In the monopolist creditor case, we observe that above a threshold on correlation and variability that is similarly downward sloping, the flexible technology is preferred, except when \(\rho \approx 1\), where the dedicated technology can be preferred. The biggest difference that we observe is that the dedicated technology can be preferred at low variability and negative enough correlation. This is because the equilibrium financing cost is lower with the dedicated technology at these instances.

5 Discussion of Assumptions and Extensions

In this section, we present some extensions to our model and discuss the robustness of our results to some of our assumption. For brevity, we only discuss the selected results qualitatively. The formal analysis is relegated to §C of the Technical Appendix.

5.1 Liquid Physical Assets \(P\)

In this section, we analyze the case where the physical assets \(P\) of the firm can be liquidated immediately. This means that defaults occurs only when the firm uses an unsecured loan. Therefore, in Tables 1, 2 and 3, the results that refer to the equilibria where the firm uses a secured loan with default possibility do not exist. In the single-product firm analysis, the other results in Tables 1 and 2 continue to hold with one exception. With the monopolist creditor, an increase in the demand variability may increase \(\dot{K}\) and \(\dot{\pi}\) at equilibria where the firm uses a secured loan. This happens when the equilibrium financing cost switches from one local maximizer (in the secured lending region) to the other one (in the unsecured lending region).

In the two-product firm analysis, if we continue to assume a sufficiently large \(P\), the firm always optimally uses a secured loan for a given financing cost and the results in Table 3 that
refer to equilibria where the firm uses a secured loan without default possibility continue to hold. On the equilibrium technology choice, in a perfectly competitive credit market (with $U \geq 0$), we can prove $\hat{a}_F \leq \hat{a}_D$ for $c_F \in [c_D, \tau^P_F(c_D)]$ where the equality only holds for $c_F = \tau^P_F(c_D)$. Therefore, the equilibrium technology choice is identical to the perfect capital market benchmark case. With the monopolist creditor, we can prove $\hat{a}_D \leq \hat{a}_F$ where the equality only holds for $c_F = \tau^P_F(c_D)$. Therefore, the equilibrium technology choice is determined by the trade-off between the lower financing cost of the dedicated technology and the capacity-pooling benefit of the flexible technology.

5.2 Unsecured Lending in the Two-Product Firm Analysis

In the two-product firm analysis, we have assumed a sufficiently large $P$ such that the firm uses a secured loan in equilibrium. In this section, we relax this assumption and focus on the equilibria where the firm uses an unsecured loan with each technology. Since we cannot guarantee the unimodality of the firm’s problem with bivariate normal demand uncertainty, we resort to numerical experiments. We use the same parameter sets as before except we set $P = 0$.\footnote{Another implication of the $P = 0$ choice is that the results of this section are also relevant for §5.1 in which we relax the liquid physical asset assumption.} We focus on a perfectly competitive credit market with $U = 0$.

With a secured loan, the impact of demand uncertainty with the dedicated technology is determined by its impact on the equilibrium financing cost. With an unsecured loan, it is determined by the change in the value of the limited liability option of the firm in addition to the equilibrium financing cost. A higher $\sigma$ or $\rho$ increases the value of the limited liability option of the firm (for $\rho$ this happens due to a lower value of financial pooling). In our experiments, $\hat{a}_D$ increases in $\sigma$ and $\rho$. For the effect on $\hat{K}_D$, we observe that either effect may dominate, and unlike the secured loan case, $\hat{K}_D$ may increase or decrease. For the effect on $\hat{\pi}_D$, we observe in our experiments that the latter factor dominates and, similar to the secured loan case, $\hat{\pi}_D$ decreases (although an increase is not precluded under other conditions).

With a secured loan, the impact of demand uncertainty with the flexible technology is determined by its impact on the value of capacity pooling and the equilibrium financing cost. With an unsecured loan, in addition to these two factors, the impact of demand uncertainty is determined by how it changes the value of the limited liability option of the firm. A higher $\sigma$ increases the value of the limited liability option and the value of capacity pooling. In our experiments, $\hat{a}_F$ decreases in $\sigma$. Therefore, unlike the secured loan case, $\hat{K}_F$ and $\hat{\pi}_F$ increase. A higher $\rho$ increases the value of the limited liability option but decreases
the value of capacity pooling. In our experiments, we observe that $\dot{a}_F$ may decrease or increase. As a result, similar to the secured loan case, $\dot{K}_F$ may increase or decrease. Unlike the secured loan case, $\dot{\pi}_F$ may decrease, and this happens even when $\dot{a}_F$ decreases. In other words, the declining value of the capacity-pooling benefit may outweigh the increase in the value of the limited liability option as well as the impact of a lower financing cost.

5.3 The Equilibrium Technology Choice When the Flexible Technology Has a Higher Salvage Value

In perfect capital markets, as follows from Remark 2, a higher salvage value $\gamma_F > \gamma_D$ always favors the flexible technology over the dedicated technology because of the reduction in the capital investment risk. In imperfect capital markets, an increase in $\gamma_F$ from $\gamma_F = \gamma_D$ alters the equilibrium financing cost with the flexible technology. At a minimum, this leads to a higher collateral value of the flexible technology in the case of default, and this works to decrease the financing cost with the flexible technology in equilibrium. However, we cannot immediately conclude that for $\gamma_F > \gamma_D$, we have $\dot{a}_F < \dot{a}_D$ as the creditor’s expected return is non-trivially affected by the change in the optimal capacity level (which also depends on $\gamma_F$). Nevertheless, interesting observations can be made that result from the asymmetry in the salvage value of capacity with each technology. For example, we can show that when the product markets are perfectly positively correlated ($\rho = 1$), and when the flexible technology is not strictly preferred in perfect capital markets, it can be strictly preferred in imperfect capital markets. This is because the firm can secure a lower financing cost with the flexible technology in equilibrium due to a higher salvage rate.

6 Conclusion

This paper contributes to the stochastic capacity investment literature by relaxing the (often implicit) perfect capital market assumption and analyzing the impact of endogenous credit terms under capital market imperfections. Our results summarized in Tables 1, 2, 3 and Figure 4 provide insights on how stochastic capacity investment in imperfect capital markets differs from that in perfect capital markets.

In a single-product setting, we analyze the impact of the demand variability and the internal budget level on the capacity investment decision and the performance of the firm in equilibrium. In a two-product setting, we analyze the impact of demand uncertainty (variability and correlation) on the same, as well as the choice between flexible and dedicated technologies in equilibrium. Our single-product analysis contributes to the literature by providing new comparative statics results under different capital market conditions. Except
for a numerical analysis in Lederer and Singhal (1994), there is no formal treatment of the two-product case in the literature, therefore, the two-product analysis is a distinct contribution of our research.

We show that the comparative statics results in imperfect capital markets depend on the firm’s loan type in equilibrium (unsecured versus secured, with or without default possibility) and the capital market conditions. The existence of capital market imperfections coupled with the endogenous credit terms frequently modifies or reverses conclusions concerning capacity investment and technology choice that would be obtained under the perfect capital market assumption. These can be explained by the effect of the changing parameter on the creditor’s expected return, and in turn, on the equilibrium financing cost. These findings are consistent with the fundamental message of this paper (and of others that analyze stochastic capacity investment with financial considerations): namely, that there is real value in the integration of financial and operations decisions.

This paper brings constructs and assumptions motivated by the finance literature into a classical operations management problem and develops new insights. In turn, by a stronger formalization of operational decisions than in the finance literature (the sequential nature of technology choice, capacity investment and production decisions and the impact of demand uncertainty), we provide novel insights on issues discussed in this literature. For example, Melnik and Plaut (1986) derive several relations among the parameters of loan contracts based on the assumption that the borrowing level is independent of the unit financing cost and that the default probability increases in the unit financing cost. As argued in MacKay (2003), firms with higher operational flexibility are assumed to have a lower default risk due to the option value of operational flexibility. Our analysis demonstrates that these assumptions may not be valid with a more formal representation of operational decisions.

Our results provide the basis for potential empirical research in this domain. These include comparisons of the performance and the optimal capacity investment level of firms based on firm characteristics (firms using secured loan versus unsecured loan) and the competitiveness of the credit market (perfectly competitive versus monopoly). While a formal development of empirical hypotheses is beyond the scope of this paper, the following predictions of our model would be interesting to explore empirically. From the single-product firm analysis, we have:

1. For firms with low internal budget levels, the capacity investment level is higher for firms that use an unsecured loan than firms that use a secured loan.

2. When the competition in the credit market is high, the higher the demand variabil-
ity, the lower (higher) will be the capacity investment in firms that use a secured (unsecured) loan.

3. When the competition in the credit market is high, the higher the internal budget level, the lower (higher) will be the capacity investment (performance) of firms that use an unsecured loan.

From the two-product firm analysis, we can hypothesize the following:

1. Unless product demands are highly negatively correlated, the higher the demand variability, the lower the capacity investment level will be, regardless of the firm’s technology choice and the competitiveness of the credit market.

2. When the product demands are negatively (positively) correlated, the higher the demand variability, the lower (higher) will be the capacity investment and performance of firms that use flexible technology.

3. The higher the competitiveness of the credit market, the more prevalent flexible technology choice will be. This prevalence will be more pronounced when the product demands are negatively correlated.

There are a number of limitations to the present study that lead to open research questions. First, we focus on a particular type of financing contract and only a few types of capital market imperfections. The firm can also issue equity or raise external capital by other forms of loan contracts, or the firm may be exposed to other capital market imperfections such as financial distress cost, taxes, agency costs etc. As the different capital market imperfections examined here show, the operational implications are expected to be model-specific.

Among these other market imperfections, agency costs arising from asymmetric information between the creditor and the firm are worth discussing. In our model, we assume that the creditor has perfect information about the firm. In reality, the creditor may not have perfect information about the risk-profile of the operational investments, nor be able to perfectly monitor the firm after the loan is taken, nor have the same valuation of the collateralized assets as the firm. Each of these would create agency costs and impose additional financing frictions. Our analysis provides partial answers for this case. For example, if there is asymmetric information, and if there is no signalling by the firm or screening by the creditor, the creditor would offer identical financing costs for each technology if they have symmetric salvage rates. In this case, as we discussed in §4.3, the technology choice
in imperfect capital markets is identical to the technology choice in perfect capital markets. With a proper modeling of the interaction between the creditor and the firm under asymmetric information, new trade-offs and new implications will arise as discussed in, for example, Brunet and Babich (2009).

Relaxing the assumptions we made on the production environment gives rise to a number of interesting possibilities, both in the theory of capacity management and integrated risk management. For example, we assume that second stage production is costless. With a positive production cost, the optimal production decision is limited by the cash availability of the firm (financial capacity constraint) in addition to the physical capacity constraint. This brings an additional facet to the problem: The allocation of the financial capacity between the two stages; and between the products at stage 2 in a two-product setting. We also assume that the internal budget of the firm is deterministic. This budget may depend on some economic factors and can be random. Moreover, if the internal budget depends on a tradable asset, then the firm can engage in financial risk management to engineer the budget as discussed in Froot et al (1993). The optimal technology choice (flexible versus dedicated) together with the decision of engaging in financial risk management form the optimal integrated risk management portfolio of the firm. In Boyabatlı and Toktay (2010), we analyze the effect of budget variability and financial risk management on the stochastic capacity investment problem with a more detailed model of firm’s production environment (that includes positive production cost and engaging in financial risk management) with hard financial constraints (no borrowing). It would be interesting to analyze the impact of endogenous credit terms on the integrated risk management portfolio of the firm (that consists of financial risk management and flexible versus dedicated technology choice).

We assume a stylized firm that is liquidated at the end of a single period. Li et al. (2003) model the dynamic capital structure choice of the firm without bankruptcy. Extending our analysis to a multi-period setting is certainly a non-trivial task and requires further research. Besides the usual operational dynamics of multi-period models (capacity and inventory carryover), there also exist additional dynamics coming from capital carryover: The firm may decide to hold some of the earlier loans as internal capital for future expenditures. In addition, the indirect costs of bankruptcy should be incorporated because the financing cost in equilibrium depends on the earlier loan performance of the firm.

Acknowledgement. We thank Dang Quang (Jason) Nguyen of Singapore Management University (SMU) for his assistance in the numerical experiments. Financial support from SMU under project fund C207-SMU-015 is acknowledged by the first author. We are grateful
to our three referees and the associate editor for their invaluable comments that helped us to improve the content of the paper considerably.

References


This is the technical appendix “Stochastic Capacity Investment and Flexible versus Dedicated Technology Choice in Imperfect Capital Markets.” In this note, we provide the proofs for the technical statements in the paper as well as the detailed analysis of the summary results presented in the paper. The outline of this appendix is similar to the main paper. §A illustrates the single-product firm analysis, whereas §B provides the two-product firm analysis. Table 4 summarizes the decision variables by stage. Table 5 summarizes all the other notation. Superscript * is appended to denote the optimal value; ′ is appended to denote the equilibrium level.

<table>
<thead>
<tr>
<th>Decision Maker</th>
<th>Stage</th>
<th>Name</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Creditor</td>
<td>Stage 0</td>
<td>( a_T )</td>
<td>Unit financing cost for technology ( T )</td>
</tr>
<tr>
<td>Firm</td>
<td>Stage 1</td>
<td>( T \in {D,F} )</td>
<td>Technology choice, dedicated or flexible</td>
</tr>
<tr>
<td></td>
<td>Stage 1</td>
<td>( e_T )</td>
<td>Borrowing level with technology ( T )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( K_T )</td>
<td>Capacity investment level with technology ( T )</td>
</tr>
<tr>
<td></td>
<td>Stage 2</td>
<td>( Q_T )</td>
<td>Production quantity with technology ( T )</td>
</tr>
</tbody>
</table>

Table 4: Decision variables by stage: Since the unit financing cost is determined before the firm makes any decisions, we denote that as “Stage 0.”
Table 5: Summary of Notation: The notation related to the single-product firm analysis is obtained by removing the subscript $T$.

### A The Single-Product Firm Analysis

#### A.1 Analysis of The Firm’s Problem for A Given Financing Cost

**Proof of Proposition 1:** To prove the result, we first demonstrate the optimal capacity investment for the benchmark case in which the firm is liable from negative cash flows, i.e. the firm does not have limited liability option.

**Lemma A.1** If the firm does not have the limited liability option, then the unique optimal capacity investment level $K^*$ is given by

\[
K^* = \begin{cases} 
K^0 &= \left( \frac{\xi(1+\frac{1}{b})}{(1-\gamma)c} \right)^{-b} 
& \text{if } B \geq cK^0 \\
\frac{B}{c} &= \text{if } cK^1 \leq B < cK^0 \\
K^1 &= \left( \frac{\xi(1+\frac{1}{b})}{(1+a-\gamma)c} \right)^{-b} 
& \text{if } B < cK^1
\end{cases}
\]
Proof of Lemma A.1: Without limited liability, the optimal expected equity value of the firm, \( \pi^* \), is given by:

\[
\pi^* = \max_K \xi K^{(1 + \frac{1}{b})} + \gamma cK + P + B - cK - a (cK - B)^+ \tag{7}
\]

subject to \( K \geq 0 \).

Let \( g(K) \) and \( K^* \) be the objective function and the optimal capacity investment for (7), respectively. As the form of \( g(K) \) depends on whether the firm borrows or not, we will analyze two sub-problems separately, and then put them together. Sub-problem 1 (2) is the restriction of the problem to the no borrowing (borrowing) regions. Let \( g_i(K) \) denote the objective function and \( K_i^* \) be the optimal solution of sub-problem \( i \). We have:

\[
g(K) = \begin{cases} 
g^1(K) = \xi K^{(1 + \frac{1}{b})} + B + P - c (1 - \gamma) K & \text{if } B \geq cK \\
g^2(K) = \xi K^{(1 + \frac{1}{b})} + B(1 + a) + P - c (1 + a - \gamma) K & \text{if } B < cK \end{cases}
\]

The remainder of the proof has the following structure:

1. We show that \( g_i(K) \) is strictly concave and solve each sub-problem \( i \) for \( K_i^* \).

2. We show that \( g(K) \) is strictly concave; and hence \( K^* \) is unique and characterized by \( K_0^* \) or \( K_1^* \).

1. Derivation of \( K_i^* \)

The first- and second-order conditions with respect to \( g^i(K) \) are

\[
\frac{\partial g^i(K)}{\partial K} = \left(1 + \frac{1}{b}\right) \xi K^{\frac{1}{b}} - c (1 + a^i - \gamma),
\]

\[
\frac{\partial^2 g^i(K)}{\partial K^2} = \frac{1}{b} \left(1 + \frac{1}{b}\right) \xi K^{\frac{1}{b} - 1},
\]

where \( a^1 = 0 \) and \( a^2 = a \). With \( b < -1 \), it follows that \( \frac{\partial^2 g^i}{\partial K^2} < 0 \) for \( K \geq 0 \) and \( g^i(K) \) is strictly concave for \( i = 1, 2 \). Since \( B > 0 \), sub-problem \( i \) always has a non-empty feasible region, thus the optimal solution for \( g^i(K) \) is either the solution of \( \frac{\partial g^i(K)}{\partial K} = 0 \),

\[
K_i^* = \left(\frac{\xi^{(1 + \frac{1}{b})}}{(1 + a_i - \gamma)c}\right)^{-b}, \text{ or is a boundary solution. Since } \lim_{K \to 0^+} \frac{\partial g^i(K)}{\partial K} \rightarrow \infty, \text{ the non-negativity constraint is never binding. For } i = 1, \text{ if } \left|\frac{\partial g^1(K)}{\partial K}\right|_{K^*} > 0, \text{ then } K_0^* = \frac{B}{c}.
\]
Similarly, if \( \frac{\partial g^2(K)}{\partial K} \bigg|_{\frac{B}{c}} < 0 \), then \( K_1^* = \frac{B}{c} \). To summarize, \( K_i^* \) is characterized by

\[
K_0^* = \begin{cases} 
  K^0 = \left( \frac{\xi(1+\frac{1}{\gamma})}{(1-\gamma)c} \right)^{-b} & \text{if } B \geq cK^0, \\
  \frac{B}{c} & \text{if } B < cK^0
\end{cases}
\]

\[
K_1^* = \begin{cases} 
  \frac{B}{c} & \text{if } cK^1 \leq B \\
  K^1 = \left( \frac{\xi(1+\frac{1}{\gamma})}{(1+a-\gamma)c} \right)^{-b} & \text{if } B < cK^1
\end{cases}
\]

Here, \( K^0 \) is budget-unconstrained optimal capacity investment level, and \( K^1 \) is the credit-unconstrained optimal capacity investment level.

2. Derivation of \( K^* \)

From the previous part, we have shown that \( g(K) \) is piecewise strictly concave. It is easy to establish that \( g^1(K) \bigg| \frac{B}{c}^- = g^2(K) \bigg| \frac{B}{c}^+ \), and \( g(K) \) is continuous at \( K = \frac{B}{c} \). Thus, to show that \( g(K) \) is strictly concave, we need to show that \( \frac{\partial g^1(K)}{\partial K} \bigg|_{\frac{B}{c}^-} \geq \frac{\partial g^2(K)}{\partial K} \bigg|_{\frac{B}{c}^+} \). We obtain

\[
\frac{\partial g^1(K)}{\partial K} \bigg|_{\frac{B}{c}^-} = \left( 1 + \frac{1}{b} \right) \xi \left( \frac{B}{c} \right) \left( \frac{1}{\gamma} \right) - c (1 - \gamma),
\]

\[
\frac{\partial g^2(K)}{\partial K} \bigg|_{\frac{B}{c}^+} = \left( 1 + \frac{1}{b} \right) \xi \left( \frac{B}{c} \right) \left( \frac{1}{\gamma} \right) - c (1 + a - \gamma).
\]

For \( a \geq 0 \), \( \frac{\partial g^1(K)}{\partial K} \bigg|_{\frac{B}{c}^-} \geq \frac{\partial g^2(K)}{\partial K} \bigg|_{\frac{B}{c}^+} \); hence \( g(K) \) is strictly concave in \( K \) for \( K \geq 0 \). Combining (8) and (9), the unique optimal solution to problem (7) is given by

\[
K^* = \begin{cases} 
  K^0 = \left( \frac{\xi(1+\frac{1}{\gamma})}{(1-\gamma)c} \right)^{-b} & \text{if } B \geq cK^0, \\
  \frac{B}{c} & \text{if } cK^1 \leq B < cK^0 \\
  K^1 = \left( \frac{\xi(1+\frac{1}{\gamma})}{(1+a-\gamma)c} \right)^{-b} & \text{if } B < cK^1
\end{cases}
\]

With limited liability, when the firm borrows \((K > \frac{B}{c})\), we have

\[
l(K) \doteq \frac{1}{\xi} \left( 1 + a - \gamma \right) c - K^{\left( -1 - \frac{1}{\gamma} \right)} \left( B(1 + a) + P \right)
\]

such that the firm is able to pay back the face value of the loan if and only if \( \hat{\xi} \) is no less than \( l(K) \). For \( \hat{\xi} > l(K) \), the optimal equity value \( \Pi^* > 0 \), and for \( \hat{\xi} \leq l(K) \), \( \Pi^* = 0 \). We
obtain
$$\frac{\partial l(K)}{\partial K} = -\frac{1}{b} K^{(-\frac{1}{b}-1)} (1 + a - \gamma) c + \left(1 + \frac{1}{b}\right) K^{(-2-\frac{1}{b})} [B (1 + a) + P] > 0. \quad (10)$$

Therefore, we can identify the unique $K^l < K^u$ such that $l(K^l) \doteq \xi^l$ and $l(K^u) \doteq \xi^u$. Since $l(K)$ is strictly increasing in $K$, we have $l(K) \geq \xi^u$ for $K \geq K^u$; hence the optimal equity value $\Pi^* = 0$ at each $\xi$ and the expected optimal equity value $\pi^* = 0$ for $K \in [K^u, \infty)$. Therefore, it is sufficient to analyze the problem for $K \in [0, K^u)$.

We have three separate cases to consider:

**Case 1:** For $K \in \left[0, \frac{B}{c}\right]$, similar to the no limited liability case, the firm does not borrow, and the expected equity value of the firm is
$$\pi^* = \max_K \bar{\xi} K^{(\bar{\xi} + \frac{1}{b})} + B + P - c (1 - \gamma) K.$$ 

**Case 2:** For $K \in \left[\frac{B}{c}, K^l\right]$, similar to the no limited liability case, the firm optimally borrows, and is always able to pay back the face value of the loan.¹ The expected equity value of the firm is
$$\pi^* = \max_K \bar{\xi} K^{(\bar{\xi} + \frac{1}{b})} + B (1 + a) + P - c (1 + a - \gamma) K.$$ 

**Case 3:** For $K \in (K^l, K^u)$ the firm always borrows, and for some demand realization $\bar{\xi}$, it is not able to pay back the face value of the loan; hence the expected equity value of the firm is
$$\pi^* = \max_K \int_{l(K)}^{\xi_u} \left[\xi K^{(\xi + \frac{1}{b})} + B (1 + a) + P - c (1 + a - \gamma) K\right] f(\xi) d\xi.$$ 

Let $g(K)$ denote the objective function in the overall optimization problem and $g^i(K)$ denote the objective function in case $i$. We have

$$g(K) = \begin{cases} 
  g^1(K) = \bar{\xi} K^{(\bar{\xi} + \frac{1}{b})} + B + P - c (1 - \gamma) K & \text{if } K \in \left[0, \frac{B}{c}\right] \\
  g^2(K) = \bar{\xi} K^{(\bar{\xi} + \frac{1}{b})} + B (1 + a) + P - c (1 + a - \gamma) K & \text{if } K \in \left(\frac{B}{c}, K^l\right] \\
  g^3(K) = \int_{l(K)}^{\xi_u} \left[\xi K^{(\xi + \frac{1}{b})} + B (1 + a) + P - c (1 + a - \gamma) K\right] f(\xi) d\xi & \text{if } K \in (K^l, K^u) 
\end{cases}.$$ 

It is easy to establish that $g(K)$ is continuous at the boundaries $K = \frac{B}{c}$ and $K = K^l$; and hence $g(K)$ is continuous in $K$. It follows from Lemma A.1 that $g(K)$ is strictly concave in $K$ for $K \in \left[0, K^l\right]$ and has a kink at $K = \frac{B}{c}$. We obtain
$$\frac{\partial g^3(K)}{\partial K} = \int_{l(K)}^{\xi_u} \left[\left(1 + \frac{1}{b}\right) \xi K^{(\xi + \frac{1}{b})} - (1 + a - \gamma) c\right] f(\xi) d\xi. \quad (11)$$

¹It can be shown that for $\xi^l \geq 0$ and $\gamma \geq 0$, $K^l \geq \frac{B}{c}$, where the equality only holds if $\xi^l = 0$ and $\gamma = 0$. 

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It is easy to verify that \( \frac{\partial^2 g(K)}{\partial K^2} \bigg|_{K^l} = \frac{\partial^3 g(K)}{\partial K^3} \bigg|_{K^l} \); hence \( g(K) \) does not have a kink at \( K = K^l \). Define \( G(K, \xi) = (1 + \frac{1}{b}) \xi K^{(\xi)} - (1 + a - \gamma) c \) as the integrand of (11) (without the density function). Note that \( G(K, \xi) \) is increasing in \( \xi \), and decreasing in \( K \). The second order condition of \( g^3(K) \) is given by

\[
\frac{\partial^2 g^3(K)}{\partial K^2} = \int_{l(K)}^{\xi_u} \left[ \frac{1}{b} \left( 1 + \frac{1}{b} \right) \xi K^{(\xi) - 1} \right] f(\xi) d\xi - \frac{\partial l(K)}{\partial K} G(K, l(K)) f(l(K)).
\]

Note that the first term is negative and we obtain \( G(K, l(K)) = -\frac{B(1+a)+P}{K} < 0 \); hence the second term is positive. Therefore, the concavity of \( g^3(K) \) is not obvious. We will use the unimodality property of \( g(K) \) in the rest of the proof.

We define \( \hat{K} = \left( \frac{\xi_u (1 + \frac{1}{b})}{(1 + a - \gamma)c} \right)^{-b} \). We have \( l(\hat{K}) = (1 + \frac{1}{b}) \xi_u \left[ 1 - \frac{B(1+a)+P}{K(1+a-\gamma)c} \right] < \xi_u \), thus \( \hat{K} < K^u \) and is in the feasible region of \( K \). Note that \( G(\hat{K}, \xi_u) = 0 \) and \( G(\hat{K}, \xi) < 0 \) for \( \xi \in [l(\hat{K}), \xi_u] \) (as \( G(K, \xi) \) is strictly increasing in \( \xi \)). Therefore \( \frac{\partial g^3(K)}{\partial K^3} \bigg|_{\hat{K}} < 0 \). Since \( G(K, \xi) \) is strictly decreasing in \( K \), \( \frac{\partial g^3(K)}{\partial K^3} < 0 \) for \( K \in [\hat{K}, K^u] \).

In summary, \( g(K) \) is strictly concave in \( K \) for \( K \in [0, K^l] \) (with a kink at \( K = \frac{B}{c} \)), and is strictly decreasing in \( K \) for \( K \in [\hat{K}, K^u] \). It follows that \( g(K) \) will be unimodal if \( K^l \geq \hat{K} \). Since \( \frac{\partial g(K)}{\partial K} > 0 \) (from (10)), this is equivalent to \( l(\hat{K}) \leq \xi_l \), which gives us

\[
B \geq B^h = c\hat{K} \left[ 1 - \frac{\xi_l}{\xi_u (1 + \frac{1}{b})} \right] \left[ 1 - \frac{\gamma}{1 + a} \right] - \frac{P}{1 + a}.
\]

For \( B \geq B^h \), the optimal \( K^\ast \) is in the strictly concave part (where the firm uses a secured loan) and is unique. \( K^\ast \) is identical to Lemma A.1. ■

**Proof of Proposition 2:** Throughout the proof we will focus on the cases that we demonstrated in the proof of Proposition 1 with the same notation. In the proof of Proposition 1, we already established that the stage-1 objective function \( g(K) \) is strictly concave in \( K \) for \( K \in [0, K^l] \) and strictly decreasing in \( K \) for \( K \in [\hat{K}, K^u] \). We obtain

\[
\frac{\partial g(K)}{\partial K} \bigg|_{K^l} = \frac{\partial g^2(K)}{\partial K^2} \bigg|_{K^l} = \frac{\partial g^3(K)}{\partial K^3} \bigg|_{K^l} = \left( 1 + \frac{1}{b} \right) \xi \left( K^l \right)^{1/b} - (1 + a - \gamma) c
\]

\[
= \left( 1 + \frac{1}{b} \right) \xi \left( \frac{K^l}{K^l} \right)^{1/b} \left[ 1 - \left( \frac{K^l}{K^l} \right)^{-\frac{1}{b}} \right].
\]

where \( K^l = \left( \frac{\xi (1 + \frac{1}{b})}{(1 + a - \gamma)c} \right)^{-b} \). It follows that \( \frac{\partial g(K)}{\partial K} \bigg|_{K^l} > 0 \) if and only if \( K^l < K^1 \). In this case \( g(K) \) is strictly increasing for \( K \in [0, K^l] \) and strictly decreasing in \( K \) for \( K \in [\hat{K}, K^u] \).
Since \( g(K) \) is continuous in \( K \), there exists at least one \( K^* \in (K^l, K^u) \) such that \( \frac{\partial g(K)}{\partial K} |_{K^*} = 0 \). We cannot guarantee the uniqueness of \( K^* \) in this case. Since \( \frac{\partial l(K)}{\partial K} > 0 \) (from (10)), \( K^l < K^1 \) is equivalent to \( l(K^1) > \xi^1 \), which gives us
\[
B < B^l \doteq cK^1 \left[ 1 - \frac{\xi^1}{\xi (1 + \frac{1}{b})} \right] \left[ 1 - \frac{\gamma}{1 + a} \right] - \frac{P}{1 + a}.
\]

To prove that \( K^* \in (K^1, K^u) \), it is sufficient to show that \( \frac{\partial g(K)}{\partial K} > 0 \) for \( K \in (K^l, K^1) \). For \( K > K^1 \), as follows from (11) we have
\[
\frac{\partial g(K)}{\partial K} = \frac{\partial g^3(K)}{\partial K} = \int_{l(K)}^{\xi^u} \left[ \left( 1 + \frac{1}{b} \right) \xi K^{(\frac{1}{b})} - (1 + a - \gamma) c \right] f(\xi)d\xi
\]
\[
= \left( 1 + \frac{1}{b} \right) K^{\frac{1}{b}} \int_{l(K)}^{\xi^u} \left[ \xi - \frac{(1 + a - \gamma)cK^{\frac{1}{b}}}{1 + \frac{1}{b}} \right] f(\xi)d\xi
\]
\[
= \left( 1 + \frac{1}{b} \right) K^{\frac{1}{b}} \int_{l(K)}^{\xi^u} \left[ \xi - \xi \left( \frac{K}{K^1} \right)^{-\frac{1}{b}} \right] f(\xi)d\xi.
\]

Let \( H(K) = \int_{l(K)}^{\xi^u} \left[ \xi - \xi \left( \frac{K}{K^1} \right)^{-\frac{1}{b}} \right] f(\xi)d\xi \). Note that, for \( K > K^l \), \( H(K) \) and \( \frac{\partial g^3(K)}{\partial K} \) have the same sign, so we can use \( H(K) \) to characterize the sign of \( \frac{\partial g^3(K)}{\partial K} \). Define \( M(K, \xi) = \xi - \xi \left( \frac{K}{K^1} \right)^{-\frac{1}{b}} \) as the integrand in \( H(K) \) (without the density function). We obtain \( M(K, l(K)) = K^{-\frac{1}{b}} \left[ \frac{(1 + a - \gamma)c}{(b + 1)} - \frac{B(1 + a + P)}{K} \right] < 0 \) since \( b < -1 \). As \( M(K, \xi) \) is strictly increasing in \( \xi \), \( M(K, \xi) < 0 \) for \( \xi \in [\xi^l, l(K)] \). Therefore, we have
\[
H(K) > \int_{\xi^l}^{\xi^u} \left[ \xi - \xi \left( \frac{K}{K^1} \right)^{-\frac{1}{b}} \right] f(\xi)d\xi = \xi \left[ 1 - \left( \frac{K}{K^1} \right)^{-\frac{1}{b}} \right].
\]

For \( K \leq K^1 \), we have \( \xi \left[ 1 - \left( \frac{K}{K^1} \right)^{-\frac{1}{b}} \right] \geq 0 \); and hence \( H(K) > 0 \) for \( K \in (K^l, K^1) \). This concludes the proof. □

**Proof of Proposition 3:** Throughout the proof we will focus on the cases that we demonstrated in the proof of Proposition 1 with the same notation. In the proof of Proposition 1, we already established that the stage-1 objective function \( g(K) \) is strictly concave in \( K \) for \( K \in [0, K^l] \) and strictly decreasing in \( K \) for \( K \in [\hat{K}, K^u] \). Also, as discussed in the proof of Proposition 2; for \( K \geq K^l \), we have \( \text{sgn} \left( \frac{\partial g^3(K)}{\partial K} \right) = \text{sgn}(H(K)) \) where \( H(K) = \int_{l(K)}^{\xi^u} \left[ \xi - \xi \left( \frac{K}{K^1} \right)^{-\frac{1}{b}} \right] f(\xi)d\xi \). Therefore we will focus on \( H(K) \) to prove the unimodality of \( g(K) \).

From integration by parts, we obtain
\[
H(K) = \int_{l(K)}^{\xi^u} \mathcal{F}(\xi)d\xi - \mathcal{F}(l(K)) \left[ K^{-\frac{1}{b}} \left( \frac{(1 + a - \gamma)c}{(b + 1)} - \frac{B(1 + a + P)}{K} \right) \right].
\]
Define \( \Delta(K) \equiv K^{-\frac{1}{b}} \left( \frac{(1+a-\gamma)c}{-b} + \frac{B(1+a)+P}{K} \right) \). We obtain

\[
\frac{\partial \Delta(K)}{\partial K} = \left( 1 + \frac{1}{b} \right) K^{-\left(\frac{1}{b}+1\right)} \left( \frac{(1+a-\gamma)c}{(b+1)^2} + \frac{B(1+a)+P}{K} \right)
\]

\[
= \left( 1 + \frac{1}{b} \right) K^{-1} \left( \frac{-b(b+2)}{(b+1)^2} (1+a-\gamma)c \right).
\]

Note that for \( K > K^l \), \( l(K) > \xi \geq 0 \); hence for \( b \geq -2 \) the second term is positive and

\[
\frac{\partial \Delta(K)}{\partial K} > 0 \text{ for } K > K^l.
\]

We have

\[
H(K) = \int_{l(K)}^{\xi_u} \overline{F}(\xi)d\xi - \overline{F}(l(K))\Delta(K)
\]

\[
= \overline{F}(l(K)) \left[ \int_{l(K)}^{\xi_u} \overline{F}(\xi)d\xi - \Delta(K) \right].
\]

As \( \Delta(K) \) is increasing in \( K \), and since \( \int_{l(K)}^{\xi_u} \overline{F}(\xi)d\xi \) is decreasing in \( K \) (which follows from Assumption 1), then for \( K > K^l \), \( H(K) \) can only change sign once, which is from positive to negative. It follows that \( g^3(K) \) can only change sign once, which is from positive to negative.

In summary, \( g(K) \) is

i) strictly concave in \( K \) for \( K \in [0, K^l] \) with a kink at \( K = \frac{B}{c} \),

ii) unimodal in \( K \) for \( K \geq K^l \), and is not kinked at \( K = K^l \),

iii) strictly decreasing in \( K \) for \( K \in [\hat{K}, K^u] \).

It follows that

1. If \( \left. \frac{\partial g^2(K)}{\partial K} \right|_{K^l} \leq 0 \), then \( \left. \frac{\partial g(K)}{\partial K} = \frac{\partial g^3(K)}{\partial K} \right|_{K^l} < 0 \) for \( K \geq K^l \), and \( g(K) \) is unimodal. \( K^* \) is unique and is characterized by the strictly concave part (similar to Proposition 1).

2. If \( \left. \frac{\partial g^2(K)}{\partial K} \right|_{K^l} > 0 \), then for \( K > K^l \), \( \left. \frac{\partial g(K)}{\partial K} = \frac{\partial g^3(K)}{\partial K} \right|_{K^l} \) is first positive and then changes sign once and then is negative, therefore \( g(K) \) is unimodal in \( K \). \( K^* \) is unique and is characterized by \( \left. \frac{\partial g^3(K^*)}{\partial K} \right| = 0 \) (or \( MP(K^*) = 0 \), as defined in \( MP(K) = \int_{l(K)}^{\xi_u} \left[ \frac{(1+\frac{1}{b})K}{K^{\frac{1}{b}}} - (1+a-\gamma)c \right] f(\xi)d\xi \)). Let \( \overline{K} \) denote the optimal solution in this case. From Proposition 2, we have \( \overline{K} \geq K^l \).
As it follows from the proof of Proposition 2, $\frac{\partial y^2(K)}{\partial K}|_{K^1} > 0$ is equivalent to $B < cK^1 \left[1 - \frac{\xi^l}{\xi(1 + b)} \right] \left[1 - \frac{\gamma}{1 + a} \right] - \frac{P}{1+a}$, and the optimal solution $K^*$ to the overall problem is given by

$$K^* = \begin{cases} 
K^0 = \left(\frac{\xi(1 + \frac{1}{b})}{(1 - \gamma)c}\right)^{-b} & \text{if } B \geq cK^0 \\
\frac{B - \gamma}{c} & \text{if } cK^1 \leq B < cK^0 \\
K^1 = \left(\frac{\xi(1 + \frac{1}{b})}{(1 + a - \gamma)c}\right)^{-b} & \text{if } cK^1 \leq B < cK^0 \\
\mathcal{K} & \text{if } 0 \leq B < cK^1 \left[1 - \frac{\xi^l}{\xi(1 + b)} \right] \left[1 - \frac{\gamma}{1 + a} \right] - \frac{P}{1+a}
\end{cases} \quad (12)$$

The optimal expected equity value of the firm with a given budget level $B$, $\pi^*(B)$, follows directly:

$$\pi^*(B) = \begin{cases} 
\frac{cK^0(1 - \gamma)}{-(b+1)} + B + P & \text{if } B \geq cK^0 \\
\left(\frac{B - \gamma}{c}\right)^{1+b} + \gamma B + P & \text{if } cK^1 \leq B < cK^0 \\
\frac{cK^1(1 + a - \gamma)}{-(b+1)} + B(1 + a) + P & \text{if } cK^1 \left[1 - \frac{\xi^l}{\xi(1 + b)} \right] \left[1 - \frac{\gamma}{1 + a} \right] - \frac{P}{1+a} \leq B < cK^1 \\
E\left[\xi K(1 + \frac{1}{b}) - (1 + a - \gamma)\mathcal{K} + B(1 + a) + P\right]^+ & \text{if } 0 \leq B < cK^1 \left[1 - \frac{\xi^l}{\xi(1 + b)} \right] \left[1 - \frac{\gamma}{1 + a} \right] - \frac{P}{1+a}
\end{cases}$$

A.2 Characterization of the Creditor’s Expected Return

**Proof of Proposition 4:** In (5), as follows from Proposition 3, $K^*(a)$ can take two forms, $K^1$ or $\mathcal{K}$. Moreover, the bankruptcy threshold $b(K^*(a))$ and the limited liability threshold $l(K^*(a))$ are strictly greater than $\xi^l$ for some $a$; hence in characterizing $\Lambda(a)$, we should consider these arguments.

We define $S^1(a) = cK^0 \left(1 - \gamma\right)^{-b} \left(1 - \frac{\xi^l}{\xi(1 + b)} \right) \frac{(1 + a - \gamma)(b+1)}{(1 + a)}$ such that for a given $a$, for $B \geq S^1(a)$, the firm borrows to invest in $K^*(a) = K^1(a)$ and does not default. $B \geq S^1(a)$ is equivalent to $b(K^1) \leq \xi^l$ such that the firm does not default. Hence, both the default cost and the expected loss due to the unsecured part of the loan are 0 in (5). Since the firm can generate some returns in stage 2 through salvaging $cK$ and the minimum demand realization is $\xi^l$, it is possible that the firm Borrows without default possibility. Notice that for $\gamma = 0$, and $\xi^l = 0$, $S^1(a) = cK^1$ and this case is not feasible, as the firm does not Borrow for $B \geq cK^1$.

Similarly, we define $S^2(a) = S^1(a) - \frac{P}{(1+a)}$ such that for $S^1(a) > B \geq S^2(a)$, the firm borrows to invest in $K^*(a) = K^1(a)$ and defaults in some demand realization but the
creditor always retrieves the face value of the loan by liquidating physical assets. $B \geq S^2(a)$ is equivalent to $l(K^1) \leq \xi^l$ such that the firm uses a secured loan to finance $K^1$. Hence, the default cost is strictly positive but the expected loss due to the unsecured part of the loan is 0 in (5).

For $B < S^2(a)$, the firm optimally borrows to invest in $K^*(a) = \mathcal{K}(a)$. In this case, as follows from Proposition 3, the firm uses an unsecured loan; hence both the default cost and the expected loss due to the unsecured part of the loan are strictly positive in (5).

In summary, for any given $a$, the ordering of $B$ and thresholds $S^1(a)$ and $S^2(a)$ determine the optimal borrowing level of the firm, and hence the form of the creditor’s expected return $\Lambda(a)$. We now analyze this ordering as a function of $a$ to characterize $\Lambda(a)$. We obtain

$$\frac{\partial S^1(a)}{\partial a} = cK^0 (1 - \gamma)^{-b} \left[ 1 - \frac{\xi^l}{\xi (1 + \frac{1}{b})} \right] \left( 1 + \frac{a - \gamma}{(1 + a)^2} \right) \left[ (1 + a) b + \gamma \right] < 0,$$

as $a \geq 0$, $b < -1$ and $\gamma \leq 1$. Since $S^1(a)$ is strictly decreasing for $a \in [0, a^{max})$, we can analyze the problem in two cases.

**Case 1:** $B \geq S^1(0) = cK^0(1 - \gamma) \left[ 1 - \frac{\xi^l}{\xi (1 + \frac{1}{b})} \right]$ As $S^1(a)$ is strictly decreasing, we have $B \geq S^1(a)$ (and hence $B > S^2(a)$ as $S^2(a) = S^1(a) - \frac{P}{1 + a}$), $\forall a \in [0, a^{max})$, and thus, the firm optimally borrows to invest in $K^1(a)$ without default possibility. Consequently, the creditor’s expected return is given by

$$\Lambda(a) = (cK^1 - B)a \quad 0 \leq a < a^{max}.$$

**Case 2:** $B < S^1(0) = cK^0(1 - \gamma) \left[ 1 - \frac{\xi^l}{\xi (1 + \frac{1}{b})} \right]$ In this case, we have to check for the ordering of $B$ and $S^2(a)$ to characterize $\Lambda(a)$. We obtain

$$S^2(0) = cK^0 (1 - \gamma)^{-b} \left[ 1 - \frac{\xi^l}{\xi (1 + \frac{1}{b})} \right] - P,$$

$$\frac{\partial S^2(a)}{\partial a} = cK^0 (1 - \gamma)^{-b} \left[ 1 - \frac{\xi^l}{\xi (1 + \frac{1}{b})} \right] \frac{(1 + a - \gamma)^b}{(1 + a)^2} \left[ (1 + a) b + \gamma \right] + \frac{P}{(1 + a)^2}$$

$$= \frac{1}{(1 + a)^2} \left[ P - cK^0 (1 - \gamma)^{-b} \left( 1 - \frac{\xi^l}{\xi (1 + \frac{1}{b})} \right) (1 + a - \gamma)^b \right] \left[ -b(1 + a) - \gamma \right].$$

Notice that $S^2(0)$ is positive (negative) if $P$ is less (greater) than $cK^0(1 - \gamma) \left[ 1 - \frac{\xi^l}{\xi (1 + \frac{1}{b})} \right]$. Since $(1 + a - \gamma)^b \left[ -b(1 + a) - \gamma \right]$ is strictly decreasing in $a$, for $P \geq cK^0(-b - \gamma) \left[ 1 - \frac{\xi^l}{\xi (1 + \frac{1}{b})} \right]$, we have $\frac{\partial S^2(a)}{\partial a} \geq 0$ for $a \geq 0$. For $P < cK^0(-b - \gamma) \left[ 1 - \frac{\xi^l}{\xi (1 + \frac{1}{b})} \right]$, there exists a unique $a$
such that $\frac{\partial S^2(a)}{\partial a} \leq 0$ for $a \leq a$ and $\frac{\partial S^2(a)}{\partial a} > 0$ for $a > a$. Since the signs of $S^2(0)$ and $\frac{\partial S^2(a)}{\partial a}$ depend on $P$, we have three subcases. Before analyzing them, we first present a Lemma that we will use throughout the rest of the proof.

**Lemma A.2** We have $B \geq S^1(a^{\max}) > S^2(a^{\max})$, $\forall B \geq 0$.

**Subcase 2.1:** $P \geq cK^0 \left[1 - \frac{e^l}{\xi(1 + \frac{1}{z})}\right] (b - \gamma)$.

As follows from (13), we have $S^2(0) < 0$ and $\frac{\partial S^2(a)}{\partial a} \geq 0$, $\forall a$. For $a^{\max} = \left[(\frac{cK^0}{B})^{-\frac{1}{z}} - 1\right] (1 - \gamma)$, we obtain

$$S^2(a^{\max}) = \frac{1}{1 + a^{\max}} \left[cK^0 \left[1 - \frac{e^l}{\xi(1 + \frac{1}{z})}\right] \frac{(1 - \gamma)}{(\frac{cK^0}{B})^{1 + \frac{1}{z}}} - P\right] < 0, \quad (14)$$

as follows from $b < -1$, $cK^0 > B$ and the definition of the subcase. Hence, for $a \in [0, a^{\max})$, we have $S^2(a) < 0 < B$. It follows that the firm always borrows to invest in $K^1(a)$. For $B < S^1(0)$ (which follows from the definition of Case 2), since $S^1(a)$ is strictly decreasing in $a$ and $B \geq S^1(a^{\max})$ (from Lemma A.2), it follows that there exists a unique $a^d$, as defined by $S^1(a^d) = B$ (where the superscript $d$ refers to “default”). We have $B < S^1(a)$ for $a < a^d$, and the firm borrows to invest in $K^1(a)$ and has a strictly positive default probability, and $B \geq S^1(a)$ for $a \geq a^d$, the firm borrows to invest in $K^1(a)$ without default possibility. Therefore $\Lambda(a)$ is characterized by

$$\Lambda(a) = \begin{cases} (cK^1 - B)a - F(b(K^1))BC & \text{if } 0 \leq a < a^d \\ (cK^1 - B)a & \text{if } a^d \leq a < a^{\max}. \end{cases}$$

**Subcase 2.2:** $cK^0 \left[1 - \frac{e^l}{\xi(1 + \frac{1}{z})}\right] (1 - \gamma) \leq P < cK^0 \left[1 - \frac{e^l}{\xi(1 + \frac{1}{z})}\right] (b - \gamma)$.

As follows from (13), we have $S^2(0) \leq 0$, and $S^2(a)$ is first strictly decreasing, and then strictly increasing in $a$. From (14), we obtain $S^2(a^{\max}) < 0$; hence $S^2(a) < 0$ for $a \in [0, a^{\max})$ in this case. Therefore $\Lambda(a)$ is identical to subcase 2.1.

**Subcase 2.3:** $cK^0 \left[1 - \frac{e^l}{\xi(1 + \frac{1}{z})}\right] (1 - \gamma) > P$

As follows from (13), we have $S^2(0) > 0$, and $S^2(a)$ is first strictly decreasing, and then strictly increasing in $a$.

If $B \geq S^2(0)$ (and $B < S^1(0)$ by definition of Case 2), since $B \geq S^1(a^{\max}) > S^2(a^{\max})$ (from Lemma A.2), the creditor’s expected return is characterized in a similar fashion with the other two subcases, i.e. for $S^1(0) > B \geq S^2(0)$, we have

$$\Lambda(a) = \begin{cases} (cK^1 - B)a - F(b(K^1))BC & \text{if } 0 \leq a < a^d \\ (cK^1 - B)a & \text{if } a^d \leq a < a^{\max}. \end{cases}$$
where \( a^d \) is defined by \( S^1(a^d) = B \).

If \( B < S^2(0) \), as \( S^2(0) \) is first strictly decreasing, and then strictly increasing in \( a \) and \( B \geq S^1(a^{max}) > S^2(a^{max}) \) (from Lemma A.2), there exists a unique \( a' \in [0,a^{max}) \), as defined in \( S^2(a') = B \) (where the superscript \( l \) refers to “limited liability”). We have \( B < S^2(a) \) for \( a < a' \) and \( B \geq S^2(a) \) for \( a \geq a' \). Since \( S^2(a) = S^1(a) - \frac{P}{1+\alpha} \), it follows that \( a' \leq a^d \), with equality only holding for \( P = 0 \). Therefore, for \( cK^0 \left[ 1 - \frac{\xi^l}{\xi(1+\frac{1}{\xi})} \right] (1 - \gamma) > P > 0 \), we have the following three regions:

1. For \( a < a'(<a^d) \), we have \( B < S^2(a) \) (and \( B < S^1(a) \)), the firm uses an unsecured loan and invests in \( K^*(a) = K(a) \).

2. For \( a' \leq a < a^d \), we have \( S^2(a) \leq B < S^1(a) \), and the firm uses a secured loan (and invests in \( K^*(a) = K^1(a) \)) with default possibility.

3. For \( a \geq a^d \), we have \( S^2(a) < S^1(a) \leq B \), and the firm uses a secured loan (and invests in \( K^*(a) = K^1(a) \)) without default possibility.

Combining all the cases concludes the proof. ■

A.3 Equilibrium Characterization

Proof of Proposition 5: As follows from Proposition 4, the creditor’s problem is only relevant for \( B < cK^0 \); hence we focus on this range of \( B \). We analyze the perfectly competitive credit market and monopolist creditor cases separately.

The Perfectly Competitive Credit Market. We only provide the proof for the perfectly competitive credit market with \( U = 0 \). The proof for the case with \( U > 0 \) is similar. We first prove the existence of \( a \) such that \( \Lambda(a) = 0 \). From Proposition 4, we have the following three cases to analyze:

Case i: \( cK^0 > B \geq cK^0 (1 - \gamma) \left[ 1 - \frac{\xi^l}{\xi(1+\frac{1}{\xi})} \right] \)

We have \( \Lambda(a) = (cK^1 - B)a > 0 \) for \( 0 < a < a^{max} \) and \( \Lambda(a) = 0 \) for \( a = 0 \).

Case ii: \( cK^0 (1 - \gamma) \left[ 1 - \frac{\xi^l}{\xi(1+\frac{1}{\xi})} \right] - P \leq B < cK^0 (1 - \gamma) \left[ 1 - \frac{\xi^l}{\xi(1+\frac{1}{\xi})} \right] \)

\[
\Lambda(a) = \begin{cases} 
(cK^1 - B)a - F(b(K^1))BC & 0 \leq a < a^d \\
(cK^1 - B)a & a^d \leq a < a^{max}
\end{cases}
\]

We have \( \Lambda(a) > 0 \) for \( a^d \leq a < a^{max} \) and \( \Lambda(0) = -F\left(b\left(K^1(0)\right)\right)BC < 0 \), since \( \Lambda(a) \) is continuous in \( a \); from the Mean-value theorem there exists at least one \( a \in (0,a^d) \) such that
\( \Lambda(a) = 0. \)

**Case iii:** \( B < cK^0(1 - \gamma) \left[ 1 - \frac{\xi^l}{\xi(1 + \frac{1}{b})} \right] - P \)

\[
\Lambda(a) = \begin{cases} 
(cK - B)a - F(b(K))BC - L(K) & 0 \leq a < a^l \\
(cK^1 - B)a - F(b(K^1))BC & a^l \leq a < a^d \\
(cK^1 - B)a & a^d \leq a < a^{max}
\end{cases}
\]

Similar to Case ii, we have \( \Lambda(a) > 0 \) for \( a^d \leq a < a^{max} \) and \( \Lambda(0) = -F(b(K(0)))BC - L(K(0)) < 0 \), since \( \Lambda(a) \) is continuous in \( a \), from the Mean-value theorem there exists at least one \( a \in (0, a^d) \) such that \( \Lambda(a) = 0. \)

For each case, we have established that there always exists at least one \( a \in [0, a^{max}] \) such that \( \Lambda(a) = 0. \) In particular, we have shown that this is obtained at \( a = 0 \) for Case i, and \( a \in (0, a^d) \) for Cases ii and iii. We now prove that in the Pareto-optimal equilibrium, \( \dot{a} = \min\{a' \} \) such that \( \Lambda(a') = 0. \) This is equivalent to \( \frac{\partial \pi^*}{\partial a} < 0. \)

**Lemma A.3** The optimal expected equity value of the firm, \( \pi^* \), decreases in the unit financing cost \( a. \)

**Proof of Lemma A.3:** For \( 0 \leq a < a^{max} \) (i.e. \( B < cK^1 \)), as follows from Proposition 3, we have two cases to consider:

**Case 1:** \( cK^1 \left[ 1 - \frac{\xi^l}{\xi(1 + \frac{1}{b})} \right] \left[ 1 - \frac{\gamma}{1+a} \right] - \frac{P}{1+a} \leq B < cK^1 \)

The firm’s optimal expected equity value is given by \( \pi^* = \frac{(1+a-\gamma)cK^1(a)}{-(b+1)} + B(1+a) + P. \)

Note that \( \frac{\partial \pi^*(K^1(a))}{\partial a} = \frac{\partial \pi(K)}{\partial a} \bigg|_{K^1(a)} + \frac{\partial K^1(a)}{\partial K} \frac{\partial K^1(a)}{\partial a} \bigg|_{K^1(a)}. \) Since \( \frac{\partial \pi(K)}{\partial K} \bigg|_{K^1(a)} = 0 \), we obtain

\[
\frac{\partial \pi^*}{\partial a} = \frac{\partial \pi(K)}{\partial a} \bigg|_{K^1(a)} = -cK^1(a) + B < 0
\]

as follows from the definition of Case 1.

**Case 2:** \( 0 \leq B < cK^1 \left[ 1 - \frac{\xi^l}{\xi(1 + \frac{1}{b})} \right] \left[ 1 - \frac{\gamma}{1+a} \right] - \frac{P}{1+a} \)

The firm’s optimal expected equity value is given by \( \pi^* = \int_{l(K(a))}^{\xi} \left[ \xi \overline{K}(a)^{(1+\frac{1}{b})} + B(1+a) + P - (1+a-\gamma)cK(a) \right] f(\xi) d\xi. \) Similarly, we obtain

\[
\frac{\partial \pi^*}{\partial a} = \frac{\partial \pi(K)}{\partial a} \bigg|_{K(a)} = \int_{l(K)}^{\xi} \left[ -cK + B \right] f(\xi) d\xi < \int_{l(K)}^{\xi} \left[ -cK^1 + B \right] f(\xi) d\xi < 0
\]

as follows from \( K^1(a) < \overline{K}(a) \) and the definition of Case 2.

**The Monopolist Creditor.** Since \( \Lambda(a) > 0 \) for \( a \in [a^d, a^{max}] \), the monopolist creditor always offers a loan contract in equilibrium. Moreover, \( \Lambda(0) \leq 0 \) for all firms considered.
(B < cK^0); therefore, in equilibrium we have \( \hat{a} \in (0, a^{max}) \). In the rest of the proof, we denote the equilibrium financing cost for the monopolist creditor as \( \hat{a}^M \). We first analyze the firms that never default (case i of Proposition 4). Substituting \( K^1 = \left( \frac{\xi(1+\frac{1}{b})}{(1+\alpha-\gamma)c} \right)^{-b} \) and

\[
a^{max} = \left[ \left( \frac{cK^0}{B} \right)^{-\frac{1}{b}} - 1 \right] (1-\gamma), \text{ for } B \geq cK^0(1-\gamma) \left[ 1 - \frac{\gamma}{\xi(1+\frac{1}{b})} \right],
\]

the creditor’s optimal expected profit is given by

\[
N^* = \max_a \left( cK^0(1-\gamma)^{-b}(1 + a - \gamma)^b - B \right) a 
\quad \text{s.t.} \quad 0 \leq a < \left[ \left( \frac{cK^0}{B} \right)^{-\frac{1}{b}} - 1 \right] (1-\gamma)
\]

where “N” stands for the net gain from secured lending. Recall that, in this case we have \( a^d = 0 \). Let \( \Lambda(a) \) and \( \hat{a}^M \) denote the objective function and the optimal solution of (15), respectively. We obtain

\[
\frac{\partial \Lambda(a)}{\partial a} = cK^0(1-\gamma)^{-b}(1 + a - \gamma)^{(b-1)}(1 + a - \gamma + ab) - B = cK^0(1-\gamma)^{-b} \left[ J(a) - \frac{B}{cK^0(1-\gamma)^{-b}} \right],
\]

where \( J(a) \equiv (1 + a - \gamma)^{(b-1)}(1 + a - \gamma + ab) \). Note that \( \frac{\partial^2 \Lambda(a)}{\partial a^2} \) has the same sign as \( \frac{\partial J(a)}{\partial a} \).

We now analyze \( J(a) \) to characterize the solution for \( \Lambda(a) \). We obtain

\[
J(0) = (1-\gamma)^b > \frac{B}{cK^0(1-\gamma)^{-b}};
\]

\[
J(a^{max}) = (1-\gamma)^b \frac{B}{cK^0} \left[ 1 + \frac{ba^{max}}{\left( \frac{cK^0}{B} \right)^{-\frac{1}{b}}} \right] < \frac{B}{cK^0(1-\gamma)^{-b}},
\]

as follows from \( B < cK^0 \) and \( b < -1 \). Therefore, \( \frac{\partial \Lambda(a)}{\partial a} |_{a^0+} > 0 \) and \( \frac{\partial \Lambda(a)}{\partial a} |_{a^{max}-} < 0 \). We obtain

\[
\frac{\partial J(a)}{\partial a} = -b(1 + a - \gamma)^{(b-2)} [-a(b + 1) - 2(1 - \gamma)].
\]

As the product of the first two terms is positive (as follows from \( b < -1 \)), \( \frac{\partial J(a)}{\partial a} \) has the same sign as \( [-a(b + 1) - 2(1 - \gamma)] \). In other words, for \( a > \frac{2(1-\gamma)}{-(b+1)} \), we have \( \frac{\partial J(a)}{\partial a} > 0 \) and \( \Lambda(a) \) is strictly convex. For \( a < \frac{2(1-\gamma)}{-(b+1)} \), \( \frac{\partial J(a)}{\partial a} < 0 \) and \( \Lambda(a) \) is strictly concave. As the feasible region is \( a \in [0, a^{max}] \), we have two cases to consider.

**Case 1**: \( a^{max} > \frac{2(1-\gamma)}{-(b+1)} \), which is equivalent to \( B < \frac{cK^0}{\left[ 1 + \frac{2(1-\gamma)}{-(b+1)} \right]^{-b}} \).

We have that \( J(a) \) is decreasing for \( 0 < a < \frac{2(1-\gamma)}{-(b+1)} \) and increasing for \( \frac{2(1-\gamma)}{-(b+1)} \leq a < a^{max} \).

We also have \( J(0) > \frac{B}{cK^0(1-\gamma)^{-b}} \) and \( J(a^{max}) < \frac{B}{cK^0(1-\gamma)^{-b}} \) (from (17)). As a result, there exist a unique \( a^N < \frac{2(1-\gamma)}{-(b+1)} \), as defined by \( J(a^N) \equiv \frac{B}{cK^0(1-\gamma)^{-b}} \) such that for \( 0 \leq a < a^N \),
\( J(a) = \frac{B}{cK^{a(1-\gamma)-b}} \) (and hence \( \frac{\partial J(a)}{\partial a} < 0 \)) and for \( a^N \leq a < a^\max \), \( J(a) < \frac{B}{cK^{a(1-\gamma)-b}} \) (and hence \( \frac{\partial J(a)}{\partial a} < 0 \)).

In summary, \( \Lambda(a) \) is characterized as follows:

1. For \( 0 \leq a < a^N \left( < \frac{2(1-\gamma)}{(b+1)} \right) \), \( \Lambda(a) \) is concave increasing.

2. For \( a^N \leq a < \frac{2(1-\gamma)}{(b+1)} \), \( \Lambda(a) \) is concave decreasing.

3. For \( \frac{2(1-\gamma)}{(b+1)} < a < a^\max \), \( \Lambda(a) \) is convex decreasing.

Hence, the optimal solution \( \hat{a}^M \) is unique and is given by \( \hat{a}^M = a^N \).

Case 2: \( a^\max < \frac{2(1-\gamma)}{(b+1)} \), which is equivalent to \( B \geq \frac{cK^a}{[1+(b+1)]^{-b}} \).

We have that \( J(a) \) is decreasing for \( 0 < a < a^\max \). From (17), we have \( J(0) > \frac{B}{cK^{a(1-\gamma)-b}} \)
and \( J(a^\max) < \frac{B}{cK^{a(1-\gamma)-b}} \), therefore, similar to Case 1, there also exists a unique \( a^N < a^\max \), as defined by \( J(a^N) = \frac{B}{cK^{a(1-\gamma)-b}} \) such that for \( 0 \leq a < a^N \), \( J(a) > \frac{B}{cK^{a(1-\gamma)-b}} \) (and hence \( \frac{\partial J(a)}{\partial a} > 0 \)) and for \( a^N \leq a < a^\max \), \( J(a) < \frac{B}{cK^{a(1-\gamma)-b}} \) (and hence \( \frac{\partial J(a)}{\partial a} < 0 \)).

The \( \Lambda(a) \) is characterized as follows:

1. For \( 0 \leq a < a^N \left( < \frac{2(1-\gamma)}{(b+1)} \right) \), \( \Lambda(a) \) is concave increasing.

2. For \( a^N \leq a < a^\max \), \( \Lambda(a) \) is concave decreasing.

After combining all the cases, the optimal solution \( \hat{a}^M \) is unique and is given by \( \hat{a}^M = a^N \)
where \( a^N \) is the solution of \((1+a-\gamma)^{b-1}(1+a-\gamma + ab) = \frac{B}{cK^{a(1-\gamma)-b}} .

For the firms that may default but use a secured loan (Case ii of Proposition 4), the creditor’s expected return is given by

\[
\Lambda(a) = \begin{cases} 
(cK^{a}(a) - B)a - F(b(K^{1}(a)))BC & 0 \leq a < a^d \\
(cK^{a}(a) - B)a & a^d \leq a < a^\max
\end{cases}
\]

where \( F(.) \) denotes the cdf of \( \xi \) and \( b(K^{1}(a)) \) is the bankruptcy threshold that is given by
\( b(K^{1}(a)) = \bar{\xi}(1 + \frac{1}{b}) \left[ 1 - \frac{B(1+a)}{cK^{a(1-\gamma)-b}(1+a-\gamma)^{(b+1)}} \right] \). We obtain

\[
\frac{\partial F(b(K^{1}(a)))}{\partial a} = -f(b(K^{1}(a)))\bar{\xi}(1 + \frac{1}{b}) \frac{B}{cK^{a(1-\gamma)-b}} \left[ \frac{1 + \frac{(b+1)(1+a)}{(1+a-\gamma)(b+1)}}{(1+a-\gamma)^{(b+1)}} \right] < 0
\]
as follows from \( b < -1 \) and \( f(b(K^{1})) > 0 \). Thus, for \( 0 \leq a < a^d \), the default risk is strictly decreasing in \( a \). On the other hand, as follows from the analysis above, the creditor’s net
gain from secured lending is strictly increasing in $a$ for $0 \leq a < a^N$ and strictly decreasing in $a$ for $a > a^N$. Therefore, the location of the maximizer $\hat{a}^M$, or in other words, whether the firm uses a secured loan with or without default possibility, depends on the ordering of $a^N$ and $a^d$. For $a^N \geq a^d$, the creditor’s expected return is increasing in $a$ for $a < a^N$ (as the default term decreases and the net gain from secured lending increases), thus we have $\hat{a}^M = a^N$. For $a^N < a^d$, the creditor’s expected return is increasing in $a$ for $a < a^N$ and decreasing in $a$ for $a > a^d$. Therefore, there exists at least one maximizer between $a^N$ and $a^d$.

As depicted in Panel A of Figure 5, for sufficiently small $B$, we have $a^N < \hat{a}^M < a^d$ and the firm uses a secured loan with default possibility in equilibrium. For large $B$, the firm uses a secured loan without default possibility in equilibrium and we have $\hat{a}^M = a^N$ (Panel B).

Figure 5: The location of the equilibrium financing cost $\hat{a}^M$ for the monopolist creditor case for firms that may default but use a secured loan (case (ii) of Proposition 4) with $B \in \{500, 700\}$, $\xi \sim U[20, 140]$; For small $B$, we have $a^N < a^d$, and $a^N < \hat{a}^M < a^d$ and the firm uses a secured loan with default possibility in equilibrium (Panel A). For large $B$, we have $a^N > a^d$ and $\hat{a}^M = a^N$ and the firm uses a secured loan without default possibility in equilibrium (Panel B).
A.4 Comparative Static Analysis

Proof of Remark 1: If the capital markets are perfect, the creditor operates under perfect competition, and makes zero return on expectation for the loan. The firm’s optimal expected equity value is given by

$$\pi^* = \max_{K,e} \left( B + e - cK - (B + e - cK)^+ + \int_N \left[ (B + e - cK)^+ + \xi K^{1+\frac{1}{b}} + P + \gamma cK - e(1 + a) \right] f(\xi)d\xi \right. $$

$$+ \left. \int_D \max \left[ (B + e - cK)^+ + \xi K^{1+\frac{1}{b}} + P + \gamma cK - e(1 + a), 0 \right] f(\xi)d\xi \right) $$

\begin{equation}
(19)
\end{equation}

s.t. \quad K \geq 0, \\
\quad e \geq 0,

where \( e \) is the borrowing level, \( N \equiv \{ \xi : \xi \geq \xi^l; \xi K^{1+\frac{1}{b}} + \gamma cK \geq e(1 + a) \} \) refers to the non-default states and \( D \equiv \{ \xi : \xi \geq \xi^l; \xi K^{1+\frac{1}{b}} + \gamma cK < e(1 + a) \} \) refers to the default states. The creditor’s zero expected return constraint can be written as

$$e = \int_N e(1 + a) f(\xi)d\xi + \int_D \left[ -BC + \min \left[ (B + e - cK)^+ + \xi K^{1+\frac{1}{b}} + P + \gamma cK - e(1 + a) \right] \right] f(\xi)d\xi $$

\begin{equation}
(20)
\end{equation}

Substituting (20) in the objective function of (19), and using \( \max(x - y, 0) = x - \min(x, y) \) for \( x = (B + e - cK)^+ + \xi K^{1+\frac{1}{b}} + P + \gamma cK \) and \( y = e(1 + a) \), the objective function can be rewritten as

$$B - c(1 - \gamma)K + P + \bar{\xi}K^{1+\frac{1}{b}} - \int_D BC. $$

It turns out that when the capital markets are perfect, we have \( BC = 0 \), and the objective function of the firm does not depend on \( a \); hence the optimal capacity investment level is given by \( \hat{K} = K^0 = \left( \frac{\bar{\xi}(1+\frac{1}{b})}{(1-\gamma)c} \right)^{-b} \). Since the firm exactly borrows what it needs to cover its capacity investment, we have \( \hat{e} = (cK^0 - B)^+ \). In summary, in perfect capital markets, the operational and financial decisions decouple; the firm decides on the optimal \( \hat{K} \) without budget constraints and borrows the amount necessary to finance its optimal capacity investment \( K^0 \). Thus, the firm’s optimal expected equity value is given by \( \hat{\pi} = B + P + \frac{c(1-\gamma)K^0}{-b+1} \). The impact of demand variability and the internal budget level on \( \hat{K} \) and \( \hat{\pi} \) follow directly. ■

A.4.1 The Impact of Demand Variability.

In this section, we provide the analytical proofs and related numerical experiments that we use in developing the results summarized in Table 1. Recall that we use the mean-preserving
spread of the uniform distribution to characterize an increase in the demand variability \( \sigma \), i.e. \( \xi \sim U [\xi^l - \epsilon, \xi^u + \epsilon] \) for \( \epsilon \in (0, \xi^l) \). A higher \( \epsilon \) leads to a higher variance of \( \xi \). In perfect capital markets, the firm’s equilibrium capacity decision and expected equity value depend on the expected value of demand but not its variability, i.e. \( \hat{K} \) and \( \hat{\pi} \) are independent of \( \sigma \).

We next analyze the impact of demand variability in imperfect capital markets under the different capital market conditions.

**A.4.1.1. The Perfectly Competitive Credit Market with \( U = 0 \).**

Here, we construct column 1 of imperfect market analysis in Table 1.

**Secured loan without default possibility.** The equilibria where the firm uses a secured loan without default possibility are only feasible for firms that borrow without default possibility (Case i of Proposition 4). In this case, as follows from Proposition 5, we have \( \hat{a} = 0 \). Since the creditor’s expected return is not affected by the demand variability, the equilibrium financing cost \( \hat{a} \) is not affected either. For a given financing cost \( a \), the optimal capacity investment level and the expected equity value of the firm only depend on the mean \( \bar{\xi} \) and not the variability. Therefore, \( \hat{K} \) and \( \hat{\pi} \) do not change in demand variability either.

**Secured loan with default possibility.** At equilibria where the firm uses a secured loan (and invests in \( K^1(\hat{a}) \)) with default possibility, the creditor’s expected return in equilibrium is characterized by the net gain from secured lending minus the expected default cost. From the firm’s perspective, for a given \( \hat{a} \), an increase in the demand variability does not alter the optimal capacity investment level or the equity value. With the uniform distribution, a higher variability at the same expected demand level corresponds to a mean-preserving spread of \( \xi \) – more probability mass is transferred to the tails, and in particular, the downside risk of the firm’s operating cash flows increases. This leads to a higher expected default risk for the firm and lower expected returns for the creditor. To compensate for this reduction, the creditor charges a higher financing cost in equilibrium. Therefore, an increase in the demand variability decreases \( \hat{K} \) and \( \hat{\pi} \). This is formally proven in the following proposition.

**Proposition A.1** If \( \xi \) is uniformly distributed in \( [\xi^l - \epsilon, \xi^u + \epsilon] \) for \( \epsilon \in (0, \xi^l) \), in a perfectly competitive credit market with \( U = 0 \), at equilibria where the firm uses a secured loan (and invests in \( K^1(\hat{a}) \)) with default possibility, the expected equity value and the capacity investment level in equilibrium locally decrease in demand variability \( (\epsilon) \) through an increase in \( \hat{a} \).
Proof of Proposition A.1: We will first prove that $\dot{a}$ is increasing in demand variability $\epsilon$. Since this equilibrium is relevant for firms that may default but use a secured loan (Case \textit{ii} of Proposition 4) and firms that may use an unsecured loan (Case \textit{iii} of Proposition 4); we will analyze these two cases separately.

For firms that may default but always use a secured loan, since the expected return of the creditor, $\Lambda(a)$ is independent of $\epsilon$ for $a \geq a^d$, we can focus only on $a \in [0, a^d)$. For $a \in [0, a^d)$, we have $\Lambda(a) = (cK^1(a) - B)a - F(b(K^1(a)))BC$ where $F(.)$ is the probability distribution function of $x$ and $b(K^1(a)) = \xi(1 + \frac{1}{b}) \left(1 - \frac{B}{cK^0(1 + a)^c}\right)$ is the bankruptcy threshold. For the uniform distribution, we have $\xi = \frac{\xi^l + \xi^u}{2}$, and $F(x) = \frac{x - \xi^l}{\xi^u - \xi^l + 2\varepsilon}$ for $x \in [\xi^l - \epsilon, \xi^u + \epsilon]$. We obtain
\[
\frac{\partial \Lambda(a)}{\partial \epsilon} = \frac{BC}{(\xi^u - \xi^l + 2\varepsilon)^2} \left(\xi^l + \xi^u\right) \left(1 + \frac{1}{b}\right) \left[1 - \frac{B}{cK^1}\right] - \left(\xi^l + \xi^u\right) < 0 \quad (21)
\]
as follows from $b < -1$ and $B < cK^1$.

Let $\dot{a}(\epsilon)$ denote the Pareto-optimal equilibrium interest rate at a given variability parameter $\epsilon$. Let us assume that the unit financing cost decreases in $\epsilon$, i.e. $\dot{a}(\epsilon_0) > \dot{a}(\epsilon_1)$ for $\epsilon_0 < \epsilon_1$. We will show by contradiction that this is not possible.

We have $\frac{\partial \Lambda(a)}{\partial \epsilon} < 0$ for $a \in [0, a^d)$. As a result, for $\dot{a}(\epsilon_1) \in [0, a^d)$, we have $\Lambda(\dot{a}(\epsilon_1); \epsilon_0) > \Lambda(\dot{a}(\epsilon_1); \epsilon_1) = 0$, where the equality follows from the definition of the equilibrium. It follows from i) $\Lambda(0; \epsilon_0) < 0$, ii) the continuity of $\Lambda(a)$ in $a$, and iii) the Mean-value theorem, that there exists an $\dot{a}' < \dot{a}(\epsilon_1) (< \dot{a}(\epsilon_0))$ such that $\Lambda(\dot{a}'; \epsilon_0) = 0$. This is a contradiction with $\dot{a}(\epsilon_0)$ being the Pareto-optimal equilibrium at $\epsilon_0$.

For firms that may use an unsecured loan, for $a \in [0, a^d)$ we have
\[
\Lambda(a) = \begin{cases} 
(cK - B)a - F(b(K))BC - L(K) & 0 \leq a < a^l \\
(cK^1 - B)a - F(b(K^1))BC & a^l \leq a < a^d
\end{cases}
\]
Since $\dot{a} \in [a^l, a^d)$, it follows from (21) that $\frac{\partial \Lambda(a)}{\partial \epsilon}|_{\dot{a}} < 0$. With a small increment in $\epsilon$ from $\epsilon_0$ to $\epsilon_1$, we can guarantee that $\Lambda(a; \epsilon_1) < 0$ for $\forall \ a < \dot{a}(\epsilon_0)$ because i) $\Lambda(a; \epsilon_0) < U = 0$ for $\forall \ a < \dot{a}(\epsilon_0)$ from the definition of equilibrium, and ii) $|\frac{\partial \Lambda(a)}{\partial \epsilon}|$ and $|\frac{\partial^2 \Lambda(a)}{\partial \epsilon^2}|$ are bounded. Therefore $\dot{a}$ increases locally in $\epsilon$ in this case.

We now analyze the impact of $\epsilon$ on $\dot{K}$ and $\pi$. We have $
\dot{K} = K^1(\dot{a}) = \left(\frac{\xi(1 + \frac{1}{b})}{(1 + a)^c}\right)^{-b},
\pi = \frac{cK^1(1 + \dot{a})}{(b + 1)} + B(1 + \dot{a}) + P$. Since for a given $a$, $\pi^*$ is independent of $\epsilon$, we have $\frac{\partial \pi}{\partial \epsilon} = \frac{\partial \dot{a}}{\partial \epsilon}\frac{\partial \pi^*}{\partial a}$,
\[ \frac{\partial \pi^*}{\partial a} > 0. \] From Lemma A.3 (in the proof of Proposition 5), we have \( \frac{\partial \pi^*}{\partial a} < 0, \) hence \( \frac{\partial \dot{\pi}}{\partial \epsilon} > 0. \) To prove that \( \dot{K} \) is decreasing in \( \epsilon, \) it is sufficient to show that \( \frac{\partial K^1(a)}{\partial a} < 0. \) We now provide a more general result that we will use throughout this section.

**Lemma A.4** If \( \xi \) is uniformly distributed in \([\xi^l, \xi^u]\), the optimal capacity investment level of the firm, \( K^* \), decreases in the unit financing cost \( a. \)

**Proof of Lemma A.4:** From Proposition 3, for \( B < cK^0 \) and \( 0 \leq a < a^{max}, \) \( K^*(a) \) is given by

\[
K^*(a) = \begin{cases} 
K^1(a) = \left( \frac{\xi(1+b)}{1+a} \right)^{-b} & \text{if } cK^1(a) \left[ 1 - \frac{\xi^l}{\xi(1+b)} \right] - \frac{p}{1+a} \leq B < cK^1 \\
\overline{K}(a) & \text{if } 0 \leq B < cK^1(a) \left[ 1 - \frac{\xi^l}{\xi(1+b)} \right] - \frac{p}{1+a}.
\end{cases}
\]

We obtain

\[
\frac{\partial K^1(a)}{\partial a} = b \left( \frac{\xi(1+b)}{(1+a)c} \right)^{-b} \frac{1}{(1+a)} < 0.
\]

From Proposition 3, \( \overline{K}(a) \) satisfies

\[
\int_{l(\overline{K}(a))}^{\xi^u} \left[ \xi - \left( \frac{\xi}{\overline{K}(a)} \right)^{-b} \frac{(1+a)c}{(1+b)} \right] f(\xi) d\xi = 0.
\]

Using the assumption that \( \xi \sim U[\xi^l, \xi^u], \) this is equivalent to

\[
\left[ \frac{\xi^u - l(\overline{K}(a))}{\xi^u - \xi^l} \right] \left[ \frac{\xi^u + l(\overline{K}(a))}{2} - \left( \frac{\overline{K}(a)}{\xi(1+b)} \right)^{-b} \right] = 0. \tag{22}
\]

Substituting \( l(\overline{K}(a)) = (1+a)c \left( \frac{\overline{K}(a)}{\xi^u} \right)^{-1/b} - (B(1+a) + P) \left( \frac{\overline{K}(a)}{\xi^u} \right)^{-1/b}, \) \( (22) \) is equivalent to \( T(\overline{K}(a)) = 0 \) where

\[
T(K) = \xi^u + \frac{(1+b)}{(b+1)} (1+a)cK^{-1/b} - (B(1+a) + P) K^{-1-1/b}. \tag{23}
\]

From the implicit function theorem, we have \( \frac{\partial \overline{K}}{\partial a} = -\frac{\partial T(\overline{K})/\partial a}{\partial T(\overline{K})/\partial \overline{K}} \bigg|_{\overline{K}}. \) Since \( \overline{K} \) is the unique maximizer, we have \( \text{sgn} \left( \frac{\partial \overline{K}}{\partial a} \right) = \text{sgn} \left( \frac{\partial T(\overline{K})}{\partial a} \bigg|_{\overline{K}} \right). \) We obtain

\[
\frac{\partial T(\overline{K})}{\partial a} = c \left( \frac{1}{cK} \right)^{\frac{1}{b}} \left[ -\frac{(b-1)}{(b+1)} - \frac{B}{cK} \right] < 0
\]

as follows from \( b < -1. \) The result with respect to \( \dot{K} \) follows from Lemma A.4.
**Unsecured loan.** At equilibria where the firm uses an unsecured loan (and invests in $K(\dot{a})$), we have $\dot{a} < a^l$ and the creditor’s expected return in equilibrium is characterized by three terms: The net gain from secured lending, minus the expected loss due to unsecured part of the loan and the expected default cost. From the firm’s perspective, for a given $\dot{a}$, it can be proven that an increase in the demand variability increases $K$:

**Lemma A.5** If $\xi$ is uniformly distributed in $[\xi^l - \epsilon, \xi^u + \epsilon]$ for $\epsilon \in (0, \xi^l)$, for a given financing cost $a$, for the firm that uses an unsecured loan, $K^\ast$ and $\pi^\ast$ increase in the demand variability ($\epsilon$).

**Proof of Lemma A.5:** When the firm uses an unsecured loan we have $K^\ast = \overline{K}$ as shown in Proposition 3. As follows from (22), with the uniform distribution assumption, $\overline{K}$ is the unique solution to $T(\overline{K}) = 0$ where, similar to (23),

$$T(K) = \xi^u + \epsilon + \frac{(1 - b)}{(b + 1)}(1 + a)cK^{-1/b} - (B(1 + a) + P)K^{-1/b}.$$

From the implicit function theorem, we have $\frac{\partial \overline{K}}{\partial \epsilon} = -\frac{\partial T(K)/\partial \epsilon}{\partial T(K)/\partial K} \bigg|_{\overline{K}}$. Since $\overline{K}$ is the unique maximizer, we have $\text{sgn} \left( \frac{\partial \overline{K}}{\partial \epsilon} \right) = \text{sgn} \left( \frac{\partial T(K)}{\partial \epsilon} \bigg|_{\overline{K}} \right)$. The result with respect to $K$ follows as $T(K)$ is increasing in $\epsilon$.

When the firm uses unsecured loan, the firm’s optimal expected equity value is given by

$$\pi^\ast = \int_{l(\overline{K})}^{\xi^u} \left[ \xi \overline{K}^{(1 + \frac{1}{b})} + B(1 + a) + P - (1 + a)c\overline{K} \right] f(\xi) d\xi.$$

With the uniform distribution assumption, and using the definition of $l(\overline{K}) = (1 + a)c\left( \overline{K} \right)^{-1/b} - (B(1 + a) + P)\left( \overline{K} \right)^{-1/b}$, it is easy to show that this expression is equivalent to

$$\pi^\ast = \frac{1}{2} \left( \overline{K} \right)^{(1 + \frac{1}{b})} \left( \frac{\xi^u + \epsilon - l(\overline{K})}{\xi^u - \xi^l + 2\epsilon} \right)^2.$$

Note that $\frac{\partial \pi^\ast}{\partial \epsilon} = \frac{\partial \pi(K)}{\partial \epsilon} \bigg|_K + \frac{\partial \pi(K)}{\partial K} \bigg|_K \frac{\partial K}{\partial \epsilon}$. Since $\frac{\partial \pi(K)}{\partial K} \bigg|_K = 0$, we obtain

$$\frac{\partial \pi^\ast}{\partial \epsilon} = 2 \left( \frac{\xi^u + \epsilon - l(\overline{K})}{\xi^u - \xi^l + 2\epsilon} \right) \left( l(\overline{K}) - \left( \xi^l - \epsilon \right) \right) > 0.$$

The optimal capacity investment level increases because as the likelihood of low demand states increases, the value of the limited liability option of the firm increases; and the firm optimally takes more investment risk and invests more in capacity. The optimal expected equity value also increases as the value of the limited liability option of the firm increases.

From the creditor’s perspective, an increase in the demand variability has three distinct effects:
Figure 6: The effect of the demand variability on the capacity investment level and the expected equity value in equilibria where the firm uses an unsecured loan (and invests in $K(\dot{a})$) in imperfect capital markets (in a perfectly competitive credit market with $U = 0$) with $B = 40$, $\xi \sim U[20 - \epsilon, 120 + \epsilon]$ and $\epsilon \in [0, 20]$ with 1-unit increments: A higher $\epsilon$ increases the default probability (Panel C) and decreases the creditor’s expected return without the expected default cost (Panel B). The total effect is that $\Lambda(\dot{a})$ decreases (Panel A) and $\dot{a}$ increases (Panel D). A higher $\dot{a}$ reduces $\dot{\pi}$ (Panel F). Despite the increase in $\dot{a}$, $\dot{K}$ increases (Panel E) as higher demand variability increases the value of the limited liability option of the firm in equilibrium.

Lemma A.6 If $\xi$ is uniformly distributed in $[\xi^l - \epsilon, \xi^u + \epsilon]$ for $\epsilon \in (0, \xi^l)$, when the firm uses an unsecured loan (i.e. $0 \leq a < a^l$), the creditor’s net gain from secured lending, its expected default cost and its expected loss due to the unsecured part of the loan increase in the demand variability ($\epsilon$).

Proof of Lemma A.6: For $0 \leq a < a^l$, the creditor’s expected return $\Lambda(a)$ is given by $(cK - B)a - F(b(K))BC - L(K)$ as shown in Case iii of Proposition 4. With the uniform distribution assumption, we obtain

$$\Lambda(a) = (cK - B)a - \left[ \frac{b(K) - \xi^l + \epsilon}{\xi^u - \xi^l + 2\epsilon} \right] BC - \frac{1}{2} \left( \frac{\xi^u - \xi^l + 2\epsilon}{(K)(1+\frac{1}{b})} \right) \left( \frac{l(K) - \xi^l + \epsilon}{\xi^u - \xi^l + 2\epsilon} \right)^2$$  \hspace{1cm} (24)$$

where $b(K) = (1 + a)c(K)^{-1/b} - B(1 + a)K^{-1-1/b}$ and $l(K) = b(K) - \frac{P}{(K)^{1+1/b}}$. In (24), the first term is the net gain from secured lending, the second term is the expected default
cost, and the third term is the expected loss due to the unsecured part of the loan. Since $\frac{\partial K}{\partial \epsilon} > 0$ from Lemma A.5, it follows that the net gain from secured lending increases in the demand variability.

For the impact of $\epsilon$ on the default probability (hence the expected default cost), by taking the derivative of the default probability in (24) with respect to $\epsilon$, we obtain

$$\frac{\xi^u + \xi^l - 2b(K)}{(\xi^u - \xi^l + 2\epsilon)^2} + \frac{\partial b(K)}{\partial \epsilon} \frac{\xi^u}{\xi^u - \xi^l + 2\epsilon}.$$ (25)

In (25), the first term represents the effect of demand variability on the default risk of the firm for a fixed $K$, whereas the second term captures the effect of change in $K$ on the default probability. We have

$$\xi^u + \xi^l - 2b(K) > \xi^u + \xi^l - 2b(\hat{K}) = \xi^u + \xi^l - 2\xi^u (1 + 1/b) \left( 1 - \frac{B}{c\hat{K}} \right) > \xi^l > 0$$

where $\hat{K} = \left( \frac{\xi^u (1 + \frac{1}{b})}{(1 + a)c} \right)^{-b}$. The first inequality is due to $b(K)$ increasing in $K$ and $K < \hat{K}$ (as follows from Proposition 3). The second inequality follows from Assumption 1 ($b \geq -2$). Therefore, the first term in (25) is positive. It is easy to establish that $b(K)$ is increasing in $\epsilon$ as $\frac{\partial K}{\partial \epsilon} > 0$ from Lemma A.5. Therefore, the expected default cost of the firm increases in demand variability.

For the impact of $\epsilon$ on the expected loss due to the unsecured part of the loan, by taking the derivative of last term in (24) with respect to $\epsilon$, we obtain

$$\frac{K^{(1+1/b)} (l(K) - (\xi^l - \epsilon)) (\xi^u + \epsilon - l(K))}{(\xi^u - \xi^l + 2\epsilon)^2} + \frac{\partial K}{\partial \epsilon} \frac{(l(K) - (\xi^l - \epsilon))}{2(\xi^u - \xi^l + 2\epsilon)} \left( (1 + 1/b)K^{1/b} (l(K) - (\xi^l - \epsilon)) + 2K^{(1+1/b)} \frac{\partial l(K)}{\partial K} \right).$$

The first term represents the effect of the demand variability on the expected loss for a fixed $K$ and is positive (as $l(K) > \xi^l - \epsilon$ by definition). The second term represents the effect of demand variability on the expected loss through altering $K$. Since $l(K)$ is increasing in $K$ and $\hat{K}$ is increasing in $\epsilon$, this term is also positive; therefore the expected loss due to the unsecured part of the loan increases in the demand variability. □

In Lemma A.6, the default risk increases because i) for a fixed $K$, the downside risk of the firm’s operating cash flows increases, and ii) $K$ increases and the firm borrows more. The expected loss due to the unsecured part of the loan increases due to a similar reasoning. The net gain from secured lending increases as the the firm invests and borrows more. The first two effects work to increase $\dot{a}$, whereas the third effect works to decrease it. As depicted
in Panel D of Figure 6, our numerical experiments reveal that the first two effects may dominate and \( \dot{a} \) may increase in the demand variability.

For the impact of the demand variability on \( \dot{K}(\dot{a}) \), the two drivers work in opposite directions: As follows from Lemma A.5, a higher demand variability induces the firm to invest more in capacity for a given \( a \) due to the increasing value of the limited liability option, but a higher \( \dot{a} \) induces the firm the invest less (as follows from Lemma A.4 in the proof of Proposition A.1). As depicted in Panel E of Figure 6, the first effect may dominate and \( \dot{K} \) may increase in the demand variability. For the effect on \( \dot{\pi} \), a higher demand variability increases the limited liability option of the firm and increases the expected equity value for a given \( a \) (as follows from Lemma A.5), but a higher \( \dot{a} \) decreases the equity value. As depicted in Panel F of Figure 6, the second effect may dominate and \( \dot{\pi} \) may decrease in the demand variability.

### A.4.1.2. The Perfectly Competitive Credit Market with \( U > 0 \).

Here, we construct column 2 of imperfect market analysis in Table 1.

**Secured loan without default possibility.** The equilibrium where the firm uses a secured loan without default possibility is feasible for all three cases of Proposition 4. At this equilibrium, we have \( \dot{K} = K^1(\dot{a}) = \left( \frac{c(1+\frac{1}{a+\gamma})}{(1+\gamma)c} \right)^{-b}, \dot{\pi} = \frac{cK^1(1+\dot{a}-\gamma)}{-(b+1)} + B(1+\dot{a}) + P, \) and \( \dot{a} > a^d \) where \( a^d \) is as defined in Proposition 4. Since the creditor’s expected return in equilibrium, \( \Lambda(\dot{a}) = (cK^1(\dot{a}) - B)\dot{a} \), is independent of \( \epsilon \), \( \Lambda(a) < U = 0 \) for \( a < \dot{a} \) by the definition of the equilibrium, and \( |\frac{\partial}{\partial \epsilon} \Lambda(a)| \) is bounded for \( a \in [0, a^{max}] \), with a small change in \( \epsilon \), the equilibrium financing cost \( \dot{a} \) is not affected. It follows that for a given financing cost \( a \), the optimal capacity investment level and the expected equity value of the firm only depend on the mean \( \overline{\xi} \) and not the variability. Therefore, \( \dot{K} \) and \( \dot{\pi} \) do not change in the demand variability either.

**Secured loan with default possibility.** In parallel to Proposition A.1, it can be proven that at equilibria where the firm uses a secured loan (and invests in \( K^1(\dot{a}) \)) with default possibility, the expected equity value and the capacity investment level in equilibrium strictly decrease in the demand variability through an increase in \( \dot{a} \).

**Unsecured loan.** At equilibria where the firm uses an unsecured loan (and invests in \( K(\dot{a}) \)), we investigate the impact of the demand variability on \( \dot{a}, \dot{K} \) and \( \dot{\pi} \) numerically. We replot Figure 6 by changing \( U = 0 \) to \( U = 20 \). As depicted in Figure 7, we observe similar pattern with the \( U = 0 \) case.
Figure 7: The effect of the demand variability on the capacity investment level and the expected equity value in equilibria where the firm uses an unsecured loan (and invests in $\bar{K}(\hat{a})$) in imperfect capital markets (in a perfectly competitive credit market with $U > 0$) with $B = 40$, $U = 20$, $\xi \sim U[20 - \epsilon, 120 + \epsilon]$ and $\epsilon \in [0, 20]$ with 1-unit increments. A higher $\epsilon$ ($\sigma$) increases the default probability (Panel C) and decreases the creditor’s expected return without the default cost (Panel B). The total effect is that $\Lambda(a)$ decreases (Panel A) and $\hat{a}$ increases (Panel D and Panel E). As depicted in Panel D, we have $\hat{a} < a^l$ and the equilibrium is always in the unsecured lending region. Panel E illustrates $\hat{a}$ of Panel D with a smaller scale. A higher $\hat{a}$ reduces $\bar{\pi}$ (Panel F). Despite the increase in $\hat{a}$, $\bar{K}$ increases (Panel E) as a higher demand variability increases the value of the limited liability option of the firm in equilibrium.

A.4.1.3. The Monopolist Creditor.

Here, we construct column 3 of imperfect market analysis in Table 1.

Secured loan with default possibility.

Proposition A.2 At equilibria where the firm uses a secured loan (and invests in $\bar{K} = K^1(\hat{a})$) without default possibility, the expected equity value and the capacity investment level in equilibrium do not change with a small increase in the demand variability.

Proof of Proposition A.2: Let $\hat{a}^M$ denote the equilibrium financing cost for the monopolist creditor. $\hat{a}^M$ satisfies $FOC(\hat{a}^M) = 0$ where $\frac{\partial \Lambda(a)}{\partial a} = FOC(a) = cK^0(1 + a)(b - 1)(1 + a + ab) - B$. We will first prove that $\hat{a}^M$ does not change with an increase in the demand variability $\epsilon$. Since this equilibrium is relevant for firms that never default (Case $i$ of Proposition 4), firms that may default but use a secured loan (Case $ii$ of Proposition 4) and firms
that may use an unsecured loan (Case iii of Proposition 4); we will analyze these cases separately.

For firms that never default and firms that may default but use a secured loan, it follows from the proof of Proposition 5 that we have \( a^N \geq a^d \) and the expected return of the creditor, \( \Lambda(a) \) is unimodal in \( a \), thus \( \dot{a}^M = a^N \) is the unique maximizer. Since \( a^N \) is independent of the demand variability, from the implicit function theorem it follows that, \( \dot{a}^M \) is also independent of the demand variability.

For firms that may use an unsecured loan, as follows from the proof of Proposition 5 that the creditor’s expected return is unimodal in \( a \) for \( a \in [a^l, a^\max] \). From the Pareto-optimality of the equilibrium, we also know that \( \Lambda(a) < \Lambda(\dot{a}^M) \) for \( a < a^l \). Since \( \dot{a}^M \) is independent of demand variability, we can guarantee that with a sufficiently small increase in \( \epsilon \), \( \dot{a}^M \) does not change. This result is relevant for local changes in \( \epsilon \), but in our numerical experiments, we observe that this result is not confined to local changes in demand variability.

Similar to the proof of Proposition A.1, it is easy to establish that \( \dot{K} \) and \( \dot{\pi} \) are increasing in \( \epsilon \) through the decrease in \( \dot{a}^M \).

**Secured loan with default possibility.** In determining \( \dot{a} \), the monopolist creditor is concerned with the marginal profit from lending, i.e. the derivative of the expected return. The marginal profit from lending is given by

\[
FOC(a) = cK^0(1 - \gamma)^{-b}(1 + a - \gamma)(1 + a - \gamma + ab) - B - BC \times f(b(K^1(a))) \frac{\partial b(K^1(a))}{\partial a},
\]

where \( f(.) \) is the probability density function of \( \xi \) and \( b(K^1(a)) \) is the bankruptcy threshold calculated at \( K^1(a) \). The optimal solution satisfies \( FOC(\dot{a}) = 0 \). It is easy to establish that \( \frac{\partial b(K^1(a))}{\partial a} < 0 \), and the last term is positive. In other words, in the optimal solution, the creditor equates the marginal cost (that is the reduction in the net gain of the creditor with an increase in \( a \)) with the marginal revenue (that is the reduction in the expected default cost with an increase in \( a \)). Only the marginal revenue term is affected by the change in the demand variability. It follows that marginal revenue decreases with the uniform distribution assumption, and the creditor charges lower \( \dot{a} \). Since \( \dot{K} \) and \( \dot{\pi} \) are only affected by the demand variability through \( \dot{a} \), they increase in equilibrium. This is formally stated in Proposition 6.

**Proof of Proposition 6:** Let \( \dot{a}^M \) denote the equilibrium financing cost for the monopolist creditor. \( \dot{a}^M \) satisfies \( FOC(\dot{a}^M) = 0 \) where

\[
\frac{\partial \Lambda(a)}{\partial a} = FOC(a) = cK^0(1 + a)^{(b-1)}(1 + a + ab) - B - BC \frac{B}{\xi^u - \xi^l + 2\epsilon} \frac{\xi(1 + \frac{b}{B})}{cK^0} \left[ \frac{B}{(1 + a)^{(b+1)}} \right],
\]
We will first prove that \( \hat{a}^M \) is decreasing in the demand variability \( \epsilon \). Since this equilibrium is relevant for firms that may default but use a secured loan (Case ii of Proposition 4) and firms that may use an unsecured loan (Case iii of Proposition 4); we will analyze these two cases separately.

For firms that may default but always use a secured loan, Proposition 5 shows that there exists a global maximizer \( \hat{a}^M \in (a^N, a^d) \) where \( a^N \) is given in Proposition 5 and \( a^d \) is given in Proposition 4. We have two subcases to consider.

If the creditor’s expected return \( \Lambda(a) \) is unimodal in \([0, a^d)\), \( \hat{a}^M \) denotes the unique maximizer. In this case, from the implicit function theorem, we have

\[
\frac{\partial \Lambda}{\partial \epsilon} = -\frac{\partial FOC(a)/\partial \epsilon}{\partial FOC(a)/\partial a} \bigg|_{\hat{a}^M}.
\]

Since \( \hat{a}^M \) is the maximizer, we have \( \text{sgn} \left( \frac{\partial \Lambda}{\partial \epsilon} \right) = \text{sgn} \left( \frac{\partial FOC(a)/\partial \epsilon}{\partial FOC(a)/\partial a} \bigg|_{\hat{a}^M} \right) \). We obtain

\[
\frac{\partial FOC(a)}{\partial \epsilon} = BC \frac{-2B}{(\xi^u - \xi^l + 2\epsilon)^2} \frac{\xi(1 + \frac{1}{b})}{cK^0} \left[ \frac{-b}{(1 + a)(b+1)} \right] < 0
\]

(26)
as follows from \( b < -1 \). Therefore, \( \hat{a}^M \) is decreasing in \( \epsilon \).

If the creditor’s expected return \( \Lambda(a) \) is not unimodal in \([0, a^d)\), since \( \Lambda(a) \) is increasing in \( a \) for \( a < a^N \) (as follows from the proof of Proposition 5), there exist at least two local maximizers. Let \( a^N < a_1, \ldots, a_k, \ldots, a_n < a^d \) denote the \( n \) local maximizers in increasing order of \( a \). Without loss of generality let \( a_k \) denote the global maximizer for a given \( \epsilon_0 \), i.e. \( \hat{a}^M(\epsilon_0) = a_k(\epsilon_0) \). It follows from (26) that all the local maximizers decrease as we increase \( \epsilon \) from \( \epsilon_0 \) to \( \epsilon_1 \). To prove that \( \hat{a}^M(\epsilon_1) < \hat{a}^M(\epsilon_0) \), it is sufficient to show that with an increase in \( \epsilon \) the global maximizer does not switch to the other local maximizers with a higher index than \( k \). We have

\[
\frac{\partial \Lambda(a)}{\partial \epsilon} = \frac{BC}{(\xi^u - \xi^l + 2\epsilon)^2} \left[ (\xi^l + \xi^u) \left( 1 + \frac{1}{b} \right) \left[ 1 - \frac{B}{cK^1} \right] - (\xi^l + \xi^u) \right] < 0
\]

and \( \left| \frac{\partial}{\partial a} \Lambda(a) \right| \) is increasing in \( a \). We have \( \Lambda(a_k(\epsilon_0), \epsilon_0) \geq \Lambda(a, \epsilon_0) \) for \( a > a_k(\epsilon_0) \) from the optimality of \( a_k(\epsilon_0) \). Since \( \Lambda(a) \) is decreasing in \( \epsilon \), and the rate of absolute change is increasing in \( a \), it follows that \( \Lambda(a_k(\epsilon_0), \epsilon_1) \geq \Lambda(a, \epsilon_1) \) for \( a > a_k(\epsilon_0) \), hence \( \hat{a}^M(\epsilon_1) > a_k(\epsilon_0) \) cannot hold.

In summary, either the global maximum is at the same local maximum, i.e. \( \hat{a}^M(\epsilon_1) = a_k(\epsilon_1) \), and we have \( \hat{a}^M(\epsilon_1) < \hat{a}^M(\epsilon_0) \) from the implicit function theorem; or the global maximum switches to another local maximum \( a_j \) for \( j < k \), and we have \( \hat{a}^M(\epsilon_1) < \hat{a}^M(\epsilon_0) \) (as \( a_j < a_k \) by definition).

For firms that may use an unsecured loan, by definition of the equilibrium we have \( \hat{a}^M(\epsilon_0) \in [a^l, a^d) \). We already established above that with an increase in \( \epsilon \) from \( \epsilon_0 \) to \( \epsilon_1 \), the equilibrium will not switch to a larger local maximizer in the secured lending region. The
only difference is that at $\epsilon_1$, the new global maximizer may be in the unsecured lending region. In this case, we still have $\dot{a}^M(\epsilon_1) < \dot{a}^M(\epsilon_0)$ as follows from Proposition 4.

Similar to the proof of Proposition A.1, it is easy to establish that $\dot{K}$ and $\dot{\pi}$ are increasing in $\epsilon$ through the decrease in $\dot{a}$. ■

**Unsecured loan.** At equilibria where the firm uses an unsecured loan (and invests in $K(\dot{a})$), we investigate the impact of the demand variability on $\dot{a}$, $\dot{K}$ and $\dot{\pi}$ numerically. We replot Figure 6 for the monopolist creditor with the same parameter setting. As depicted in Panel A of Figure 8, for low demand variability, the firm uses a secured loan (and invests in $K^1(\dot{a})$) with default possibility. In this case, in line with Proposition 6, $\dot{a}$ decreases and $\dot{K}$ and $\dot{\pi}$ increase with an increase in the demand variability. After sufficient increase in $\sigma$, the firm uses an unsecured loan (and invests in $K(\dot{a})$) in equilibrium. In this case the optimal $\dot{a}$ switches from the secured lending region to the unsecured lending region, i.e. from one local maximizer to the other local maximizer as can be seen from Panel A (and also from Panel B with a smaller scale). In this case, there is a drastic drop in $\dot{a}$ which induces a drastic increase in $\dot{K}$ and $\dot{\pi}$. As we increase the demand variability, $\dot{a}$ stays in the unsecured lending region, and then increases. Although a higher demand variability increases $\pi^*$ for a given financing cost due to higher value of limited liability option (as follows from Lemma A.5), the increase in the financing cost may outweigh this effect and $\dot{\pi}$ may decrease (Panel D).
Figure 8: The effect of the demand variability on the capacity investment level and the expected equity value in equilibria where the firm uses a secured loan (and invests in $K^1(\dot{a})$) with default possibility and the firm uses an unsecured loan (and invests in $K(\dot{a})$) in imperfect capital markets (for the monopolist creditor) with $B = 40$, $\xi \sim U[20 - \epsilon, 120 + \epsilon]$ and $\epsilon \in [0, 20]$ with 1-unit increments: When the firm uses a secured loan in equilibrium, in parallel to Proposition 6, $\dot{a}$ decreases and $\dot{K}$ increases. The jumps in Panels C and E illustrate the transition from a secured loan to an unsecured loan in equilibrium. When the firm uses an unsecured loan, $\dot{\pi}$ decreases in $\sigma$ (Panel E) whereas $\dot{K}$ first increases and then decreases (Panel D).

For the effect on $K(\dot{a})$, two drivers work in opposite directions: A higher demand variability induces the firm to invest more in capacity for a given $a$ due to the higher value of the limited liability option (as follows from Lemma A.5), but a higher $\dot{a}$ induces the firm to invest less (as follows from Lemma A.4). As depicted in Panel D of Figure 8, both effects may dominate: $K(\dot{a})$ first increases and then decreases.

A.4.2 The Impact of the Internal Budget Level.

In this section, we provide the analytical proofs and related numerical experiments that we use in developing the results summarized in Table 2. We first provide a Lemma that we will use throughout this section.

**Lemma A.7** For a given financing cost $a$, the firm’s optimal expected equity value $\pi^*$ strictly increases in the internal budget level $B$. For a given financing cost $a$, if the firm
uses an unsecured loan, the optimal capacity investment level $K^*$ strictly decreases in the internal budget level $B$; whereas if the firm does not borrow or uses a secured loan, then the optimal capacity investment level $K^*$ increases in the internal budget level $B$.

**Proof of Lemma A.7:** For the impact of $B$ on $\pi^*$, from Proposition 3 it is easy to establish that $\frac{\partial}{\partial B} \pi^* \geq 0$ for $B \geq cK^1 \left[ 1 - \frac{\xi^l}{\xi(1+\frac{a}{b})} \right] - \frac{P}{1+a}$, i.e. for the firms that do not borrow or use secured loans, $K^*$ increases in $B$. For the firms that use unsecured loans, i.e. $B < cK^1 \left[ 1 - \frac{\xi^l}{\xi(1+\frac{a}{b})} \right] - \frac{P}{1+a}$, the optimal $K^*$ satisfies

$$\int_{l(K)}^{\xi^u} \left[ \xi - (K)^{-1/b} \left( (1+a)c \right) \left( \frac{1}{1+\frac{a}{b}} \right) \right] f(l(K)) d\xi = 0.$$  

Let $V(K)$ denote the left-hand side of the optimality condition. From the implicit function theorem, we have $\frac{\partial V(K)}{\partial B} \bigg|_K = -\frac{\partial l(K)}{\partial B} \bigg|_K \left[ l(K) - (K)^{-1/b} \left( (1+a)c \right) \left( \frac{1}{1+\frac{a}{b}} \right) \right] f(l(K))$. We obtain

$$\frac{\partial V(K)}{\partial B} \bigg|_K = -\frac{\partial l(K)}{\partial B} \bigg|_K \left[ l(K) - (K)^{-1/b} \left( (1+a)c \right) \left( \frac{1}{1+\frac{a}{b}} \right) \right] f(l(K)).$$

After substituting $l(K) = (1+a)c \left( K \right)^{-1/b} - (B(1+a) + P) \left( K \right)^{-1/b}$, since $l(K)$ decreases in $B$, it follows that $\frac{\partial V(K)}{\partial B} \bigg|_K < 0$ and $K$ strictly decreases in $B$.  

In perfect capital markets, the effect of an increase in the internal budget level on the equilibrium capacity investment level is zero (because the firm is not budget-constrained), and the effect on the expected equity value is positive. We next analyze the impact of internal budget level in imperfect capital markets under three different capital market conditions.
A.4.2.1. The Perfectly Competitive Credit Market with $U = 0$.

Here, we construct column 1 of imperfect market analysis in Table 2.

**Secured loan without default possibility.** At equilibria where the firm uses a secured loan without default possibility, we have $\dot{a} = 0$ as follows from Proposition 5. Therefore, $\dot{a}$ is not affected by the internal budget level. We have $\dot{K} = K^1(\dot{a}) = \left(\frac{\xi (1 + \frac{1}{\xi})}{(1 + a)c}\right)^{-b}$, $\dot{\pi} = \frac{cK^1(1 + \dot{a})}{(b + 1)} + B(1 + \dot{a}) + P$. For a given financing cost, $K^*$ does not change, $\pi^*$ increases in the internal budget level, therefore $\dot{K}$ does not change and $\dot{\pi}$ increases in $B$.

**Secured loan with default possibility.** At equilibria where the firm uses a secured loan (and invests in $K^1(\dot{a})$) with default possibility, the creditor’s expected return is characterized by the net gain from secured lending minus the expected default cost. From the firm’s perspective, an increase in the internal budget level does not alter the optimal capacity investment level for a given $a$. From the creditor’s perspective, an increase in the internal budget level decreases both the net gain and the default probability (as the firm optimally borrows less). The first effect works to increase $\dot{a}$ and the second effect to decrease it. With a uniform distribution of demand (and with $U \leq BC \frac{\xi l}{\xi u - \xi l}$), the second effect dominates the first, and $\dot{a}$ decreases in the internal budget level. As the equilibrium financing cost decreases, the firm invests more in capacity in equilibrium. The expected equity value also increases as the internal budget increases and financing becomes cheaper. This result is formally stated in the following proposition:

**Proposition A.3** In a perfectly competitive credit market with $U \leq BC \frac{\xi l}{\xi u - \xi l}$, at equilibria where the firm uses a secured loan (and invests in $K^1(\dot{a})$) with default possibility, $\dot{a}$ decreases and the expected equity value and the capacity investment level in equilibrium strictly increase in the internal budget level.

**Proof of Proposition A.3:** We will first prove that $\dot{a}$ is decreasing in the internal budget level $B$. Since this equilibrium is relevant for firms that may default but use a secured loan (Case ii of Proposition 4) and firms that may use an unsecured loan (Case iii of Proposition 4); we will analyze these two cases separately.

With the uniform distribution assumption of $\xi$, in equilibrium, we have

$$(cK^1(\dot{a}) - B) \left(\dot{a} - BC \frac{\xi (1 + \frac{1}{\xi})}{\xi u - \xi l} \frac{1}{cK^1(\dot{a})}\right) = U - BC \frac{\xi l}{\xi u - \xi l}$$

for $\dot{a} \in [a^l, a^d)$. 

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For firms that may default but always use a secured loan, we have \( a^i = 0 \) and
\[
(cK^1(a) - B) \left( a - BC \frac{\xi^l (1 + \frac{1}{\xi})}{\xi_u - \xi^l} \frac{1}{cK^1(a)} \right) \leq U - BC \frac{\xi^l}{\xi_u - \xi^l}
\]
for \( 0 \leq a \leq \hat{a} \) where equality only holds at \( \hat{a} \) (as follows from the Pareto-optimality of \( \hat{a} \)). For \( U \leq BC \frac{\xi^l}{\xi_u - \xi^l} \), we obtain
\[
\frac{\partial}{\partial B} \Lambda(a) = \left( a - BC \frac{\xi^l (1 + \frac{1}{\xi})}{\xi_u - \xi^l} \frac{1}{cK^1(a)} \right) > 0 \quad \text{for} \quad 0 \leq a \leq \hat{a}
\]
as follows from \( B < cK^1(a) \).

Let \( \hat{a}(B) \) denote the equilibrium financing cost at a given internal budget level \( B \). Let us assume that the unit financing cost increases in \( B \), i.e. \( \hat{a}(B_0) < \hat{a}(B_1) \) for \( B_0 < B_1 \). We will show by contradiction that this is not possible. We have \( \frac{\partial \Lambda(a)}{\partial B} > 0 \) for \( a \in [0, \hat{a}(B_0)] \).

As a result, we have \( U = \Lambda(\hat{a}(B_0); B_0) < \Lambda(\hat{a}(B_0); B_1) \), where the equality follows from the definition of the equilibrium. It follows from i) \( \Lambda(0; B_1) < 0 \), ii) the continuity of \( \Lambda(a) \) in \( a \), and iii) the Mean-value theorem, that there there exists an \( \hat{a}' < \hat{a}(B_0) \) (\( < \hat{a}(B_1) \)) such that \( \Lambda(\hat{a}'; B_1) = U \). This is a contradiction with \( \hat{a}(B_1) \) being the Pareto-optimal equilibrium at \( B_1 \).

For firms that may use unsecured loan, for \( a \in [0, a^d] \) we have
\[
\Lambda(a) = \begin{cases} 
(cK - B)a - F(b(K))BC - L(K) & 0 \leq a < a^i \\
(cK^1 - B)a - F(b(K^1))BC & a^i \leq a < a^d
\end{cases}
\]

We already know that for \( a \in [a^i, a^d] \), \( \Lambda(a) \) is increasing in \( B \) for \( U \leq BC \frac{\xi^l}{\xi_u - \xi^l} \), but we do not know the effect of \( B \) on \( \Lambda(a) \) for \( a < a^i \). With an increment in \( B \) from \( B_0 \) to \( B_1 \), if there exists \( a' < a'(B_0) \) such that \( \Lambda(a', B_1) > U \), i.e. expected return of the creditor when the firm uses an unsecured loan increases in \( B \) and crosses over \( U \), then we have \( \hat{a}(B_1) < \hat{a}(B_0) \) (as \( \hat{a}(B_0) \) is in the secured lending region, and \( \hat{a}(B_1) \) is in the unsecured lending region) and the equilibrium financing cost decreases in \( B \). If such an \( a' \) does not exist, then since \( \Lambda(a) \) is increasing in \( B \) for \( a \in [a^i, a^d] \), it follows that the equilibrium financing cost decreases in \( B \).

We now analyze the impact of \( B \) on \( \hat{K} \) and \( \hat{\pi} \). Since i) \( \hat{a} \) is decreasing in \( B \), ii) \( \pi^* \) and \( K^* \) increase in \( B \) for a given \( a \) (as follows from Lemma A.7), and iii) \( \pi^* \) and \( K^* \) decrease in \( a \) for a given \( B \) (as follows from Lemma A.3 and Lemma A.4, respectively), it follows that \( \hat{\pi} \) and \( \hat{K} \) increase in \( B \).

**Unsecured loan.** At equilibria where the firm uses an unsecured loan (and invests in \( K(\hat{a}) \)), we have \( \hat{a} < a^i \) and the creditor’s expected return in equilibrium is characterized by three terms: Net gain from secured lending, minus the expected loss due to the unsecured
part of the loan and the expected default cost. For a given \( a \), as follows from Lemma A.7, an increase in the internal budget level decreases \( K \). This is because as \( B \) increases, the value of the limited liability option of the firm decreases; and the firm optimally takes less investment risk and invests less in capacity. From the creditor’s perspective, increasing \( B \) has three distinct effects:

**Lemma A.8** If \( \xi \) is uniformly distributed in \( [\xi^l, \xi^u] \), when the firm uses unsecured lending (i.e. \( 0 \leq a < a^l \)), the creditor’s net gain from secured lending, the expected default cost and the expected loss due to the unsecured part of the loan decrease in the internal budget level \( B \).

**Proof of Lemma A.8:** As we discussed in the proof of Lemma A.6, for \( 0 \leq a < a^l \), with the uniform distribution assumption, the creditor’s expected return \( \Lambda(a) \) is given by

\[
\Lambda(a) = (cK - B)a - \left[ \frac{b(K) - \xi^l}{\xi^u - \xi^l} \right] BC - \frac{1}{2} \left( \frac{1}{b} \right) \frac{(l(K) - \xi^l)^2}{\xi^u - \xi^l} \tag{27}
\]

where \( b(K) = (1 + a)c(K)^{-1/b} - B(1 + a)(K)^{-1-1/b} \) and \( l(K) = b(K) - \frac{P}{(K)^{1+1/b}} \). In (27), the first term is the net gain from secured lending, the second term is the expected default cost, and the third term is the expected loss due to the unsecured part of the loan.

Since \( \frac{\partial K}{\partial B} < 0 \) from Lemma A.7, it follows that the net gain from secured lending decreases in the internal budget level \( B \).

For the impact of \( B \) on the default probability (and hence on the expected default cost), by taking the derivative of the default probability in (27) with respect to \( B \), we obtain

\[
\frac{1}{\xi^u - \xi^l} \left[ \frac{\partial b(K)}{\partial B} \right] + \frac{\partial b(K)}{\partial K} \left[ \frac{\partial K}{\partial B} \right] \tag{28}
\]

The first term represents the effect of \( B \) on the default risk of the firm for a fixed \( K \), whereas the second term captures the effect of the change in \( K \) on the default probability. Since \( b(K) \) is decreasing in \( B \), is increasing in \( K \) and \( K \) is decreasing in \( B \) from Lemma A.7, the expected default cost of the firm decreases in the internal budget level \( B \).

For the impact of \( B \) on the expected loss due to the unsecured part of the loan, by taking the derivative of the last term in (27) with respect to \( B \), we obtain

\[
-\left( \frac{(l(K) - \xi^l)(1 + a)}{\xi^u - \xi^l} \right)
+ \frac{\partial K}{\partial B} \left( \frac{1}{2} \frac{(l(K) - \xi^l)^2}{\xi^u - \xi^l} \right) \tag{29}
\]

The first term represents the effect of \( B \) on the expected loss for a fixed \( K \) and is negative (as \( l(K) > \xi^l \) by definition). The second term represents the effect of \( B \) on the expected
loss through altering $\overline{K}$. Since $l(K)$ is increasing in $K$ and $\overline{K}$ is decreasing in $B$ (from Lemma A.7), this term is also negative; therefore the expected loss due to the unsecured part of the loan decreases in the internal budget $B$. ■

The default risk decreases as i) for a fixed $\overline{K}$, the downside risk of the firm’s operating cash flows decreases; and ii) $\overline{K}$ decreases and the firm borrows less. The expected loss due to the unsecured part of the loan decreases due to a similar reasoning. The net gain from secured lending decreases as the firm borrows less. The first two effects work to decrease $\dot{a}$, whereas the third effect works to increase it.

We investigate the impact of the internal budget level $B$ on $\dot{a}$, $\dot{K}$ and $\dot{\pi}$ numerically. As depicted in Panel D of Figure 9, our numerical experiments show that the first two effects may dominate and $\dot{a}$ may decrease in the internal budget of the firm. Since $\dot{a}$ decreases and $B$ increases, $\dot{\pi}$ also increases (Panel F of Figure 9). For the effect on $\overline{K}(\dot{a})$, two drivers work in opposite directions: As follows from Lemma A.7, a higher $B$ induces the firm to invest less in capacity for a given $a$ due to the decreasing value of the limited liability option, but a lower $\dot{a}$ induces the firm to invest more due to the lower financing cost as follows from Lemma A.4. As depicted in Panel E of Figure 9, the first effect may dominate and $\dot{K}$ may decrease in the internal budget level. It is interesting to observe that a higher internal budget leads to a reduction in the equilibrium capacity investment level. In a perfectly competitive credit market with $U = 0$, this result is driven by the decreasing value of the limited liability option of the firm.

A.4.2.2. The Perfectly Competitive Credit Market with $U > 0$.

Here, we construct column 2 of imperfect market analysis in Table 2. We first provide the proof for Proposition 7 that states that $\dot{a}$ strictly increases and $\dot{K}$ strictly decreases at i) equilibria where the firm uses a secured loan without default possibility and ii) equilibria where the firm uses a secured loan with default possibility if $U > BC\frac{\xi^i}{\xi - \xi^i}$.

**Proof of Proposition 7:** We first provide the proof for the equilibria where the firm uses a secured loan without default possibility. We will first prove that $\dot{a}$ increases in the internal budget level $B$. Since this equilibrium is relevant for all the firms (Cases i, ii and iii of Proposition 4); we will analyze these cases separately.

For firms that never default (Case $i$ of Proposition 4), we have $\Lambda(a) = (cK^1(a) - B) a$ for $a \in [0, a^{max})$. We obtain $\frac{\partial}{\partial a} \Lambda(a) = -\dot{a} < 0$. With an increase in the internal budget level $B$, if there exists a feasible $a' < a^{max}$ such that $\Lambda(a') = U$, then in line with the proof of Proposition A.1, it is easy to prove that $\dot{a}$ increases in $B$. If such an $a'$ does not exist,
Figure 9: The effect of the internal budget level on the capacity investment level and the expected equity value in equilibria where the firm uses an unsecured loan (and invests in $K(\hat{a})$) in imperfect capital markets (in a perfectly competitive credit market with $U = 0$) with $\xi \sim U[20, 200]$ and $B \in [0, 150]$ with 1-unit increments: A higher $B$ decreases the default probability (Panel C) and increases the creditor’s expected return without the default cost (Panel B). The total effect is that $\Lambda(a)$ increases (Panel A) and $\hat{a}$ decreases (Panel D). A lower $\hat{a}$ and a higher $B$ increase $\hat{\pi}$ (Panel F). Despite the decrease in $\hat{a}$, $\hat{K}$ decreases (Panel E) as a higher internal budget level decreases the value of the limited liability option of the firm.

then the loan contract is not offered in equilibrium; then by definition, $\hat{a} = a^{max}$ and the equilibrium financing cost also increases.

For firms that may default but always use a secured loan (Case ii of Proposition 4) and firms that may use an unsecured loan (Case iii of Proposition 4), we already established that for $a \in [a^d, a^{max})$, $\Lambda(a)$ is decreasing in $B$, but we do not specify the effect of $B$ on $\Lambda(a)$ for $a < a^d$. Let $\hat{a}(B)$ denote the equilibrium financing cost for a given $B$. Since $\hat{a}(B) \in [a^d, a^{max})$, with a small increment in $B$ from $B_0$ to $B_1$, we can guarantee that $\Lambda(a; B_1) < U$ for $\forall a < \hat{a}(B_0)$ because i) $\Lambda(a; B_0) < U$ for $\forall a < \hat{a}(B_0)$ from the definition of the equilibrium, and ii) $|\frac{\partial}{\partial B} \Lambda(a)|$ and $|\frac{\partial}{\partial a} \Lambda(a)|$ are bounded. Therefore $\hat{a}$ increases locally in $B$ in these cases.
At the equilibria where the firm uses a secured loan with default possibility, with the uniform distribution assumption of $\xi$, the creditor’s expected return is given by

$$(cK^1(\dot{a}) - B) \left( \dot{a} - BC \frac{\bar{\xi}(1 + \frac{1}{b})}{\xi_u - \xi_l} \frac{1}{cK^1(\dot{a})} \right) = U - BC \frac{\xi_l}{\xi_u - \xi_l}$$

for $\dot{a} \in [0, \dot{a}^d)$. This equilibrium is relevant for firms that may default but use a secured loan (Case ii of Proposition 4), and for firms that may use an unsecured loan (Case iii of Proposition 4). Since $B < cK^1(\dot{a})$ and $U > BC \frac{\xi_l}{\xi_u - \xi_l}$, we have $\left( \dot{a} - BC \frac{\bar{\xi}(1 + \frac{1}{b})}{\xi_u - \xi_l} \frac{1}{cK^1(\dot{a})} \right) > 0$. It follows that $\frac{\partial}{\partial B} \Lambda(a) |_{\dot{a}} = - \left( \dot{a} - BC \frac{\bar{\xi}(1 + \frac{1}{b})}{\xi_u - \xi_l} \frac{1}{cK^1(\dot{a})} \right) < 0$.

Similar to the arguments above, with a small increment in $B$ from $B_0$ to $B_1$, we can guarantee that $\Lambda(a; B_1) < U$ for $\forall \ a < \dot{a}(B_0)$ because i) $\Lambda(a; B_0) < U$ for $\forall \ a < \dot{a}(B_0)$ by the definition of equilibrium, and ii) $\left| \frac{\partial}{\partial B} \Lambda(a) \right|$ and $\left| \frac{\partial}{\partial a} \Lambda(a) \right|$ are bounded. Since $\Lambda(\dot{a}(B_0); B_1) < U$, we have $\dot{a}(B_1) > \dot{a}(B_0)$ and the equilibrium financing cost increases locally in $B$.

For the impact of $B$ on $K$, since we have $\dot{K} = K^1(\dot{a})$ and it is independent of $B$ for a given $\dot{a}$, a higher $\dot{a}$ strictly decreases $\dot{K}$.

**Secured loan without default possibility.** At equilibria where the firm uses a secured loan (and invests in $K^1(\dot{a})$) without default possibility, the creditor’s expected return is given by the net gain from secured lending. As $B$ increases, the firm borrows less and the creditor’s expected return decreases. Therefore, the creditor increases $\dot{a}$ to satisfy his reservation return. As the equilibrium financing cost increases, $\dot{K}$ decreases. With a sufficient increase in $B$, the creditor may not be able to satisfy $U$; hence does not offer any loan in equilibrium (which we denote by $\dot{a} = a^{max}$). For the effect on the expected equity value of the firm at this equilibrium, there exists a positive effect of a higher internal budget for a given financing cost, and a negative effect of a higher financing cost. As depicted in Panel C (and in Panel D with a smaller scale) of Figure 10, as long as the loan contract is offered in equilibrium, the first effect may dominate the second and $\pi$ may increase in $B$. If the loan contract is not offered in equilibrium, with a further increase in $B$, then there may be a sharp decline in $\pi$. This is because going from an equilibrium with a loan contract to another equilibrium without a loan contract creates a discontinuous increase in the equilibrium financing cost (Panel D).

**Secured loan with default possibility.** At equilibria where the firm uses a secured loan (and invests in $K^1(\dot{a})$) with default possibility, with the uniform distribution assumption,
Figure 10: The effect of the internal budget level on the capacity investment level and the expected equity value in equilibria where the firm uses a secured loan (and invests in $K^1(\tilde{a})$) without default possibility in imperfect capital markets (in a perfectly competitive credit market with $U > 0$) with $U = 10$, $\xi \sim U[20, 120]$ and with internal budget $B \in [2000, 2800]$ with 5-unit increments: A higher $B$ increases the equilibrium financing cost as long as the firm borrows in equilibrium, i.e. $\tilde{a} < a^{max}$ (Panel A). In Panel A, when $\tilde{a} = a^{max}$, $\tilde{a}$ decreases in $B$ but the firm does not borrow in equilibrium, so the decreasing part is irrelevant to our analysis. Since $\tilde{a}$ increases, the firm’s equilibrium capacity investment level decreases for $\tilde{a} < a^{max}$ (Panel B). Except for one case, despite the increase in $\tilde{a}$, $\hat{\pi}$ increases (Panel D) as the positive effect of an increased internal budget dominates the negative effect of higher $\tilde{a}$. The only exception is the case in which with a further increase in $B$, the creditor does not offer any loan ($\tilde{a} = a^{max}$). In this case, there is a decrease in $\hat{\pi}$.

The creditor’s expected return $\Lambda(\tilde{a})$ can be rewritten as

$$\left(CK^1(\tilde{a}) - B\right)\left(\tilde{a} - BC\frac{\zeta \left(1 + \frac{1}{\xi}\right)}{(\xi^u - \xi^l)} \frac{1}{cK^1(\tilde{a})}\right) + BC \frac{\xi^l}{\xi^u - \xi^l}$$

(29)

for $\tilde{a} \in [0, a^{\tilde{a}}]$. In (29), the first term in parenthesis is the amount of lending and the second term in parenthesis is the unit marginal profit of lending. For each unit of the loan $(cK^1(\tilde{a}) - B)$, the creditor earns $\tilde{a}$ minus the expected default cost. The third term is the saving from the default cost due to the minimum demand realization. At a given $\tilde{a}$, the unit marginal profit of the creditor is independent of the internal budget of the firm, but a higher budget level means a smaller loan as $B$ is larger. With a sufficiently small creditor’s reservation return ($U \leq BC\frac{\xi^l}{\xi^u - \xi^l}$), the unit marginal profit is negative and the
A creditor charges lower financing cost in equilibrium (and the capacity investment level of the firm increases) as proven in Proposition A.3. For sufficiently high $U$ ($U > BC \frac{\xi^l}{\xi_u - \xi^l}$), the marginal profit is positive at $a = \dot{a}$. Hence the creditor’s expected return decreases in $B$, and to compensate for the reduction in the loan size, the creditor increases the unit financing cost in equilibrium and the capacity investment level decreases in $B$ as proven in Proposition 7. In parallel with our analytical results, as depicted in Panel B of Figure 11, for sufficiently small $U$, $\dot{a}$ decreases in $B$ and the equilibrium capacity investment level increases (Panel C). For sufficiently large $U$, as depicted in Panel B of Figure 12, $\dot{a}$ increases in $B$ and the equilibrium capacity investment level decreases (Panel C).

Figure 11: The effect of the internal budget level on the capacity investment and the expected equity value in equilibria where the firm uses a secured loan (and invests in $K^1(\dot{a})$) with default possibility in imperfect capital markets (in a perfectly competitive credit market with $U > 0$) with $U = 5$, $\xi \sim U[20, 120]$, $BC = 75$ and with internal budget $B \in [1600, 1900]$ with 5-unit increments: Since $U = 5$ is smaller than the threshold value $\left(BC \frac{\xi^l}{\xi_u - \xi^l} = 8.33\right)$, a higher $B$ decreases the equilibrium financing cost (Panel B) and increases the firm’s equilibrium capacity investment level (Panel C). Since $\dot{a}$ decreases in $B$, together with the direct positive effect of the increased internal budget, the expected equity value in equilibrium increases (Panel D).
Figure 12: The effect of the internal budget level on the capacity investment and the expected equity value in equilibria where the firm uses a secured loan (and invests in $K^1(\dot{a})$) with default possibility in imperfect capital markets (in a perfectly competitive credit market with $U > 0$) with $U = 20$, $\xi \sim U[20, 120]$, $BC = 75$ and with internal budget $B \in [1600, 1900]$ with 5-unit increments:

For sufficiently low $B$, the firm uses a secured loan with default possibility, i.e. $\dot{a} \leq a^d$ as depicted in Panel A. For sufficiently high $B$, the loan contract is still offered but the firm uses a secured loan without default possibility in equilibrium, i.e. $\dot{a} \in (a^d, a^{max})$ (Panel A). Since $U = 20$ is larger than the threshold value $\left(BC \frac{\xi^l}{\xi - \xi^l} = 8.33\right)$, a higher $B$ increases the equilibrium financing cost when there is default possibility (Panel A and Panel B drawn with a smaller scale). Similarly, a higher $B$ increases $\dot{a}$ when there is no default possibility in equilibrium (Panel A). Therefore, the firm’s equilibrium capacity investment level decreases (Panel C). Despite the increase in $\dot{a}$, $\dot{\pi}$ increases (Panel D) as the positive effect from an increased internal budget dominates the negative effect of an increase in $\dot{a}$.

For the effect of the internal budget level $B$ on the expected equity value in equilibrium, we can prove that $\dot{\pi}$ increases in $B$ if $U \leq BC \frac{\xi^l}{\xi - \xi^l}$. This is because there exists a positive effect of a higher internal budget for a given financing cost (as follows from Proposition A.3), and another positive effect of a lower financing cost in equilibrium (as follows from Lemma A.3). For $U > BC \frac{\xi^l}{\xi - \xi^l}$, there exists a positive effect of a higher internal budget for a given financing cost, and a negative effect of a higher financing cost in equilibrium. As depicted in Panel D of Figure 12, the first effect may dominate and $\dot{\pi}$ may increase.
**Unsecured loan.** Before analyzing the impact of the internal budget level $B$ at equilibria where the firm uses an unsecured loan (and invests in $\mathcal{K}(\hat{a})$), we first analyze the impact of $B$ on the creditor’s expected return. As depicted in Panel A of Figure 13, for $U = 0$, the equilibrium financing cost is in the unsecured lending region ($\hat{a} < a^l$) and $\hat{a}$ may decrease in $B$. A similar behavior can be observed for a sufficiently small $U > 0$. For sufficiently large $U$, it follows from Panel A that not only the equilibrium financing cost can be in the secured lending region ($\hat{a} \geq a^l$), but also, regardless of its location, $\hat{a}$ may increase in $B$. This is because the creditor’s expected return without default cost decreases in $B$ in that range (as depicted in Panel B).

**Figure 13:** The effect of the internal budget level on the expected return of the creditor at equilibria where the firm uses an unsecured loan (and invests in $\mathcal{K}(\hat{a})$) in imperfect capital markets (in a perfectly competitive credit market with $U > 0$) with $\xi \sim U[20, 140]$ and $B \in \{125, 285, 445\}$: A higher $B$ decreases the default probability (Panel D) and increases (decreases) the creditor’s expected return without the default cost for sufficiently low (high) $a$ (Panel C). The total effect is that $\Lambda(a)$ increases (decreases) in $B$ for sufficiently low (high) level of $a$ (Panel A, and Panel B with a larger scale). Therefore, for a sufficiently small $U$ (for example, $U < 20$), $\hat{a}$ decreases in $B$. For a sufficiently large $U$ (for example, $U > 100$), $\hat{a}$ increases in $B$.

We investigate the impact of the internal budget level $B$ on $\hat{a}$, $\hat{K}$ and $\hat{\pi}$ numerically. Figure 14 illustrates a numerical example for this case in a perfectly competitive credit market with $U = 0$ case. Similar to Figure 9, $\hat{a}$ is always in the unsecured lending region and decreases with an increase in $B$ (Panel D). Therefore, $\hat{\pi}$ increases (Panel G). For the
effect on $\dot{K}$, a higher $B$ induces the firm to invest less in capacity for a given financing cost (as the value of limited liability option decreases); a lower $\dot{a}$ to increase it. It follows from Panel F that the former effect may outweigh the latter and $\dot{K}$ may decrease.

Figure 14: The effect of the internal budget level on capacity investment and expected equity value in equilibria where the firm uses an unsecured loan (and invests in $\overline{K}(\dot{a})$) in imperfect capital markets (in a perfectly competitive credit market with $U = 0$) with $\xi \sim U[20, 140]$ and $B \in [200, 450]$ with 5-unit increments: A higher $B$ decreases the default probability (Panel C) and increases the creditor’s expected return without the default cost (Panel B). The total effect is that $\Lambda(a)$ increases (Panel A) and $\dot{a}$ decreases (Panel D). A lower $\dot{a}$ increases $\dot{\pi}$ (Panel F). Despite the decrease in $\dot{a}$, $\dot{K}$ decreases (Panel E) as a higher internal budget decreases the value of the limited liability option of the firm.

Figure 15 and Figure 16 replot Figure 14 with $U = 5$ and $U = 160$ respectively. As depicted in Figure 15, when the firm uses an unsecured loan in equilibrium, i.e. $\dot{a} < a^l$, the results are identical with $U = 0$ case: $\dot{a}$ and $\dot{K}$ decrease and $\dot{\pi}$ increases with an increase in $B$. With sufficiently large $U$, as depicted in Panel D of Figure 16, when the firm uses an unsecured loan in equilibrium, the only difference is that an increase in $B$ may increase $\dot{a}$. This is consistent with our earlier observation in Panel A of Figure 13: For a sufficiently high $U$, $\Lambda(a)$ decreases in $B$. Despite the increase in the financing cost, a higher $B$ increases $\dot{\pi}$ (Panel G). Since $\dot{a}$ is increasing and the value of the limited liability option is decreasing with an increase in $B$, $\dot{K}$ decreases (Panel F).
In summary, in our numerical experiments we observe that at equilibria where the firm uses an unsecured loan (and invests in $K(\dot{a})$), $\dot{a}$ may increase or decrease with an increase in $B$, but regardless of this effect, $\dot{K}$ decreases and $\dot{\pi}$ increases.

Figure 15: The effect of the internal budget level on the capacity investment level and the expected equity value in equilibria where the firm uses an unsecured loan (and invests in $K(\dot{a})$) in imperfect capital markets (in a perfectly competitive credit market with $U > 0$) with $U = 5$, $\xi \sim U[20, 140]$ and $B \in [200, 450]$ with 5-unit increments: Similar to $U = 0$ case, a higher $B$ decreases the default probability (Panel C) and increases the creditor’s expected return without the default cost (Panel B). The total effect is that $\Lambda(a)$ increases (Panel A) and $\dot{a}$ decreases (Panel D). Lower $\dot{a}$ increases $\dot{\pi}$ (Panel H). When the firm uses an unsecured loan in equilibrium, i.e. $\dot{a} < a^l$, despite the decrease in $\dot{a}$, $\dot{K}$ decreases (Panel F) as a higher internal budget decreases the value of the limited liability option of the firm. When the firm uses a secured loan with default possibility in equilibrium, i.e. $a^l < \dot{a} < a^d$, $\dot{a}$ still decreases as depicted in Panel D and Panel E with a larger scale. In this case, as follows from Proposition A.3, since $U = 5$ is smaller than the threshold value $BC \frac{\xi_u}{\xi_l} = 12.5$, $\dot{K}$ increases in $B$ (Panel G).
Figure 16: The effect of the internal budget level on the capacity investment level and the expected equity value in equilibria where the firm uses an unsecured loan (and invests in $K(\hat{a})$) in imperfect capital markets (in a perfectly competitive credit market with $U > 0$) with $U = 160$, $\xi \sim U[20, 140]$ and $B \in [200, 450]$ with 5-unit increments: A higher $B$ decreases the default probability (Panel C) and decreases the creditor’s expected return without the default cost (Panel B). The total effect is that $\Lambda(a)$ decreases (Panel A) and $\hat{a}$ increases (Panel D). As $B$ increases, the equilibrium moves from the unsecured lending region to the secured lending with default possibility region, then to the secured lending without default possibility region. For a sufficiently large $B$, $\hat{a}$ decreases in $B$, but in this case we have $\hat{a} = a^{\text{max}}$ and the creditor does not offer any contract; hence these data points are irrelevant for our discussion. When the firm uses a secured loan with default possibility in equilibrium, i.e. $\hat{a}^d < \hat{a} \leq \hat{a}^d$, since $U = 160$ is larger than the threshold value $\left(BC\frac{\xi}{\xi - \xi} = 12.5\right)$, $\dot{K}$ decreases in $B$ as follows from Proposition 7.

A.4.2.3. The Monopolist Creditor.

Here, we construct column 3 of imperfect market analysis in Table 2.

**Secured loan without default possibility.**

**Proposition A.4** At equilibria where the firm uses a secured loan (and invests in $K^1(\hat{a})$) without default possibility, $\hat{a}$ strictly decreases and the firm’s capacity investment level and expected equity value in equilibrium strictly increase in the internal budget level.

**Proof of Proposition A.4:** Let $\hat{a}^M$ denote the equilibrium financing cost for the monopolist creditor. $\hat{a}^M$ satisfies $FOC(\hat{a}^M) = 0$ where $\frac{\partial \Lambda(a)}{\partial a} = FOC(a) = cK^0(1 + a)^{(b-1)}(1 + a + ab) - B$. We will first prove that $\hat{a}^M$ decreases with an increase in the internal budget level.
Since this equilibrium is relevant for firms that never default (Case i of Proposition 4), firms that may default but use a secured loan (Case ii of Proposition 4) and firms that may use an unsecured loan (Case iii of Proposition 4); we will analyze these cases separately.

For firms that never default and firms that may default but use a secured loan, it follows from the proof of Proposition 5, we have $a_N \geq a^d$ and the expected return of the creditor, $\Lambda(a)$, is unimodal in $a$, thus $\dot{a}^M = a^N$ is the unique maximizer. From the implicit function theorem, we obtain $\frac{\partial \dot{a}^M}{\partial B} = -\frac{\partial FOC}{\partial a} \bigg|_{\dot{a}^M}$. Since $\dot{a}^M$ is the unique maximizer, we have $\text{sgn} \left( \frac{\partial \dot{a}^M}{\partial B} \right) = \text{sgn} \left( \frac{\partial FOC}{\partial a} \right)$. We obtain $\frac{\partial FOC}{\partial B} = -\frac{1}{cK^0(1-\gamma)^b} < 0$. Hence $\dot{a}^M$ strictly decreases in $B$.

For firms that may use an unsecured loan, as follows from the proof of Proposition 5, the creditor’s expected return is unimodal in $a$ for $a \in [a^l, a^{max})$, but we cannot guarantee unimodality for $a < a^l$. In fact, our numerical observations demonstrate that $\Lambda(a)$ is not unimodal in this case. With an increase in $B$, two cases can happen. Either the global maximum is still in the secured lending without default region, i.e. $\dot{a}^M = a^N$, and $\dot{a}^M$ decreases in $B$ as follows from the implicit function theorem; or the global maximum switches to the unsecured lending region, i.e. $\dot{a}^M < a^l$ and $\dot{a}^M$ decreases by definition of this region.

Similar to the proof of Proposition A.1, it is easy to establish that $\dot{K}$ increases in $B$ through the decrease in $\dot{a}^M$, and $\dot{\pi}$ increases in $B$ through the decrease in $\dot{a}^M$, as well as through the direct effect of $B$.

**Secured loan with default possibility.** At equilibrium where the firm uses a secured loan (and invests in $K^1(\dot{a})$) with default possibility, in determining $\dot{a}^M$, the creditor equates the marginal cost (that is the reduction in the net gain of the creditor with an increase in $a$) with the marginal revenue (that is the reduction in the expected default cost with an increase in $a$). With an increase in $B$, the marginal cost term decreases, and the marginal revenue term increases. As depicted in Panel A of Figure 17, the decrease in the marginal revenue of lending may dominate the decrease in the marginal cost of lending at the optimal solution; hence $\dot{a}^M$ decreases. As depicted in Panel B, at equilibrium where the firm uses a secured loan with default possibility, an increase in $B$ may decrease $\dot{a}$; thus $\dot{K}$ (Panel C) and $\dot{\pi}$ (Panel D) increase. With a sufficiently large increase in $B$, the firm uses a secured loan without default possibility, and the impact of $B$ is consistent with Proposition A.4.

**Unsecured loan.** At equilibrium where the firm uses an unsecured loan (and invests in $\overline{K}(\dot{a})$), as depicted in Panel A of Figure 18, the creditor’s expected return is not unimodal in $a$. As follows from Panel B, for small $B$, the firm uses an unsecured loan in equilibrium,
Figure 17: The effect of the internal budget level on the capacity investment level and the expected equity value in equilibria where the firm uses a secured loan (and invests in $K^1(\hat{a}^M)$) in imperfect capital markets (for the monopolist creditor) with $\xi \sim U[20, 120]$ and $B \in [180, 300]$ with 5-unit increments.

i.e. $\hat{a}^M < \hat{a}^l$. In this case, an increase in $B$ decreases $\hat{a}^M$. As $B$ increases, the value of the limited liability option of the firm decreases for a given $a$. This induces the firm to decrease the investment level $\bar{K}$ as follows from Lemma A.7. On the other hand, a lower $\hat{a}^M$ induces the firm to increase the investment level as follows from Lemma A.4. As can be seen from Panel C of Figure 18, the latter may dominate the former and $\hat{K}$ may increase. This is different from the perfectly competitive credit market case. The main driver is the fact that $\hat{a}^M$ is sufficiently larger than the one in a perfectly competitive credit market equilibrium $\hat{a}$; and a reduction in $\hat{a}^M$ has a more significant positive effect on $\hat{K}$. Since the expected equity value increases in $B$ for a given financing cost $a$ (as follows from Lemma A.7), and financing becomes cheaper, $\hat{\pi}$ increases as depicted in Panel D. After a sufficient increase in the internal budget level $B$, the optimal $\hat{a}^M$ switches from the unsecured lending region to the secured lending region, i.e. from one local maximizer to the other local maximizer. In this case, there is a sharp increase in $\hat{a}^M$ which induces a sharp decline in $\hat{K}$ (Panel C) and $\hat{\pi}$ (Panel D). As we increase $B$ further, $\hat{a}^M$ stays in the secured lending with default possibility region, i.e. $\hat{a}^l < \hat{a}^M < \hat{a}^d$. For a sufficiently high $B$, the firm uses a secured loan without default possibility in equilibrium. When the firm uses a secured loan, $\hat{a}^M$ decreases and $\hat{K}$ and $\hat{\pi}$ increase.
Figure 18: The effect of the internal budget level on the capacity investment level and the expected equity value in equilibria where the firm uses an unsecured loan (and invests in $\bar{K}(a^M)$) in imperfect capital markets (for the monopolist creditor) with $\xi \sim U[20,200]$ and $B \in [800,1500]$ with 5-unit increments.

B The Two-Product Firm Analysis

B.1 Analysis for a Given Technology

In this section, we provide the detailed analysis for the two-product case for a given technology $T \in \{D,F\}$. In particular, §B.1.1 provides the detailed analysis for the firm’s problem with each technology $T$. The characterization of the creditor’s expected return is in §B.1.2. In §B.1.3, we analyze the equilibrium for a given technology $T$ in the two-product case.

B.1.1 Analysis of the Firm’s Problem for A Given Technology $T$

Let $K_T$ denote the firm’s capacity investment vector with technology $T \in \{D,F\}$, where (with a little abuse of notation) $K_F = K_F$ and $K'_D = (K'_D^1, K'_D^2)$. We adopt this notation to save space by presenting the analysis of both both flexible and dedicated technology at once. In stage 1, the firm will have borrowed $e_T$ and invested in $K_T$ to maximize its expected equity value with respect to demand uncertainty. In stage 2, the firm observes the demand realization $\tilde{\xi}$ and determines the production quantities $Q_T = (q_T^1, q_T^2)$ within the existing capacity limit $K_T$ to maximize the stage 2 equity value. We now solve the firm’s problem using backward induction starting from stage 2.
Stage 2, Production Decision: Let $\Gamma^*_T(K_T, \xi)$ denote the optimal stage 2 operating profit. The production decision only affects the operating profit in stage 2. Thus, maximizing the stage 2 equity value is equivalent to maximizing the operating profit. Since we assume that production is costless, the optimal stage 2 operating profit is equal to the maximum sales revenue that can be obtained using the existing capacity $K_T$:

$$\Gamma^*_T(K_T, \xi) = \max_{Q_T \in \Theta_T} Q_T' p(Q_T; \tilde{\xi}) = \max_{Q_T \in \Theta_T} \tilde{\xi}' Q_T^{1+\frac{1}{b}},$$

(30)

where $\Theta_T = \{ Q_T : Q_T \geq 0; 1' Q_T \leq K_T \}$ and $\Theta_D = \{ Q_D : Q_D \geq 0; Q_D \leq K_D \}$ are the feasibility sets for production quantity levels for each technology $T$.

**Proposition B.1** The optimal production quantity vector in stage 2 with technology $T \in \{D, F\}$ for given $K_T$ and $}\tilde{\xi}$ is given by

$$Q^*_D = K_D, \quad Q^*_F = \frac{K_F}{\tilde{\xi}_1 + \tilde{\xi}_2} \tilde{\xi}^{-b}.\]$$

Since the unit production cost is zero, the firm optimally utilizes the entire available capacity. With dedicated technology, the optimal production quantities are equal to the available capacity levels for each product. With flexible technology, the firm allocates the available capacity $K_F$ in such a way that the marginal profits for each product are equal.

**Proof of Proposition B.1:** Let $f(Q_T) = \xi' Q_T^{1+\frac{1}{b}}$ and $Q^*_T$ denote the optimal production vector that solves (30) for technology $T \in \{F, D\}$. It is easy to establish that $f(Q_T)$ is strictly concave in $Q_T'$ and $Q^*_T$ is unique. Since $\frac{\partial f}{\partial q_i} = (1 + 1/b) \tilde{\xi}_i q_T^{1/b} > 0$ and with $b \in (-\infty, -1)$, $\lim_{q_T \to 0^+} \frac{\partial f}{\partial q_T} = \infty$, the non-negativity constraints will be non-binding and the capacity constraint will be binding at optimality. With the dedicated technology, this yields $Q^*_D = K_D$ and

$$\Gamma_D^*(K_D, \tilde{\xi}) = f(Q^*_D) = \tilde{\xi}' K_D^{1+1/b}.\]$$

With the flexible technology, according to the KKT conditions, $Q^*_F$ solves $\frac{\partial f}{\partial q_1} |_{q_1^*} = \frac{\partial f}{\partial q_2} |_{K_F - q_1^*}$. After some algebra, we obtain $Q^*_F = \frac{K_F}{\tilde{\xi}_1 + \tilde{\xi}_2} \tilde{\xi}^{-b}$ and

$$\Gamma_F^*(K_F, \tilde{\xi}) = \frac{K_F^{1+\frac{1}{b}}}{\left(\tilde{\xi}_1^{-b} + \tilde{\xi}_2^{-b}\right)^{1+\frac{1}{b}}} \left(\tilde{\xi}_1^{-b} + \tilde{\xi}_2^{-b}\right)^{-\frac{1}{b}} K_F^{1+\frac{1}{b}}.\]$$
Stage 1, Capacity Choice and External Financing: In stage 1, for given $B \geq 0$, the firm determines the optimal capacity investment level $K_T$ and the optimal external borrowing level $e_T$. The optimal expected equity value of the firm, $\pi_T^*$, is

$$
\pi_T^* = \max_{K_T, e_T} B + e_T - c_T 1'K_T - (B + e_T - c_T 1'K_T)^+ + \mathbb{E}\left[\Pi_T^*(K_T, e_T, B, \tilde{\xi})\right]
$$

s.t. $K_T \geq 0$

$$
e_T \geq (c_T 1'K_T - B)^+.
$$

The firm has available budget $B$ and borrows $e_T$ from the creditor. Out of this sum $B + e_T$, the firm invests $c_T 1'K_T$ in capacity and places the remainder $(B + e_T - c_T 1'K_T)^+$ into the cash account at the risk-free rate. The cash holdings and the operating profits from the capacity investment are included in $\mathbb{E}\left[\Pi_T^*(K_T, e_T, B, \tilde{\xi})\right]$.

To derive $\Pi_T^*$, note that two outcomes are possible in stage 2: If the firms final cash position (optimal operating profits, the cash account holdings and the salvage value of capacity) is sufficient to cover the face value of the loan, i.e. $\Gamma_T^*(K_T, B) + (B + e_T - c_T 1'K_T)^+ + \gamma_T c_T 1'K_T \geq e_T(1 + a_T)$, then the firm does not default; otherwise, it does. If the firm does not default, it repays the face value of its loan and liquidates the physical assets, generating $P$. If the firm defaults, the cash on hand and the ownership of the collateralized physical asset are transferred to the creditor. The firm receives the remaining cash (if any) after the face value of the loan is deducted from its seized assets. The optimal equity value can be written as

$$
\Pi_T^*(K_T, e_T, B, \tilde{\xi}) = \left[\Gamma_T^*(K_T, \tilde{\xi}) + (B + e_T - c_T 1'K_T)^+ + \gamma_T c_T 1'K_T - e_T(1 + a_T) + P\right]^+,
$$

where we invoke the assumptions that the bankruptcy cost $BC$ is borne by the creditor as an out-of-pocket expenditure, the risk-free rate is 0, and the shareholders have limited liability.

Before characterizing the optimal capacity investment vector $K_T^*$, we make two observations. First, in parallel with the single-product analysis, for any $K_T \geq 0$, we have $e_T^* = (c_T 1'K_T - B)^+$ (i.e. the firm only borrows for financing the capacity investment). Second, since $\xi$ has a symmetric bivariate normal distribution ($\sigma_1 = \sigma_2 = \sigma$), in the optimal solution, we have $K_D^1 = K_D^2$, and we can use a single capacity $K_D^*$ to characterize $K_D^* = (K_D^*, K_D^*)$. In the rest of the analysis, we drop the vector notation and use $K_D^*$ and $K_F^*$.

For a sufficiently large value of the physical asset $P_T$, the unique optimal capacity investment level $K_T^*$ is given by the analogue of Proposition 1.
Proposition B.2 For the firm with \( P \geq P_T \doteq 2c_T^{b+1} \left[ \xi_u \left( 1 + \frac{1}{b} \right) \right]^{-b} \left[ 1 - \frac{\xi_i}{\xi_u(1+b)} \right] [1 - \gamma_T]^{b+1} \), the optimal capacity investment level \( K_T^* \), \( T \in \{ D, F \} \) is given by

\[
K_T^* = \begin{cases} 
K_T^0 = \left( \frac{M_T(1+\frac{1}{b})}{(1-\gamma_T)c_T} \right)^{-b} & \text{if } B \geq \eta_T K_T^0 \\
\frac{B}{\eta_T} \left( \frac{M_T(1+\frac{1}{b})}{(1-\gamma_T)c_T} \right)^{-b} & \text{if } \eta_T K_T^1 \leq B < \eta_T K_T^0 , \\
K_T^1 = \left( \frac{M_T(1+\frac{1}{b})}{(1-\gamma_T)c_T} \right)^{-b} & \text{if } B < \eta_T K_T^1 
\end{cases}
\] (32)

where \( M_D = M_T, M_F = E \left[ \left( \xi_1^{-b} + \xi_2^{-b} \right)^{-\frac{1}{b}} \right], \eta_D = 2c_D, \eta_F = c_F \). The firm’s optimal expected equity value is given by

\[
\pi_T^* = \begin{cases} 
\eta_T K_T^0 \left( \frac{1-\gamma_T}{b+1} \right) + B + P & \text{if } B \geq \eta_T K_T^0 \\
\eta_T M_T \left( \frac{B}{\eta_T} \right) \left( \frac{1+\frac{1}{b}}{1-\gamma_T} \right) + P + \gamma_T B & \text{if } \eta_T K_T^1 \leq B < \eta_T K_T^0 \\
\eta_T K_T^1 \left( \frac{1+\gamma_T}{1-(b+1)} \right) + B(1+a) + P & \text{if } B < \eta_T K_T^1. 
\end{cases}
\]

Proof of Proposition B.2: We only demonstrate the proof for the flexible technology.

The proof for dedicated technology follows from a similar argument. When the firm borrows \( (B < c_F K_F) \), we have \( \Pi_F^* (\xi, K_F, B) = \left( \tilde{\xi}_1^{-b} + \tilde{\xi}_2^{-b} \right)^{-\frac{1}{b}} K_F^{(1+\frac{1}{b})} + B(1+a_F) + P - c_F (1+a_F-Gamma) K_F \) and is non-negative for

\[
\left( \tilde{\xi}_1^{-b} + \tilde{\xi}_2^{-b} \right)^{-\frac{1}{b}} \geq l_F(K_F) \doteq K_F^{\frac{1}{b}} \left( 1 + a_F - \gamma_F \right) c_F - K_F^{-\left( -\frac{1}{b} \right)} [B (1+a_F) + P],
\]

and \( \Pi_F^* (\xi, K_F, B) = 0 \) for \( \left( \tilde{\xi}_1^{-b} + \tilde{\xi}_2^{-b} \right)^{-\frac{1}{b}} \leq l_F(K_F) \). As \( \left( \xi_1^{-b} + \xi_2^{-b} \right)^{-\frac{1}{b}} \) is strictly increasing in \( \xi \), we can define unique minimum and maximum realization of \( \left( \xi_1^{-b} + \xi_2^{-b} \right)^{-\frac{1}{b}} \);

min\( \left( \xi_1^{-b} + \xi_2^{-b} \right)^{-\frac{1}{b}} = 2^{-\frac{1}{b}} \xi^1 \) and max\( \left( \xi_1^{-b} + \xi_2^{-b} \right)^{-\frac{1}{b}} = 2^{-\frac{1}{b}} \xi^u \). We obtain

\[
\frac{\partial l_F(K_F)}{\partial K_F} = -\frac{1}{b} K_F^{-\left( \frac{1}{b} - 1 \right)} (1 + a_F - \gamma_F) c_F + \left( 1 + \frac{1}{b} \right) K_F^{-\left( -\frac{1}{b} \right)} [B (1+a_F) + P] > 0. \] (33)

Therefore, we can define unique \( K_F^l < K_F^u \) such that \( l_F(K_F^l) \doteq 2^{-\frac{1}{b}} \xi^l \) and \( l_F(K_F^u) \doteq 2^{-\frac{1}{b}} \xi^u \).

Since \( l_F(K_F) \) is strictly increasing in \( K_F \), we have \( \Pi_F^* (\xi, K_F, B) \geq 2^{-\frac{1}{b}} \xi^u \) for \( K_F \geq K_F^u \); hence the optimal equity value \( \Pi_F^* (\xi, K_F, B) = 0 \) at each \( \tilde{\xi} \) and the optimal expected equity value \( \pi_F^* = 0 \) for \( K_F \in \left[ K_F^u, \infty \right) \). Therefore, it is sufficient to analyze the problem for \( K_F \in \left[ 0, K_F^u \right) \).

We have three separate cases to consider:

**Case 1:** For \( K_F \in \left[ 0, \frac{B}{c_F} \right] \), the firm does not borrow, and the expected equity value of the firm, \( \pi_F^* \), is

\[
\pi_F^* = \max\left( \left( \xi_1^{-b} + \xi_2^{-b} \right)^{-\frac{1}{b}} \right) K_F^{(1+\frac{1}{b})} + B + P - c_F (1-\gamma_F) K_F.
\]
Case 2: For $K_F \in \left[ \frac{B}{c_F}, K_F^l \right]$, the firm optimally borrows, and is always able to pay back the face value of the loan.\(^6\) The expected equity value of the firm, $\pi_F^\star$, is

$$
\pi_F^\star = \max_{K_F} \mathbb{E} \left[ \left( \xi_1 - b + \xi_2 - b \right)^{-\frac{1}{b}} K_F^{(1+\frac{1}{b})} + B(1+a_F) + P - c_F(1+a_F-\gamma_F)K_F \right].
$$

Case 3: For $K_F \in (K_F^l, K_F^u)$ the firm always borrows, and for some demand realization $\hat{\xi}$, is not able to pay back the face value of the loan; hence the expected equity value of the firm, $\pi_F^\star$, is

$$
\pi_F^\star = \max_{K_F} \int \int_{\mathcal{Y}_F(K_F)} \left[ \left( \xi_1 - b + \xi_2 - b \right)^{-\frac{1}{b}} K_F^{(1+\frac{1}{b})} + B(1+a_F) + P - c_F(1+a_F-\gamma_F)K_F \right] f(\xi_1, \xi_2) d\xi_1 d\xi_2,
$$

where $f(\xi_1, \xi_2)$ is the probability density function of $\xi$ and

$$
\mathcal{Y}_F(K_F) \doteq \left\{ \xi : \xi' \geq (\xi^l, \xi^l); l_F(K_F) \leq \left( \xi_1 - b + \xi_2 - b \right)^{-\frac{1}{b}} \right\} \leq 2\frac{\xi}{b}^u.
$$

Let $g_F(K_F)$ denote the objective function in the overall optimization problem and $g_F^1(K_F)$ denote the objective function in case $i$. We have

$$
g_F(K_F) = \begin{cases} 
g_F^1(K_F) & \text{if } K_F \in \left[ 0, \frac{B}{c_F} \right], \\
g_F^2(K_F) & \text{if } K_F \in \left[ \frac{B}{c_F}, K_F^l \right], \\
g_F^3(K_F) & \text{if } K_F \in (K_F^l, K_F^u). 
\end{cases}
$$

Paralleling Proposition 1, it is easy to verify that $g_F(K_F)$ is strictly concave in $K_F$ for $K_F \in [0, K_F^l]$ and is kinked at $K_F = \frac{B}{c_F}$. We obtain

$$
\frac{\partial g_F^1(K_F)}{\partial K_F} = \int \int_{\mathcal{Y}_F(K_F)} \left[ \left( 1 + \frac{1}{b} \right) \left( \xi_1 - b + \xi_2 - b \right)^{-\frac{1}{b}} K_F^{(\frac{1}{b})} - (1+a_F-\gamma_F) c_F \right] f(\xi_1, \xi_2) d\xi_1 d\xi_2. \tag{34}
$$

It is easy to verify that $\frac{\partial^2 g_F^1(K_F)}{\partial K_F} = \frac{\partial^2 g_F^2(K_F)}{\partial K_F} = \frac{\partial^2 g_F^3(K_F)}{\partial K_F}$; hence $g_F(K_F)$ does not have a kink at $K_F = K_F^l$. Define $G_F(K_F, \xi) \doteq (1 + \frac{1}{b}) \left( \xi_1 - b + \xi_2 - b \right)^{-\frac{1}{b}} K_F^{(\frac{1}{b})} - (1+a_F-\gamma_F) c_F$ as the integrand of (34) without $f(\xi_1, \xi_2)$. Note that, $G_F(K_F, \xi)$ is strictly increasing in $\xi$, and strictly decreasing in $K_F$. We will use the unimodality property of $g_F(K_F)$ in the rest of the proof.

We define $\hat{K}_F \doteq \left( 2\frac{\xi}{b}^u (1 + \frac{1}{b}) \right)^{-\frac{1}{b}}$. We have $l_F(\hat{K}_F) = (1 + \frac{1}{b}) 2\frac{\xi}{b}^u \left[ 1 - \frac{B(1+a_F) + P}{K_F(1+a_F-\gamma_F)c_F} \right] < 2\frac{\xi}{b}^u$, thus $\hat{K}_F < K_F^u$ and $\hat{K}_F$ is in the feasible region of $K_F$. Note that for $\xi' = (\xi^u, \xi^u)$, $G_F \left( \hat{K}_F, \xi^u \right) = 0$. Since $\left( \xi_1 - b + \xi_2 - b \right)^{-\frac{1}{b}}$ takes its maximum value at $\xi = \xi^u$, and $G_F(K_F, \xi)$ is

---

\(^6\)It can be shown that for $2\frac{\xi}{b}^l \geq 0$ and $\gamma_F \geq 0, K_F^l \geq \frac{B}{c_F}$, where the equality only holds if $\xi^l = 0$ and $\gamma_F = 0$. 

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strictly increasing in $\xi$, we have $G_F(K_F, \xi) < 0$ for $\xi \in \mathcal{Y}_F(K_F)$. Therefore $\frac{\partial g_F^b(K_F)}{\partial K_F} |_{K_F} < 0$. As $G_F(K_F, \xi)$ is strictly decreasing in $K_F$, $\frac{\partial g_F^b(K_F)}{\partial K_F} < 0$ for $K_F \in [\hat{K}_F, K_F^u]$.

In summary, $g_F(K_F)$ is strictly concave in $K_F$ for $K_F \in [0, K_F^l]$ (with a kink at $K_F = \frac{B}{c_F}$), and is strictly decreasing in $K_F$ for $K_F \in [\hat{K}_F, K_F^u]$. It follows that $g_F(K_F)$ will be unimodal if $K_F^l \geq \hat{K}_F$. Since $\frac{\partial g_F^b(K_F)}{\partial K_F} > 0$ (from (33)), this is equivalent to $l_F(\hat{K}_F) \leq 2^{-\frac{1}{2}} \xi^l$, which gives us

$$B \geq B^b_F \doteq c_F \hat{K}_F \left[1 - \frac{\xi^l}{\xi^u (1 + \frac{1}{b})}\right] \left[1 - \frac{\gamma_F}{1 + a_F}\right] - \frac{P}{1 + a_F}. \quad (35)$$

For $B \geq B^b_F$, the optimal $K_F^*$ is in the strictly concave part (where the firm uses a secured loan). By substituting $\hat{K}_F = \left(2^{-\frac{1}{2}} \xi^u (1 + \frac{1}{b})\right)^{-b}$, we obtain $B^b_F = \frac{1}{(1 + a_F)} [T_F(a_F) - P]$, where $T_F(a_F) = 2 F^{b+1} \left[\xi^u (1 + \frac{1}{b})\right]^b \left[1 - \frac{\xi^l}{\xi^u (1 + \frac{1}{b})}\right] [1 - \gamma_F]^{b+1}$. Since $T_F(a_F)$ is strictly decreasing in $a_F$, we can define $\overline{T}_F \doteq T_F(0) = 2 c_F^{b+1} \left[\xi^u (1 + \frac{1}{b})\right]^b \left[1 - \frac{\xi^l}{\xi^u (1 + \frac{1}{b})}\right] [1 - \gamma_F]^{b+1}$ such that, for $P \geq \overline{T}_F$, $B^b_F \leq 0 \forall a_F \geq 0$. It follows from (35) that $B \geq B^b_F$ is satisfied for $B \geq 0$ and $a_F \geq 0$.

As a result, the optimal capacity investment level $K_F^*$ is in the strictly concave part. In that case, the firm’s optimal equity value $\Pi_F^*$ is given by

$$\Pi_F^*(\xi, K_F, B) = \left(\bar{\xi}_1^{-b} + \bar{\xi}_2^{-b}\right)^{-\frac{1}{b}} K_F^{(1 + \frac{1}{b})} + \gamma_F c_F K_F + B + c_F K_F - a_F (c_F K_F - B)^+. $$

Therefore, the optimal expected equity value of the firm, $\pi^*$, is given by:

$$\pi^*_F = \max_{K_F} \mathbb{E} \left[\left(\xi_1^{-b} + \xi_2^{-b}\right)^{-\frac{1}{b}}\right] K_F^{(1 + \frac{1}{b})} + \gamma_F c_F K_F + B + c_F K_F - a_F (c_F K_F - B)^+ \quad \text{s.t.} \quad K_F \geq 0$$

Paralleling the proof of Proposition 1 of the single product case, it can be proven that the optimal capacity investment level for flexible technology, $K_F^*$, is unique and is given by

$$K_F^* = \begin{cases} 
K_F^0 \doteq \left(\mathbb{E} \left[\xi_1^{-b} + \xi_2^{-b}\right]^{-\frac{1}{b}}\right)^{-b} \left(\frac{1 + \frac{1}{b}}{1 - \gamma_F c_F}\right) \quad & \text{if } B \geq c_F K_F^0 \\
\frac{B}{c_F} \quad & \text{if } c_F K_F^1 \leq B < c_F K_F^0 \\
K_F^1 \doteq \left(\mathbb{E} \left[\xi_1^{-b} + \xi_2^{-b}\right]^{-\frac{1}{b}}\right)^{-b} \left(\frac{1 + \frac{1}{b}}{1 + a_F - \gamma_F c_F}\right) \quad & \text{if } B < c_F K_F^1 
\end{cases}$$

The optimal expected equity value $\pi^*_F$ can be obtained by substituting $K_F^*$ in the objective function. \(\blacksquare\)
B.1.2 Characterization of the Creditor’s Expected Return

Anticipating the firm’s actions, the creditor determines the unit financing cost $a_T \geq 0$. As follows from Proposition B.2, for any $a_T \geq 0$ a firm with an internal budget $B \geq \eta_T K_T^0$ where $\eta_D = 2c_D, \eta_F = c_F$ never borrows from the creditor and the creditor does not have any returns. Throughout the analysis, we will exclude this case and focus on the firm with $B < \eta_T K_T^0$. Since the firm does not borrow from the creditor if $B \geq \eta_T K_T^0$, the maximum unit financing cost that can be offered is given by $a_T^{\max} = \left[ \left( \frac{\eta_T K_T^0}{B} \right)^{-\frac{1}{b}} - 1 \right] (1 - \gamma_T)$. Let $\Lambda_T(a_T)$ denote the creditor’s expected return from the lending business with technology $T \in \{D, F\}$.

**Proposition B.3** For the firm with $B < \eta_T K_T^0$ and $P \geq \overline{P}_T$ as defined in Proposition B.2, the creditor’s expected return with technology ($T \in \{D, F\}$), $\Lambda_T(a_T)$, is characterized by

i) $(\eta_T K_T^1 - B) a_T \quad 0 \leq a_T < a_T^{\max} \quad$ if $B \geq \eta_T K_T^0 (1 - \gamma_T) \left[ 1 - \frac{N_T}{M_T(1 + \frac{1}{\xi})} \right]$,

ii) $(\eta_T K_T^1 - B) a_T - F(b_T(K_T^1))BC \quad 0 \leq a_T < a^d_T \quad$ if $B < \eta_T K_T^0 (1 - \gamma_T) \left[ 1 - \frac{N_T}{M_T(1 + \frac{1}{\xi})} \right]$,

$(\eta_T K_T^1 - B) a_T \quad a^d_T \leq a_T < a_T^{\max} \quad$ otherwise,

where $b_T(K_T) = K_T^{-\frac{1}{b}} \left( (1 + a_T - \gamma_T) \eta_T - \frac{B(1 + a_T)}{K_T} \right)$, $N_D = \xi^l$, $N_F = 2 - \xi^l$ with $M_D = \bar{\xi}, M_F = \mathbb{E} \left[ \left( \xi_1^{-b} + \xi_2^{-b} \right)^{-\frac{1}{b}} \right], \eta_D = 2c_D, \eta_F = c_F, a_T^{\max} = \left[ \left( \frac{\eta_T K_T^0}{B} \right)^{-\frac{1}{b}} - 1 \right] (1 - \gamma_T)$ and $a^d_T$ is the unique solution to $B = \eta_T K_T^0 (1 - \gamma_T)^{-b} \left[ 1 - \frac{N_T}{M_T(1 + \frac{1}{\xi})} \right] \left( \frac{1 + a_T - \gamma_T}{1 + a_T^{\max}} \right)$.

**Proof of Proposition B.3:** The proof is similar to the proof of Proposition 4 and is omitted.

B.1.3 Equilibrium Analysis

**Remark B.1** In the perfect capital market equilibrium, for any firm with $B \geq 0$, we have $K_T = K_T^0 = \left( \frac{M_T(1 + \frac{1}{\xi})}{(1 - \gamma_T)^{\xi_T}} \right)^{-b}$ and $e_T = (\eta_T K_T^0 - B)^+ \quad$ for $M_D = \bar{\xi}, M_F = \mathbb{E} \left[ \left( \xi_1^{-b} + \xi_2^{-b} \right)^{-\frac{1}{b}} \right], \eta_D = 2c_D, \eta_F = c_F$. The expected equity value of the firm is given by $\pi_T = B + P + \frac{\eta_T K_T^0 (1 - \gamma_T)^{-b}}{(1 + \frac{1}{\xi})}$. 

**Proof of Remark B.1:** The proof is similar to the proof of Remark 1 and is omitted.

**Proposition B.4** In a perfectly competitive credit market with $U = 0$, $\dot{a}_T = 0$ for case (i) and $\dot{a}_T \in (0, a_T^{d})$ for case (ii) of Proposition B.3. For $U > 0$, if the contract is offered, $\dot{a}_T$ can take any value in $(0, a_T^{\max})$. The monopolist creditor always offers a contract, and $\dot{a}_T \in (0, a_T^{d})$. In particular, $\dot{a}_T = a_T^N$ if $a_T^N \geq a_T^d$ and $\dot{a}_T \in (a_T^N, a_T^d)$ otherwise, where $a_T^N$ is the unique solution to $(1 + a_T^N - \gamma_T)^{(b - 1)} (1 + a_T^N - \gamma_T + a_T^N b) = \frac{B}{\eta_T K_T^0 (1 - \gamma_T)^{-\frac{1}{b}}}$. 

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**Proof of Proposition B.4:** The proof is similar to the proof of Proposition 5 and is omitted. ■

B.2 The Effect of Demand Uncertainty on Each Technology

In this section, we provide the analytical proofs and related numerical experiments that we use in developing the results summarized in Table 3.

B.2.1 Dedicated Technology

In perfect capital markets, $\dot{K}_D$ and $\dot{\pi}_D$ do not depend on the demand variability or the demand correlation as follows from Remark B.1.

B.2.1.1. The Perfectly Competitive Credit Market with $U \geq 0$.

Here, we construct the results related to dedicated technology in the imperfect market analysis column of Table 3.

**Secured loan without default possibility.**

**Proposition B.5** In a perfectly competitive credit market with $U \geq 0$ and with the monopolist creditor, at equilibria where the firm uses a secured loan (and invests in $\dot{K}_D = K_D^1(\dot{a}_D)$) without default possibility, the expected equity value and the capacity investment level with the dedicated technology do not change with a small increase in the demand variability $\sigma$ or the demand correlation $\rho$.

**Proof of Proposition B.5:** In a perfectly competitive credit market with $U = 0$, since $\dot{a}_D = 0$ and is independent of $\rho$ and $\sigma$; and for a given $a_D$, $K_D^*$ and $\pi_D^*$ are also independent of $\sigma$ and $\rho$, the result holds globally. The proof for the perfectly competitive credit market with $U > 0$ case is similar to the proof of Proposition 7 and is omitted. The proof for the monopolist creditor case is similar to the proof of Proposition A.2 and is omitted. ■

**Secured loan with default possibility.** As follows from Proposition 8, at equilibria where the firm uses a secured loan with default possibility, the equilibrium capacity investment level and the expected equity value decrease in the demand variability and the demand correlation through an increase in $\dot{a}_D$. We now provide the proof for Proposition 8. **Proof of Proposition 8:** We first provide the proof for the perfectly competitive credit market with $U = 0$ and $U > 0$ cases. For $a_D \in [0,a_D^{\text{max}})$, the creditor’s expected return with the dedicated technology is given by

$$\Lambda_D(a_D) = (2c_D K_D^1 - B) a_D - BC \Pr \left( \xi_1 + \xi_2 < 2 \xi \left( 1 + \frac{1}{b} \right) \left[ 1 - \frac{B(1 + a_D)}{2c_D K_D^1 (1 + a_D - \gamma_D)} \right] \right),$$

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where the default term is non-zero for \( a_D < a_D^d \). Since \((\xi_1, \xi_2)\) is bivariate normal with \( N(\xi, \Sigma) \), \( \xi_1 + \xi_2 \) is also Normally distributed with mean \( 2\xi \) and standard deviation \( \sigma = \sigma \sqrt{2(1 + \rho)} \). Let \( C_D \) denote the right-hand side of the default probability. Since \( b < -1 \) and \( B < 2c_DK_D^1[1 - \gamma_D] \), we obtain \( C_D < 2\xi \). We have \( Pr(\xi_1 + \xi_2 < C_D) = \Phi \left( \frac{C_D - 2\xi}{\sigma} \right) \) where \( \Phi(.) \) is the cumulative distribution function of the standard normal random variable.

For \( a_D < a_D^d \), from \( \frac{\partial}{\partial \rho} \sigma = \frac{\sigma}{\sigma} > 0 \), \( \frac{\partial}{\partial \sigma} \sigma = \sqrt{2(1 + \rho)} > 0 \), and \( C_D < 2\xi \) we obtain

\[
\begin{align*}
\frac{\partial \Lambda_D(a_D)}{\rho} &= -BC \phi \left( \frac{C_D - 2\xi}{\sigma} \right) \left( \frac{2\xi - C_D}{\sigma^2} \right) \frac{\partial \sigma}{\partial \rho} < 0, \\
\frac{\partial \Lambda_D(a_D)}{\sigma} &= -BC \phi \left( \frac{C_D - 2\xi}{\sigma} \right) \left( \frac{2\xi - C_D}{\sigma^2} \right) \frac{\partial \sigma}{\partial \sigma} < 0
\end{align*}
\]

where \( \phi(.) \) is the density function of the standard normal random variable.

Let \( \dot{a}_D(\sigma_0), \dot{a}_D(\sigma_1) (\dot{a}_D(\rho_0), \dot{a}_D(\rho_1)) \) denote the equilibrium financing cost with demand variability (correlation) \( \sigma_0 \) and \( \sigma_1 \) (\( \rho_0 \) and \( \rho_1 \)) respectively with \( \sigma_1 > \sigma_0 \) (\( \rho_1 > \rho_0 \)). We will focus on \( \dot{a}_D(\sigma_0) < a_D^d \) and \( \dot{a}_D(\rho_0) < a_D^d \) such that the firm defaults in equilibrium. Since the expected return of the creditor strictly decreases in \( \sigma \) and \( \rho \) for \( a_D < a_D^d \), it follows from the definition of the Pareto-optimal equilibrium that the equilibrium level of financing cost increases in \( \sigma \) and \( \rho \), i.e. \( \dot{a}_D(\sigma_1) > \dot{a}_D(\sigma_0) \) and \( \dot{a}_D(\rho_1) > \dot{a}_D(\rho_0) \). Similar to the proof of Proposition 6, it is easy to establish that the optimal expected equity value of the firm and the optimal capacity investment level \( K_D^1 \) strictly decrease in \( a_D \).

For the monopolist creditor, let \( \dot{a}_D^M \) denote the equilibrium financing cost. Proposition B.4 shows that there exists a global maximizer \( \dot{a}_D^M \in (a_D^N, a_D^d) \) where \( a_D^N \) is given in Proposition B.4 and \( a_D^d \) is given in Proposition B.3. \( \dot{a}_D^M \) satisfies \( FOC_D(\dot{a}_D^M) = 0 \) where \( FOC_D(a_D) \) is the first order derivative of \( \Lambda_D(a_D) \) and is given by

\[
2c_DK_D^0(1 - \gamma_D)^{-b}(1 + a_D - \gamma_D)^{(b-1)}(1 + a_D - \gamma_D + a_Db) - B - BC \phi \left( \frac{C_D - 2\xi}{\sigma} \right) \frac{\partial C_D}{\partial a_D}
\]

where \( \phi(.) \) is the probability density function of the standard normal random variable. We will only provide the proof for \( \sigma \). The result with respect to \( \rho \) can be proven in a similar fashion and is omitted. We have two subcases to consider.

If the creditor’s expected return \( \Lambda_D(a_D) \) is unimodal in \([0, a_D^d]\), \( \dot{a}_D^M \) denotes the unique maximizer. In this case, from the implicit function theorem, we have \( sgn \left( \frac{\partial FOC_D}{\partial \sigma} \right) = sgn \left( \frac{\partial FOC_D}{\partial \sigma} \right) \bigg|_{\dot{a}_D^M} \).

We obtain

\[
\frac{\partial FOC_D(a_D)}{\partial \sigma} = BC \frac{\partial C_D}{\partial a_D} \phi' \left( \frac{C_D - 2\xi}{\sigma} \right) \left( \frac{C_D - 2\xi}{\sigma^2} \right) \frac{\partial \sigma}{\partial a_D}
\]

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Using the identity \( \phi'(z) = -z\phi(z) \) of the standard normal random variable, the derivative expression above can be rewritten as

\[
\frac{\partial \varphi C_D(a_D)}{\partial \sigma} = -BC \frac{\partial C_D}{\partial a_D} \phi \left( \frac{C_D - 2\xi}{\sigma} \right) \left( \frac{C_D - 2\xi}{\sigma} \right)^2 \frac{1}{\sigma^2} \frac{\partial \sigma}{\partial \sigma}.
\]

From \( \frac{\partial \sigma}{\partial \sigma} = \sqrt{2(1 + \rho)} > 0 \) and

\[
\frac{\partial C_D}{\partial a_D} = -\frac{2\xi(1 + \frac{1}{2})B}{2cD K_D^0(1 - \gamma_D)^{-b}} \left[ 1 + \frac{-b b (1 + a_D - \gamma_D)}{(1 + a_D - \gamma_D)^{b+1}} \right] < 0
\]

(as follows from \( b < -1 \), it follows that \( \frac{\partial \varphi C_D(a_D)}{\partial \sigma} > 0 \). Therefore, \( \dot{a}_D^M \) is strictly increasing in \( \sigma \). We note here that although we cannot prove the unimodality of \( \Lambda_D(a_D) \), it is unimodal in our numerical experiments.

If the creditor’s expected return \( \Lambda_D(a_D) \) is not unimodal in \([0, a_D^N] \), since \( \Lambda_D(a_D) \) is increasing in \( a_D \) for \( a_D < a_D^N \), there exist at least two local maximizers. Let \( a_D^N < a_1, a_k, ..., a_n < a_D^L \) denote the \( n \) local maximizers in increasing order of \( a_D \). Without loss of generality let \( a_k \) denote the global maximizer for a given \( \sigma_0 \), i.e. \( \dot{a}_D^M(\sigma_0) = a_k(\sigma_0) \). It follows from the implicit function theorem result above that all the local maximizers increase as we increase \( \sigma \) from \( \sigma_0 \) to \( \sigma_1 \). To prove that \( \dot{a}_D^M(\sigma_1) < \dot{a}_D^M(\sigma_0) \), it is sufficient to show that with an increase in \( \sigma \) the global maximizer does not switch to the other local maximizers with a lower index than \( k \). We have

\[
\frac{\partial \Lambda_D(a_D)}{\sigma} = -BC \phi \left( \frac{C_D - 2\xi}{\sigma} \right) \left( \frac{2\xi - C_D}{\sigma^2} \right) \frac{\partial \sigma}{\partial \sigma} < 0.
\]

Similar to the proof of Proposition 6, if \( | \frac{\partial}{\partial \sigma} \Lambda_D(a_D) | \) is decreasing in \( a_D \), then \( \Lambda(a_k(\sigma_0), \sigma_1) \geq \Lambda(a_D, \sigma_1) \) for \( a_D < a_k(\sigma_0) \), hence \( \dot{a}_D^M(\sigma_1) < a_k(\sigma_0) \) cannot hold. \( | \frac{\partial}{\partial \sigma} \Lambda_D(a_D) | \) is decreasing in \( a_D \), if and only if \( Z(a_D) = \phi \left( \frac{C_D - 2\xi}{\sigma} \right) \left( 2\xi - C_D \right) \) is decreasing in \( a_D \). We obtain

\[
\frac{\partial Z(a_D)}{a_D} = \phi \left( \frac{C_D - 2\xi}{\sigma} \right) \frac{\partial C_D}{\partial a_D} \left[ -1 + \left( \frac{2\xi - C_D}{\sigma} \right)^2 \right].
\]

It is not possible to determine the sign of the last term in brackets except some special cases based on parameter restrictions\(^7\). However, since \( | \frac{\partial}{\partial \sigma} \Lambda_D(a_D) | \) is bounded and \( \Lambda_D(a_k(\sigma_0), \sigma_0) \geq \Lambda_D(a_D, \sigma_0) \) for \( a_D < a_k(\sigma_0) \) from the optimality of \( a_k(\epsilon_0) \), for a sufficiently small increase in \( \sigma \), we can guarantee that \( \Lambda(a_k(\sigma_0), \sigma_1) \geq \Lambda(a_D, \sigma_1) \) for \( a_D < a_k(\sigma_0) \), hence \( \dot{a}_D^M(\sigma_1) < a_k(\sigma_0) \) cannot hold. Therefore \( \dot{a}_D^M \) increases in \( \sigma \).

Similar to the proof of Proposition 6, it is easy to establish that the optimal expected equity

\(^7\)For sufficiently small \( \sigma \), this term is negative; hence \( \dot{a}_D^M \) increases in \( \sigma \).
value of the firm and the optimal capacity investment level $K_D^1$ locally decreases in $a_D$. ■

B.2.1.2. The Monopolist Creditor.

**Secured loan without default possibility.** As follows from Proposition B.5, at equilibria where the firm uses a secured loan without default possibility, the equilibrium capacity investment level and the expected equity value locally decrease in the demand variability and the demand correlation through an increase in $a_D$.

**Secured loan with default possibility.** As follows from Proposition 8, at equilibria where the firm uses a secured loan with default possibility, the equilibrium capacity investment level and the expected equity value locally decrease in the demand variability and the demand correlation through an increase in $a_D$.

B.2.2 Flexible Technology

In perfect capital markets, the firm’s equilibrium capacity decision and the expected equity value with the flexible technology increases with an increase in demand variability and a decrease in demand correlation as, using Assumption 2, the term $M_F = E \left[ \left( \xi_1^{-b} + \xi_2^{-b} \right)^{-\frac{1}{b}} \right]$ increases. We now analyze the impact of $\rho$ and $\sigma$ on $K_F^*$ and $\pi_F^*$ in imperfect capital markets.

B.2.2.1. The Perfectly Competitive Credit Market with $U \geq 0$.

Here, we construct the results related to dedicated technology in the imperfect market analysis column of Table 3.

**Secured loan without default possibility.**

**Proposition B.6** If Assumption 2 holds, in a perfectly competitive credit market with $U \geq 0$ at equilibria where the firm uses a secured loan (and invests in $K_F = K_F^1(\hat{a}_F)$) without default possibility, the expected equity value and the capacity investment level with flexible technology strictly increase with an increase in the demand variability $\sigma$ and a decrease in the demand correlation $\rho$.

**Proof of Proposition B.6** With $U = 0$, in equilibrium we have $\hat{a}_F = 0$ and the financing cost is independent of $\rho$ and $\sigma$. For a given $a_F$, the effect of $\rho$ and $\sigma$ on $M_F^*$ and $\pi_F^*$ are determined by the effect on $M_F$. With an increase in $\sigma$ and a decrease in $\rho$, as follows from Assumption 2, $M_F$ increases and so do $K_F^*$ and $\pi_F^*$.

With $U > 0$, in equilibrium, the creditor’s expected return is determined by the net gain from secured lending, i.e. $\Lambda_F(\hat{a}_F) = (c_F K_F^1(\hat{a}_F) - B) \hat{a}_F$. From the firm’s perspective, for a given $a_F$, a decrease in $\rho$ or an increase in $\sigma$ increases the optimal capacity investment
level $K^1_F(a_F)$ as follows from Assumption 2. Therefore, the firm borrows more, and $\Lambda_F(a_F)$ increases for $a_F \geq a^d_F$. If the effect of $\rho$ and $\sigma$ on $\Lambda_F(a_F)$ for $a_F < a^d_F$ is such that the equilibrium is still in the secured lending without default region, then it is easy to show that $\dot{a}_F$ decreases. If the effect on $\Lambda_F(a_F)$ for $a_F < a^d_F$ is such that $\Lambda_F(a_F) = U$ for $a_F < a^d_F$, then $\dot{a}_F$ still decreases. Since $\dot{a}_F$ decreases and $M_F$ increases, $\dot{K}_F$ and $\dot{\pi}_F$ increase.

Secured loan with default possibility. At equilibria where the firm uses a secured loan (and invests in $\dot{K}_F = K^1_F(\dot{a}_F)$) with default possibility, we investigate the effect of the demand variability and the correlation on $\dot{a}_F$, $\dot{K}_F$ and $\dot{\pi}_F$ numerically. We had discussed the effect of demand variability and demand correlation on the key performance measures with $U = 0$ by using Figure 3. We replot this figure with $U > 0$. As depicted in Figure 19, the only difference from the $U = 0$ case is that, with $U > 0$, for low correlation levels, $\dot{a}_F$ increases in demand variability, whereas with $U = 0$, $\dot{a}_F$ does not change (and is equal to 0). Similar to $U = 0$ case, at low correlation levels, the diversification benefit of operating in two markets is sufficiently high such that default risk is very low, and the firm uses a secured loan without default possibility in equilibrium. Because of positive reservation return of the creditor, in this case, we have $\dot{a}_F > 0$, and in parallel with Proposition B.6, with an increase in $\sigma$, the firm invests and borrows more, and thus, $\dot{a}_F$ decreases.
Figure 19: The effect of the demand correlation (\( \rho \)) and the demand variability (\( \sigma \)) on the flexible technology investment at equilibria where the firm uses a secured loan (and invests in \( \dot{K}_F = K^1_F(\dot{a}_F) \)) with default possibility in imperfect capital markets (in a perfectly competitive credit market with \( U > 0 \)) with \( U = 20 \) and \( \sigma \in [3, 6] \) with 0.5-unit increments: In perfect markets, a higher \( \sigma \) or a lower \( \rho \) increases the capacity-pooling value; thus \( \dot{K}_F \) (Panel A) and \( \dot{\pi}_F \) (Panel B) increase. In imperfect capital markets, a decrease in \( \rho \) decreases \( \dot{a}_F \) (Panel C). This is because for a given \( a_F \), the diversification benefit increases (and the default risk decreases) and the net gain of the creditor from secured lending increases as the firm borrows more due to the higher value of capacity pooling. Therefore, \( \dot{a}_F \) decreases. With a decrease in \( \rho \), since capacity-pooling value increases, and \( \dot{a}_F \) decreases, \( \dot{K}_F \) (Panel D) and \( \dot{\pi}_F \) (Panel E) increase. In imperfect capital markets, a higher \( \sigma \) decreases (increases) \( \dot{a}_F \) for low (high) correlation levels (Panel C). This is because, for a given \( a_F \), at low correlation levels the diversification benefit is sufficiently large such that default risk is low and is not significantly affected by the change in \( \sigma \). A higher \( \sigma \) increases the value of capacity-pooling, and the firm invests and borrows more. Therefore, the net gain from secured lending increases and \( \dot{a}_F \) decreases. At high correlation levels, a higher \( \sigma \) significantly increases the default risk for a given \( a_F \), and the creditor increases \( \dot{a}_F \). For low correlation levels, an increase in \( \sigma \) increases \( \dot{K}_F \) (Panel D) and \( \dot{\pi}_F \) (Panel E). This is because the value of capacity-pooling for a given \( a_F \) increases, and \( \dot{a}_F \) decreases. For high correlation levels, the negative effect of a higher \( \dot{a}_F \) dominates the positive effect of the increase in the value of capacity-pooling, thus \( \dot{K}_F \) (Panel D) and \( \dot{\pi}_F \) (Panel E) decrease.

In some of the numerical experiments we observe that, contrary to intuition, at equilibria where the firm uses a secured loan without default possibility, the firm’s capacity investment level and expected equity value in equilibrium may increase in demand correla-
tion $\rho$. Although the capacity-pooling value decreases for a given $a_F$, $\hat{a}_F$ may also decrease and may outweigh the former effect. Recall that from the firm’s perspective, for a given $a_F$, an increase in $\rho$ decreases the optimal capacity investment level $K_F^1(a_F)$ as the value of capacity-pooling decreases (under Assumption 2). From the creditor’s perspective, an increase in $\rho$ has three distinct effects: First, for a fixed $K_F^1(a_F)$ the default risk increases. This is because the firm is able to generate lower returns as the value of capacity-pooling decreases. Second, the net gain from secured lending decreases as the firm invests and, in turn, borrows less. Third, as the firm borrows less, the default risk decreases. The first two effects work to increase $\hat{a}_F$, whereas the third effect works to decrease it.

As depicted in Panel D of Figure 20, at close to perfectly positive correlation levels, the third effect may dominate the first two and an increase in $\rho$ may decrease $\hat{a}_F$. Despite the reduction in the capacity-pooling value, a reduction in $\hat{a}_F$ may increase $\dot{K}_F$ (Panel E) and $\dot{\pi}_F$ (Panel F). These results are also obtained with $U = 0$ as depicted in Panels A, B and C.

Figure 20: The effect of the demand correlation ($\rho$) and the demand variability ($\sigma$) on the flexible technology investment at equilibria where the firm uses a secured loan (and invests in $K_F^1(\hat{a}_F)$) with default possibility in imperfect capital markets (in a perfectly competitive credit market with $U > 0$) with $\sigma \in [3, 4.5]$ with 0.25-unit increments and $\rho \in [0.95, 0.99]$ with 0.005-unit increments: With $U = 20$, as depicted in Panel C, an increase in $\rho$ may decrease $\hat{a}_F$ for $\sigma \in \{3, 3.25\}$, and $\dot{K}_F$ (Panel E) and $\dot{\pi}_F$ (Panel F) may increase. The same pattern is also relevant for the $U = 0$ case as depicted in Panels A, B and C.
The Monopolist Creditor.

Secured loan without default possibility.

**Lemma B.1** With the monopolist creditor, under Assumption 2, at equilibria where the firm uses a secured loan (and invests in $K^F = K^F_1 (\hat{a}^M_F)$) without default possibility with the flexible technology, $\dot{a}_F$ increases with an increase in the demand variability $\sigma$ and a decrease in the demand correlation $\rho$.

**Proof of Lemma B.1:** We will only demonstrate the proof for $\sigma$; the proof for $\rho$ is similar and omitted. Let $\dot{a}^M_F$ denote the equilibrium financing cost. As follows from Proposition B.4, the creditor offers a unique financing cost $\hat{a}^M_F = a^N_F$ where $a^N_F$ is the unique solution to $J_F(a^N_F) = 0$ and $J_F(a_F)$ is characterized by

$$J_F(a_F) = (1 + a_F - \gamma_F)^{(b-1)} (1 + a_F - \gamma_F + a_F b) - \frac{B}{c_F K_F^0 (1 - \gamma_F)^b}.$$

Since $\hat{a}^M_F$ is the maximizer, from the implicit function theorem, we have $\text{sgn} \left( \frac{\partial \dot{a}^M_F}{\partial \sigma} \right) = \text{sgn} \left( \frac{\partial J_F}{\partial \sigma} \right) \bigg|_{\dot{a}^M_F}$. Under Assumption 2, $M_F = E \left[ \left( \xi_1^{-b} + \xi_2^{-b} \right)^{-\frac{1}{b}} \right]$ (and thus $K_F^0$) is strictly increasing in $\sigma$, it is easy to verify that $J_F(a_F)$ is strictly increasing in $\sigma$. Therefore, $\dot{a}^M_F$ strictly decreases in $\sigma$.

On the impact of the demand variability and the correlation on the key performance measures; a decrease in $\rho$ or an increase in $\sigma$ induces the firm $i$) to increase the optimal capacity investment level for a given financing cost as the value of capacity-pooling increases; $ii$) to decrease the optimal capacity investment level as $\dot{a}^M_F$ increases. As follows from Panel B of Figure 21, the first effect may dominate and $\dot{K}_F$ may increase. Similarly, expected equity value may increase (Panel C).

**Secured loan without default possibility.** The creditor’s expected return with the flexible technology is given by

$$\Lambda_F(a_F) = (c_F K_F^1 - B) a_F - BC \Pr \left( (\xi_1^{-b} + \xi_2^{-b})^{-\frac{1}{b}} < M_F \left( 1 + \frac{1}{b} \right) \left[ 1 - \frac{B(1 + a_F)}{c_F K_F^0 (1 + a_F - \gamma_F)} \right] \right),$$

where $M_F = E \left[ \left( \xi_1^{-b} + \xi_2^{-b} \right)^{-\frac{1}{b}} \right]$. Let $C_F$ denote the right hand side of the default probability. Let $\dot{a}^M_F$ denote the equilibrium financing cost. In equilibrium, $\dot{a}^M_F$ satisfies $FOC_F(\dot{a}^M_F) = 0$ where $FOC_F(a_F)$ is the first order derivative of $\Lambda_F(a_F)$ and is given by

$$c_F K_F^0 (1 - \gamma_F)^{-b} (1 + a_F - \gamma_F)^{(b-1)} (1 + a_F - \gamma_F + a_F b) - B - BC \times f(C_F) \frac{\partial C_F}{\partial a_F}.$$
Figure 21: The effect of the demand correlation (\(\rho\)) and the demand variability (\(\sigma\)) on the flexible technology investment at equilibria where the firm uses a secured loan (and invests in \(\dot{K}_F = K^*_F(\dot{\alpha}_M^F)\)) without default possibility in imperfect capital markets (with the monopolist creditor) with \(B = 240\) and \(\sigma \in [3, 6]\) with 0.5-unit increments: A higher \(\sigma\) or a lower \(\rho\) leads to a higher financing cost \(\dot{a}_M\) (Panel A). Despite a higher \(\dot{a}_M\), the increasing value of the capacity-pooling benefit for a given \(a_F\) outweighs the negative effect of higher \(\dot{a}_M\); and \(\dot{K}_F\) (Panel B) and \(\dot{\pi}_F\) (Panel C) increase.

where \(f(.)\) is the probability density function of the random variable \((\xi_1^{b} + \xi_2^{b})^{-\frac{1}{b}}\). We obtain

\[
\frac{\partial C_F}{\partial a_F} = -\frac{M_F}{c_F} K^0_F(1 - \gamma_F)^{-b} \left[ \frac{1 + \frac{-(b+1)(1+a_F)}{1+a_F-\gamma_F}}{(1+a_F-\gamma_F)^{(b+1)}} \right] < 0
\]

as follows from \(b < -1\). It follows that in the optimal solution the marginal cost (that is the reduction in the net gain of the creditor from secured lending with an increase in \(a_F\)) is equal to the marginal revenue (that is the reduction in the expected default cost with an increase in \(a_F\)). Since \(\dot{\alpha}_M^F\) is the maximizer, from the implicit function theorem, we have

\[
\text{sgn}\left(\frac{\partial \dot{a}_M^F}{\partial \sigma}\right) = \text{sgn}\left(\frac{\partial FOC_F}{\partial \sigma}\right)\bigg|_{\dot{\alpha}_M^F}. \quad \text{We have}
\]

\[
\frac{\partial FOC_F(a_F)}{\partial \sigma} = -b \frac{\partial M_F}{\partial \sigma} \left( \frac{M_F}{c_F} \right)^{(b-1)} \left( 1 + \frac{1}{b} \right)^{-b} (1 + a_F - \gamma_F)^{(b-1)} (1 + a_F - \gamma_F + a_F b) 
\]

\[
+ BC \frac{\partial^2 C_F}{\partial a_F \partial \sigma} f(C_F) + BC \frac{\partial^2 C_F}{\partial a_F^2} f'(C_F) \frac{\partial C_F}{\partial \sigma} .
\]

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Since $M_F$ is increasing in $\sigma$ (under Assumption 2), it is easy to establish that $\frac{\partial^2 C_F}{\partial a_F \partial \sigma} > 0$ and the second term is positive. It also follows that $\frac{\partial C_F}{\partial a} > 0$, but we cannot determine the sign of the first term (as we do not know the sign of $(1 + a_F - \gamma_F + a_F b)$ for $b < -1$) and the third term (as we do not know the sign of the derivative of the density function $f(.)$). It follows that effect of $\sigma$ on the marginal cost and marginal revenue terms in the optimal solution is ambiguous.

We now investigate the effect of the demand variability and the correlation on $\dot{a}_M$, $\dot{K}_F$ and $\dot{\pi}_F$ numerically. As depicted in Panel C of Figure 22, $\dot{a}_M$ may increase in the demand variability. An increase in $\sigma$ induces the firm $i$) to increase $\dot{K}_F$ as the value of capacity-pooling increases for a given $a_F$ (under Assumption 2); ii) to decrease $\dot{K}_F$ as financing becomes more expensive. For low correlation levels, the former may outweigh the latter and $\dot{K}_F$ may increase in $\sigma$. For sufficiently high correlation levels, the latter effect may dominate and $\dot{K}_F$ may decrease as depicted in Panel D. A similar behavior is observed for $\dot{\pi}_F$ (Panel E).

Similar observations can be made with respect to an increase in the demand correlation. As $\rho$ increases $\dot{a}_M$ may increase or decrease as depicted in Panel C of Figure 22. When $\dot{a}_M$ is increasing, we can conclude that $\dot{K}_F$ and $\dot{\pi}_F$ decrease as the capacity pooling value decreases (under Assumption 2) besides the increase in $\dot{a}_F$. When $\dot{a}_M$ is decreasing with an increase in $\rho$, the two effects work in opposite directions. As follows from Panels D and E, in most cases the effect of the reduction in the capacity pooling value is stronger and $\dot{K}_F$ and $\dot{\pi}_F$ decrease. As depicted in Panels B and C of Figure 23, for sufficiently large correlation levels, the reduction in $\dot{a}_F$ may outweigh the declining capacity-pooling value, and $\dot{K}_F$ and $\dot{\pi}_F$ may increase in demand correlation.

**B.3 Technology Choice**

**Proof of Remark 2** When the capital markets are perfect, it follows from Remark B.1 that the firm borrows to invest in the optimal capacity level $K_F^0 = \left(\frac{M_T (1+\frac{1}{c_T})}{(1-\gamma_T) c_T}\right)^{-b}$ where $M_F = \mathbb{E} \left[ (\xi_1^{-b} + \xi_2^{-b})^{-\frac{1}{b}} \right]$ and $M_D = \xi$. Consequently, the firm’s optimal expected equity
Figure 22: The effect of the demand correlation ($\rho$) and the demand variability ($\sigma$) on the flexible technology investment at equilibria where the firm uses a secured loan (and invests in $\hat{K}_F = K_1^F(\hat{a}_F^M)$) with default possibility in imperfect capital markets (with the monopolist creditor) with $B = 20$, $\sigma \in [3, 6]$ with 0.5-unit increments: A lower $\rho$ may increase $\hat{a}_M$ for low $\sigma$ levels (Panel C). Although $\hat{a}_M^F$ increases with a decrease in $\rho$, the increase in the capacity-pooling benefit outweighs the increase in the financing cost and $\hat{K}_F$ and $\hat{\pi}_F$ increase (Panel D and E respectively). A higher $\sigma$ increases $\hat{a}_M^F$ (Panel C) and this increase is more significant at high correlation levels. In this case, the increase in $\hat{a}_M^F$ outweighs the increase in the capacity-pooling benefit and $\hat{K}_F$ and $\hat{\pi}_F$ decrease (Panels D and E respectively). For low correlation levels, the latter dominates the former and $\hat{K}_F$ and $\hat{\pi}_F$ increase.

value for each technology is given by

$$
\pi^*_F = \left( \frac{\mathbb{E}\left[\left(\xi_1^{-b} + \xi_2^{-b}\right)^{-\frac{1}{b}}\left(1 + \frac{1}{b}\right)\right]}{(1 - \gamma_F) c_F} \right)^{-b} \frac{(1 - \gamma_F) c_F}{-(b + 1)} + B + P,
\pi^*_D = 2 \left( \frac{\mathbb{E}\left[\left(1 + \frac{1}{b}\right)\right]}{(1 - \gamma_D) c_D} \right)^{-b} \frac{(1 - \gamma_D) c_D}{-(b + 1)} + B + P.
$$

It is more profitable to invest in the flexible technology when $\pi^*_F \geq \pi^*_D$, which is equivalent to

$$
c_F \leq \pi^*_F(c_D) \equiv c_D \left( \frac{1 - \gamma_D}{1 - \gamma_F} \right)^{\frac{b}{2 - \frac{1}{b}}} \left[ \frac{\mathbb{E}\left[\left(\xi_1^{-b} + \xi_2^{-b}\right)^{-\frac{1}{b}}\right]}{2 - \frac{1}{b} \xi} \right]^{\frac{b}{2 - \frac{1}{b}}}.
$$
Figure 23: The effect of the demand correlation ($\rho$) and the demand variability ($\sigma$) on the flexible technology investment at equilibria where the firm uses a secured loan (and invests in $K_F = K_F^1(a_F^M)$) with default possibility in imperfect capital markets (with the monopolist creditor) with $B = 20$, $\sigma \in [3, 4.5]$ with 0.25-unit increments and $\rho \in [0.95, 0.99]$ with 0.005-unit increments.

We now prove the relation $\tau_F^P(c_D) \geq c_D$. Since $\gamma_F \geq \gamma_D$ by assumption, it is sufficient to show $\mathbb{E}^{-b} \left[ \left( \xi_1^{-b} + \xi_2^{-b} \right)^{-\frac{1}{b}} \right] \geq 2\xi^{-b}$. From Hardy et. al (1988, p.146), if $d \in (0, 1)$ and $X, Y$ are non-negative random variables then the following is true:

$$\mathbb{E}^{1/d} \left[ (X + Y)^d \right] \geq \mathbb{E}^{1/d} \left[ X^d \right] + \mathbb{E}^{1/d} \left[ Y^d \right]$$

where equality only holds when $X$ and $Y$ are effectively proportional, i.e. $X = \lambda Y$. In $\tau_F^P(c_D)$, we have $d = -\frac{1}{b} \in (0, 1)$ and $\xi \geq \xi_1 \geq \xi_2 \geq 0$, replacing $X$ with $\xi_1^{-b}$ and $Y$ with $\xi_2^{-b}$ gives the desired result. Notice that $\tau_F^P(c_D) = c_D$ only if $\xi_1 = \xi_2$ (since we focus on the symmetric bivariate distribution) and $\gamma_F = \gamma_D$. $\xi_1 = \xi_2$ is only possible if either $\xi$ is deterministic or the product markets are perfectly positively correlated ($\rho = 1$). □

**B.3.1. The Perfectly Competitive Credit Market with $U \geq 0$.**

We first provide the following lemma that we will use throughout this section.

**Lemma B.2** For a fixed financing cost pair $(a_F, a_D)$, there exists a unique variable cost $\tau_F$ such that when $c_F \leq \tau_F$ ($c_F > \tau_F$) it is optimal to invest in flexible (dedicated) technology. With symmetric salvage rates and symmetric financing costs, we have $\tau_F = \tau_F^P(c_D)$.

**Proof of Lemma B.2:** We first prove the existence of $\tau_F$. For a given $a_F$, the expected
equity value $\pi_F^*(c_F)$ is a continuous function of $c_F$. For a finite $c_D > 0$, $\pi_D^*(c_D)$ is finite. It is easy to prove that for a given $a_F(c_F)$

$$\lim_{c_F \to \infty} \pi_F^*(c_F) = B + P,$$

$$\lim_{c_F \to 0} \pi_F^*(c_F) \to \infty.$$  

Since the equity value is continuous in $c_F$, there exists a $c_F$ such that the equity values with both technologies coincide. This concludes the proof for the existence of $c_F$. It is easy to establish that for a given $a_T$, the optimal expected equity value of the firm with technology $T$ strictly decreases in the unit capacity investment cost ($\frac{\partial \pi_T^*}{\partial c_T} < 0$). This implies the uniqueness of $c_F$.

With symmetric salvage values ($\gamma_D = \gamma_F$) and identical financing cost ($a_D = a_F$), we have $c_F K_F^1|_{c_F = \pi_F^*(c_D)} = 2c_D K_D$ and it follows that at $c_F = \pi_F^*(c_D)$ we have $\pi_D^*(c_D) = \pi_F^*(c_F)$. 

It follows from Remark 2 that $\pi_F^*(c_D) = c_D$ for $\rho \approx 1$. It is easy to establish that at $\rho = 1$, with the technology cost pair $c_F = c_D$, $\Lambda_D(a_D) = \Lambda_F(a_F)$, and the creditor offers identical financing cost with each technology, i.e. $\dot{a}_F = \dot{a}_D$. It follows from Lemma B.2 that the firm is indifferent between the two technologies in equilibrium.

We now focus on equilibria where the firm uses a secured loan with default possibility with each technology. We first provide Proposition B.7 that shows the dominance of dedicated technology in imperfect capital markets with the technology cost pair $(c_D, \pi_F^*(c_D))$ where $\rho \approx 1$, but not equal to 1.

**Proposition B.7** Let $c_F' = \pi_F^*(c_D)$ for $\rho \approx 1$. In a perfectly competitive credit market with $U \geq 0$, for the technology cost pair $(c_D, c_F')$, and $\rho \approx 1$ we have $\dot{a}_D < \dot{a}_F$ and the firm optimally invests in the dedicated technology in equilibrium.

**Proof of Proposition B.7:** As follows from Lemma B.2, with $c_F = \pi_F^*(c_D)$, the firm would be indifferent between the two technologies if both were exposed to identical financing costs. It is easy to establish that the optimal expected equity value of the firm with technology $T$ strictly decreases in the financing cost ($\frac{\partial \pi_T^*}{\partial a_T} < 0$); therefore the equilibrium technology choice is determined by the ordering of the financing cost in equilibrium. We have

$$\Lambda_F(a_F) = (c_F K_F^1 - B) a_F - BCPr \left( \left( \xi_1^b + \xi_2^b \right)^{-\frac{1}{b}} < M_F \left( 1 + \frac{1}{b} \left[ 1 - \frac{B(1 + a_F)}{(1 + a_F - \gamma_F) c_F K_F^1} \right] \right) \right),$$

$$\Lambda_D(a_D) = (2c_D K_D^1 - B) a_D - BCPr \left( \xi_1 + \xi_2 < 2\xi(1 + \frac{1}{b}) \left[ 1 - \frac{B(1 + a_D)}{(1 + a_D - \gamma_D) 2c_D K_D^1} \right] \right),$$

(36)
where $M_F = \mathbb{E} \left[ \left( \xi_1^{-b} + \xi_2^{-b} \right)^{-\frac{1}{b}} \right]$. With the technology cost pair $(c_D, \overline{c}_F(c_D))$, and with $a_D = a_F, \gamma_D = \gamma_F$, the first terms in (36) (the net gain from secured lending) are identical for each technology as follows from the proof of Lemma B.2. Thus, the ordering of the equilibrium financing cost with each technology is determined by the ordering of the default risk calculated at identical financing costs with each technology.

We now show that for the technology cost pair $(c_D, c'_F)$ such that $c'_F = \overline{c}_F(c_D)$ at $\rho \approx 1$, the default risk of the firm is higher with flexible technology if both technologies are exposed to identical financing costs.

![Figure 24: Default regions in the ($\xi_1, \xi_2$) space with each technology: $C_D$ and $C_F$ are the right-hand side terms in the default probability in (36). The area below the straight line (curve) is the default region with the dedicated (flexible) technology. $F$ ($D$) represents the $\xi$ realizations where the firm does not default with the flexible (dedicated) technology and defaults with the other technology.](image)

Figure 24 demonstrates the default region with each technology with identical financing cost for $c_F = \overline{c}_F(c_D)$. In this figure, $F$ ($D$) represents the $\xi$ realizations where the firm does not default with the flexible (dedicated) technology and defaults with the other technology. The overall default probability is determined by superimposing the $\xi$ probability density function and taking the expectation over the regions. Let $C_T$ denote the right-hand side of the default probability with technology $T$ in (36). At the technology cost pair $(c_D, \overline{c}_F(c_D))$, since $\left( \xi_1^{-b} + \xi_2^{-b} \right)^{-1/b} < \left( \xi_1 + \xi_2 \right)^{-1/b}$ for any $\xi$ realization, it follows that $\mathbb{E} \left[ \left( \xi_1^{-b} + \xi_2^{-b} \right)^{-\frac{1}{b}} \right] < 2\xi$ and we have $C_D = C_F \frac{2\xi}{M_F} > C_F$. Therefore the region denoted
with $F$ always exists around the points $(0, C_D)$ and $(C_D, 0)$. The point $(\frac{C_F}{2}, \frac{C_F}{2})$ which is on the default line of the dedicated technology is in the default region of the flexible technology if \( \left( \left( \frac{C_F}{2} \right)^{-b} + \left( \frac{C_F}{2} \right)^{-b} \right)^{-\frac{1}{b}} \leq C_F \). At the technology cost pair \((c_D, \tilde{e}_F^C(c_D))\), this condition is equivalent to $2^{-1/b} \xi \leq M_F$. From the proof of Remark 2, it follows that this condition is satisfied with equality only if $\rho = 1$, otherwise the inequality is always satisfied for $\rho \neq 1$. Therefore, the $D$ region only exists if $\rho \neq 1$.

In summary, for $\rho \neq 1$, there exists regions in which the firm defaults with one technology and not with the other. The overall default risk is calculated by superimposing the $\xi$ probability density function and taking the expectation over the regions. With close to perfect positive correlation, all the $\xi$ realizations are located around the $\xi_1 = \xi_2$ line (which also passes through the point $(\frac{C_F}{2}, \frac{C_F}{2})$). It follows that after taking the expectation over the default regions, we have a higher default risk with the flexible technology because of the existence of region $D$.

We now prove that since the dedicated technology has lower default risk with identical financing cost, the equilibrium financing cost with the dedicated technology is lower. Let $\hat{a}_D(c_D)$ and $\hat{a}_F(c'_F)$ denote the equilibrium financing costs with the dedicated and flexible technologies respectively. Suppose that $\hat{a}_F(c'_F) \leq \hat{a}_D(c_D)$. From the definition of the equilibrium, we have $\Lambda_F(\hat{a}_F(c'_F)) = 0$. Since the default probability with flexible technology is higher, we have $\Lambda_D(\hat{a}_F(c'_F)) > 0$. This implies from the definition of the equilibrium that there exists $\hat{a}_D' < \hat{a}_F(c'_F) \leq \hat{a}_D(c_D)$. This contradicts $\hat{a}_D(c_D)$ being the Pareto-optimal equilibrium.

In summary, since $\hat{a}_D(c_D) < \hat{a}_F(c'_F)$ and since $\frac{\partial \pi_T^*}{\partial a_T} < 0$ and $\frac{\partial \pi_T^*}{\partial c_T} < 0$, the firm invests in dedicated technology in equilibrium.

**B.3.2. The Monopolist Creditor.**

Let $\hat{a}_T^M$ denote the equilibrium financing cost with technology $T \in \{F, D\}$. At the equilibria where the firm uses a secured loan with default possibility for each technology, $\hat{a}_F^M$ and $\hat{a}_D^M$ respectively satisfy

\[
\begin{align*}
  cFK_F^0(1 - \gamma_F)^{-b}(1 + a_F - \gamma_F)(b^{-1})(1 + a_F - \gamma_F + a_F b) - B &= -BC \times f \left( \frac{C_F}{\partial a_F} \right) \frac{\partial C_F}{\partial a_F}, \\
  2cDK_D^0(1 - \gamma_D)^{-b}(1 + a_D - \gamma_D)(b^{-1})(1 + a_D - \gamma_D + a_D b) - B &= -BC \phi \left( \frac{C_D - 2\xi}{\sigma} \right) \frac{\partial C_D}{\partial a_D}
\end{align*}
\]

where $f(.)$ is the probability density function of the random variable $(\xi_1^{-b} + \xi_2^{-b})^{-\frac{1}{b}}$ and $\phi(.)$ is the probability density function of the standard normal random variable. The left-hand
side terms denote the marginal cost of lending (the reduction in the net gain from secured lending with an increase in financing cost) whereas the right-hand side terms denote the marginal revenue of lending (the reduction in the expected default cost with an increase in financing cost). We have

\[ C_F = M_F \left( 1 + \frac{1}{b} \right) \left[ 1 - \frac{B(1 + a_F)}{c_F K^F_F(1 + a_F - \gamma_F)} \right], \]

\[ C_D = 2 \xi \left( 1 + \frac{1}{b} \right) \left[ 1 - \frac{B(1 + a_D)}{2c_D K^D_D(1 + a_D - \gamma_D)} \right], \]

and

\[ \frac{\partial C_F}{\partial a_F} = -\frac{M_F(1 + \frac{1}{b})B}{c_F K^F_F(1 - \gamma_F)^{-b}} \left[ 1 + \frac{- (b+1)(1+a_F)}{1+a_F-\gamma_F} \right] < 0, \]

\[ \frac{\partial C_D}{\partial a_D} = -\frac{2\xi(1 + \frac{1}{b})B}{2c_D K^D_D(1 - \gamma_D)^{-b}} \left[ 1 + \frac{- (b+1)(1+a_D)}{1+a_D-\gamma_D} \right] < 0 \]

where \( M_F = E \left( \xi_1^{-b} + \xi_2^{-b} \right)^{-\frac{1}{b}} \)

We first analyze the technology choice with \( c_F = c^D_F(c_D, \rho) \). It is easy to show that if the creditor charges identical financing cost with each technology, then the marginal cost terms in (37) are identical with each technology. With perfectly positive correlation (\( \rho = 1 \)), at \( c_F = c^P_F(c_D, \rho) = c_D \), we have \( f(C_F) \frac{\partial C_F}{\partial a_F} = \phi \left( \frac{c_D - 2\xi}{\sigma} \right) \frac{\partial C_D}{\partial a_D} \) for \( a_F = a_D \), and thus \( \dot{a}^M_F = \dot{a}^M_D \). Therefore, the firm is indifferent between the two technologies.

Analyzing the equilibrium technology choice with the technology cost pair \( (c_D, c^P_F(c_D, \rho)) \) for \( \rho \neq 1 \) requires making a comparison between the density functions \( \phi(.) \) and \( f(.) \). This comparison is not analytically tractable therefore we resort to the numerical analysis. We observe that dedicated technology is weakly preferred when the demand correlation is very high and very low; and when moderate demand correlation levels are accompanied with low demand variability. This is because \( i \) the firm would be indifferent between the two technologies if they are offered identical financing costs and \( ii \) we have \( \dot{a}^M_D \leq \dot{a}^M_F \) at these \( \sigma \) and \( \rho \) combinations.

For the equilibrium technology choice with \( c_F = c_D \); in our numerical experiments, we observe \( \dot{a}^M_D \leq \dot{a}^M_F \) for the entire range of \( (\sigma, \rho) \) combinations except a few cases for very high correlation level. Since flexible technology has a capacity-pooling benefit over dedicated technology, \( \dot{a}^M_D \) should be sufficiently lower than \( \dot{a}^M_F \) to outweigh this benefit for the dedicated technology to be chosen in equilibrium. The dominance of the dedicated technology may be observed when the demand correlation is very high. In this case, the
capacity pooling value is already very low, and even a slight advantage in financing cost may lead to a dedicated technology choice.

C Discussion of Assumptions and Extensions

C.1 Liquid Physical Assets \( P \)

We have assumed that the physical assets of the firm (\( P \)) are illiquid; they can be liquidated with a lead time. Therefore, at times \( P \) and the firm’s final cash position is sufficient to cover the face value of the debt, if the firm’s final cash position alone is not sufficient to cover the face value of the debt, default occurs. In this section, we will analyze the case where \( P \) can be liquidated immediately. We replicate our previous analysis (both analytical statements and numerical experiments) with liquid \( P \) assumption. Only a few of these are reported throughout this section.

In the single-product firm analysis, for a given financing cost \( a \), we have

\[
b(K) = l(K) = (1 + a - \gamma)cK\frac{1}{a} - \frac{B(1+a)}{K^{(1+\frac{1}{a})}} - \frac{P}{K^{(1+\frac{1}{a})}},
\]

i.e. the bankruptcy cost threshold and the limited liability threshold are identical. The implication is that the firm defaults only when the firm uses an unsecured loan. It follows that the creditor’s expected return in Proposition 4 is characterized as

\[
i = (cK - B)a
\]

for \( 0 \leq a < a^{max} \) if \( cK^0 (1 - \gamma) \frac{[(1+\frac{1}{a})-\hat{\xi}]}{\xi^{(1+\frac{1}{a})}} - P \leq B < cK^0 \),

\[
iii = \begin{cases} 
(cR(a) - B)a - F(l(R(a)))BC - L(R(a)) & \text{for } 0 \leq a < a^l \\
(cK^1(a) - B)a & \text{for } a^l \leq a < a^{max}
\end{cases}
\]

if \( B < cK^0 (1 - \gamma) \frac{[(1+\frac{1}{a})-\hat{\xi}]}{\xi^{(1+\frac{1}{a})}} - P \),

In other words, we have \( a^l = a^l \) in Proposition 4 where only \( a^l \) is relevant for the liquid \( P \) case. Using a secured loan with default possibility is not feasible anymore, and thus, such equilibrium is also not feasible. The other two equilibria can be observed: An equilibrium where the firm uses a secured loan without default possibility, and an equilibrium where the firm uses an unsecured loan. With a secured loan, the firm invests in \( K^1(\hat{a}) \); whereas with an unsecured loan, the firm invests in \( K(\hat{a}) \). It follows that in all the Tables that summarize our results, the rows that refer to the equilibria where the firm uses a secured loan with default possibility do not exist.

For the impact of the demand variability on \( \hat{K} \) and \( \hat{\pi} \), the results are identical to Table 1 in the perfectly competitive credit market (\( U \geq 0 \)) case. In the monopolist creditor case, the only change occurs at equilibria where the firm uses a secured loan. As depicted in Figure 25 below (which is the replot of Figure 8 with liquid \( P \) assumption), \( \hat{K} \) and \( \hat{\pi} \) may
increase in the demand variability at the equilibria where the firm uses a secured loan. This happens when the equilibrium financing cost switches from one local maximizer (in the secured lending region) to the other one (in the unsecured lending region). This leads to a discontinuous decrease in $\dot{a}$ and $\dot{K}$ and $\dot{\pi}$ increase. Therefore, $\dot{K}$ and $\dot{\pi}$ either increase or do not change with an increase in the demand variability at this equilibrium.

Figure 25: The effect of the demand variability on the capacity investment level and the expected equity value in equilibria where the firm uses a secured loan (and invests in $K^1(\dot{a})$) without default possibility and the firm uses an unsecured loan (and invests in $K(\dot{a})$) in imperfect capital markets (for the monopolist creditor) with liquid $P$ with $B = 40$, $\xi \sim U[20 - \epsilon, 120 + \epsilon]$ and $\epsilon \in [0, 20]$ with 1-unit increments: When the firm uses a secured loan in equilibrium, $\dot{a}$ decreases and $\dot{K}$ increases. The jumps in Panels C and E illustrate the transition from a secured loan to an unsecured loan in equilibrium. When the firm uses an unsecured loan, $\dot{\pi}$ decreases in $\sigma$ (Panel E) whereas $\dot{K}$ first increases then decreases (Panel D).

For the impact of the internal budget level, the results are identical to Table 2.

In the two-product firm analysis, if we assume $P \geq 2c_T^{b+1} \left[\xi^u (1 + \frac{b}{T})\right]^{-b} \left[1 - \frac{\xi^l}{\xi^u (1+b)}\right] \left[1 - \gamma_T\right]^{b+1}$ for $T \in \{D, F\}$, with liquid $P$ assumption it follows that the firm never uses an unsecured loan with each technology. Therefore, the creditor’s expected return is characterized by the net gain from secured lending. On the impact of demand uncertainty, in Table 3, only results that refer to equilibria where the firm uses a secured loan without default possibility are relevant.

On the equilibrium technology choice in a perfectly competitive credit market, it is easy
to show that at $c_F = \tau^p_F(c_D)$, we have $\dot{a}_F = \dot{a}_D$. Therefore, similar to the perfect market case, the firm is indifferent between the two technologies. For $c_F > \tau^p_F(c_D)$, we can prove that $\dot{a}_F > \dot{a}_D$. Therefore, similar to the perfect market case, flexible technology is strictly preferred over the dedicated technology. With the monopolist creditor, for $c_F = \tau^p_F(c_D)$, we can prove that the firm is exposed to identical financing cost with each technology; and is indifferent between the two technologies. For $c_F < \tau^p_F(c_D)$, it is easy to prove that $\dot{a}_D > \dot{a}_F$, i.e. dedicated technology is exposed to a lower financing cost in equilibrium. The equilibrium technology choice is determined by the trade-off between the lower financing cost of the dedicated technology and the capacity-pooling benefit of the flexible technology. In our numerical experiments, we observe that the lower financing cost of the dedicated technology never outweighs the capacity-pooling benefit of the flexible technology for $c_F < \tau^p_F(c_D)$; and the flexible technology is preferred in equilibrium.

C.2 Unsecured Lending in the Two-Product Firm Analysis

In the two-product firm analysis, we have assumed that the value of the physical assets $P$ is sufficiently large\(^8\) such that it is never optimal for the firm to invest in capacity levels at which the uses an unsecured loan. In this section, we relax this assumption. It follows from our two-product firm analysis that we cannot guarantee the unimodality of the firm’s problem with bivariate normal demand uncertainty. Therefore, we resort to numerical experiments in this section. We use the same parameter sets as before except we set $P = 0$. Another implication of $P = 0$ choice is that the results of this section are also relevant for the previous section in which we relax the liquid physical assets $P$ assumption. In C.1, we focus on equilibria where the firm uses a secured loan without default possibility. In this section, we focus on equilibria where the firm uses an unsecured loan. For brevity, we only analyze the perfectly competitive credit market (with $U = 0$) case.

On the firm’s optimal capacity investment decision with technology $T$ for a given financing cost, similar to the single-product firm analysis, we can prove that there exists budget thresholds $B^b_T < B^h_T$ (which we can characterize in closed-form) such that for $B \geq B^h_T$, we have the optimal capacity investment level characterized by Proposition B.2. For $B < B^b_T$, the optimal capacity investment level $K^*_T$ is a solution to $MP_T(K^*_T) = 0$ where

$$MP_T(K_T) = \int_{Y_T(K_T)} \left[ \left( 1 + \frac{1}{b} \right) \left( \xi_1^b + \xi_2^b \right)^{-\frac{1}{b}} K_T^{(\frac{1}{b})} - \left( 1 + a_F - \gamma_F \right) c_F \right] f(\xi_1, \xi_2) d\xi_1 d\xi_2$$

\(^8\)More precisely, we assume $P \geq 2c_T^{b+1} \left[ \xi^u \left( 1 + \xi^u \right) \right]^{-b} \left[ 1 - \frac{\xi^l}{\xi^u(1+\xi^u)} \right] \left[ 1 - \gamma_T \right]^{b+1}$ for $T \in \{D, F\}$. 

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with
\[ Y_F(K_F) = \left\{ \xi : \xi' \geq (\xi^1, \xi^2); l_F(K_F) \leq \left( \xi_{1}^{-b} + \xi_{2}^{-b} \right)^{-\frac{1}{b}} \leq 2^{-\frac{1}{b} \xi''} \right\}, \]
\[ l_F(K_F) = K_F^{-1} (1 + a_F - \gamma_F) c_F - K_F^{(-1 - \frac{1}{b})} [B(1 + a_F) + P], \]
and
\[ MP_D(K_D) = \left[ 1 - \Phi \left( \frac{l_D(K_D) - \bar{\mu}}{\sigma} \right) \right] \left[ \left( 1 + \frac{1}{b} \right) \frac{\bar{\pi} K_D^{(\frac{1}{b})}}{2} - 2 (1 + a_D - \gamma_D) c_D \right] + \left( 1 + \frac{1}{b} \right) \frac{\bar{\pi} K_D^{(\frac{1}{b})}}{\sigma} \frac{l_D(K_D) - \bar{\mu}}{\bar{\sigma}} \right) \]
where
\[ l_D(K_D) = K_D^{(-1 - \frac{1}{b})} 2 (1 + a_D - \gamma_D) c_D - K_D^{(-1 - \frac{1}{b})} [B(1 + a_D) + P], \]
and
\[ \bar{\pi} = 2 \xi, \bar{\sigma} = \sigma \sqrt{2(1 + \rho)}, \text{ and } \Phi(.), \phi(.) \text{ are cdf and pdf of standard normal random variable respectively.} \]

For firms with \( B \in [B_T^l, B_T^h] \), the optimal capacity investment level is found by comparing the expected equity value at the two solutions, one of Proposition B.2, and the other one, the solution to \( MP_T(K_T) = 0 \) that gives the highest expected equity value (if there exist more than one solution to this equation). Although we cannot prove the unimodality of the objective function with the bivariate normal demand distribution, in our numerical experiments we observe that when the solution to \( MP_T(K_T) = 0 \) exists, it is unique.

Since the unimodality of the firm’s problem cannot be guaranteed, the creditor’s expected return cannot be characterized as in Proposition 4. With the dedicated technology, we have
\[ \Lambda_D(a_D) = (2c_D K_D^* - B) a_D - BC \times \Phi \left( \frac{C_D(K_D^*) - \bar{\mu}}{\bar{\sigma}} \right) - L_D(K_D^*) \]
where the first term is the net gain from secured lending, the second term is the expected default cost and the last term is the expected loss due to the unsecured part of the loan.

In (38), we have
\[ C_D(K_D^*) = K_D^{\frac{1}{b} + \frac{1}{2}} (1 + a_D - \gamma_D) c_D - K_D^{(-1 - \frac{1}{b})} [B(1 + a_D)], \]
and \( L_D(K_D^*) \) is given by
\[ \left[ \frac{\bar{\pi} K_D^{(\frac{1}{b} + \frac{1}{2})} + 2c_D K_D^* (1 + a_D - \gamma_D) - B(1 + a_D) - P} {\Phi \left( \frac{l_D(K_D^*) - \bar{\mu}}{\bar{\sigma}} \right)} \frac{\bar{\sigma} K_D^{(\frac{1}{b} + \frac{1}{2})}}{\bar{\pi}} \right) + \frac{\bar{\pi} K_D^{(\frac{1}{b} + \frac{1}{2})}}{\bar{\sigma}} \Phi \left( \frac{l_D(K_D^*) - \bar{\mu}}{\bar{\sigma}} \right) . \]
The expected return of the creditor with the flexible technology can be characterized in a similar manner, and is omitted. Unlike Proposition 4, we cannot prove that as \( a_D \) increases
the firm goes from unsecured lending region \((K^*_D \text{ is the solution to } MP_D(K^*_D) = 0)\) to secured lending region \((K^*_D = K^*_1)\). Therefore, we numerically characterize \(\Lambda_T(a_T)\) by first solving for \(K^*_T\) for a given \(a_T\), and then calculating the expected return of the creditor using (38). The equilibrium \(\hat{a}_T\) in the perfectly competitive creditor market case can be easily calculated numerically.

We now analyze the effect of demand uncertainty on each technology. We will only focus on equilibria where the firm uses an unsecured loan.

**C.2.1 Dedicated Technology**

For a given \(a_D\), as the demand variability \(\sigma\) or the correlation \(\rho\) increases, the optimal capacity investment level and optimal expected equity value increase. The intuition is similar to the single-product case: The value of the limited liability option of the firm increases. With an increase in \(\sigma\), the downside risk of the firm’s cash flows increases, so does the value of the limited liability option of the firm. Therefore, the firm invests more in capacity. Similarly, \(\pi^*_D\) also increases. With an increase in \(\rho\), since the value of financial-pooling (diversification benefit of operating in two markets) decreases, downside risk of the firm’s cash flows increases. Therefore, the value of the limited liability option increases, and so do \(K^*_D\) and \(\pi^*_D\). This pattern is depicted in Figure 26 below.

From the creditor’s perspective, when the firm uses an unsecured loan, with an increase in \(\sigma\) or \(\rho\) the default risk increases as \(i\) for a fixed \(K_D\), the downside risk of the firm’s cash flows increases, and \(ii\) \(\overline{K}_D\) increases and the firm borrows more. The expected loss due to the unsecured part of the loan increases due to a similar reasoning. Net gain from secured lending increases as the firm invests and borrows more. The first two effects work to increase \(\hat{a}_D\), whereas the third effect works to decrease it. In our numerical experiments, we observe that the first two effects dominate and \(\hat{a}_D\) increases in the demand variability and the demand correlation.

For the impact of \(\sigma\) and \(\rho\) on \(\hat{K}_D\), similar to the single-product case analysis, two drivers work in the opposite directions: A higher \(\sigma\) or \(\rho\) induces the firm to invest more in capacity for a given \(a_D\) due to the increasing value of the limited liability option, but a higher financing cost induces the firm to invest less. In our numerical experiments, we observe that both effect may dominate and \(\hat{K}_D\) may increase or decrease with an increase in \(\sigma\) or \(\rho\). For the effect on \(\hat{\pi}_D\), a similar trade-off between a higher value of the limited liability option for a given financing cost and a higher equilibrium financing cost exists. In our numerical experiments, we observe that the second effect outweighs the first and \(\hat{\pi}_D\) decreases with
Figure 26: The effect of the demand variability $\sigma$ and the demand correlation $\rho$ on the optimal capacity investment level and the expected equity value of the firm with dedicated technology for a given financing cost $a_D$ with $BC = 100$, $B = 5$, $\gamma_D = 0.1$, $b = -2$, $P = 0$, $c_D = 1$, $\bar{\xi}_1 = \bar{\xi}_2 = 20$, $a_D = 0.0001$ and $\sigma \in [3, 6]$ with 0.5-unit increments: When $\rho$ is not very low, the firm uses an unsecured loan in equilibrium, and the optimal capacity investment level (Panel A) and the optimal expected equity value (Panel B) increase with an increase in $\sigma$ or $\rho$. For sufficiently low correlation levels, the firm uses a secured loan without default possibility in equilibrium, and the firm is not affected by the demand uncertainty. An increase in $\sigma$ or $\rho$.

C.2.2 Flexible Technology

For a given $a_F$, as the demand variability $\sigma$ increases, the optimal capacity investment level and optimal expected equity value increase. This is because $i)$ the downside risk of the firm’s cash flows increases, so does the the value of the limited liability option of the firm; and $ii)$ the capacity-pooling benefit of the flexible technology increases (under Assumption 2). Therefore, $K^*_F$ and $\pi^*_F$ increase. This is depicted in Figure 27 below. For a given $a_F$, as the demand correlation $\rho$ increases, two drivers work on the opposite directions: The value of financial-pooling (diversification benefit of operating in two markets) decreases, downside risk of the firm’s cash flows increases, and thus, the value of the limited liability option increases. This induces the firm to invest more in capacity. However, the value of the capacity-pooling decreases (under Assumption 2), and this induces the firm to invest
less in capacity. As depicted in Panel A of Figure 27, on the impact on $K_F^*$, both effects may dominate, and $K_F^*$ may increase or decrease. On the impact on $\pi_F^*$, we observe that (as depicted in Panel B) the second effect outweighs the first and $K_F^*$ decreases.

![Figure 27: The effect of the demand variability $\sigma$ and the demand correlation $\rho$ on the optimal capacity investment level and the expected equity value of the firm with flexible technology for a given financing cost $a_F$ with $BC = 100$, $B = 5$, $\gamma_F = 0.1$, $b = -2$, $P = 0$, $c_F = 1$, $\xi_1 = \xi_2 = 20$, $a_F = 0.0001$ and $\sigma \in [3, 6]$ with 0.5-unit increments: For a sufficiently low $\rho$, the firm uses a secured loan, and $K_F^*$ and $\pi_F^*$ increase with an increase in $\sigma$ and a decrease in $\rho$ due to the higher value of capacity-pooling. If $\rho$ is not sufficiently low, the firm uses an unsecured loan. In this case, a higher $\sigma$ increases $K_F^*$ and $\pi_F^*$ as the capacity-pooling benefit as well as the value of the limited liability option of the firm increase. A higher $\rho$ increases the value of the limited liability option of the firm, but decreases the capacity-pooling benefit. $K_F^*$ may increase or decrease depending on which one dominates. For $\pi_F^*$, the declining value of the capacity-pooling outweighs the increase in the limited liability option and $\pi_F^*$ decreases.

From the creditor’s perspective, when the firm uses an unsecured loan, with an increase in $\sigma$ the default risk increases as i) for a fixed $\overline{K}_F$, the downside risk of the firm’s cash flows increases, and ii) $\overline{K}_F$ increases and the firm borrows more. The expected loss due to the unsecured part of the loan increases due to a similar reasoning. Net gain from secured lending increases as the firm invests and borrows more. The first two effects work to increase $\hat{a}_F$, whereas the third effect works to decrease it. With an increase in $\rho$, when $\overline{K}_F$ increases, the impact on each part of the creditor’s expected return is in the same direction. With an
increase in \( \rho \), when \( \bar{K}_F \) decreases, net gain from secured lending decreases as the firm invests and borrows less. For the impact on the default probability (and the expected loss due to the unsecured part of the loan) two drivers work in opposite directions: For a fixed \( \bar{K}_F \), the downside risk of the firm’s cash flows increases (due to lower diversification benefits) but the firm borrows more. The total effect is ambiguous. In our numerical experiments, on the impact of \( \sigma \), we observe that the third effect dominates and \( \dot{a}_F \) decreases in the demand variability. On the impact of \( \rho \), we observe that \( \dot{a}_F \) may increase or decrease in the demand correlation.

For the impact of \( \sigma \) and \( \rho \) on \( \dot{K}_F \) and \( \dot{\pi}_F \), we numerically observe that a higher \( \sigma \) increases them. This is because \( \dot{a}_F \) decreases, and the value of the limited liability option of the firm as well as the capacity pooling benefit increase. A higher \( \rho \) increases the value of the limited liability option of the firm but decreases the value of the capacity-pooling. The first effect works to increase \( \dot{K}_F \) and \( \dot{\pi}_F \) whereas the second works to decrease them. If \( \dot{a}_F \) decreases (increases), this also works to decrease (increase) them. In our numerical experiments, we observe that \( \dot{K}_F \) may increase or decrease as any of these effects may dominate. In particular, we observe that \( \dot{K}_F \) mimics the pattern in \( \dot{a}_F \), i.e. \( \dot{a}_F \) is the main determinant of the impact of \( \rho \). We also observe that \( \dot{\pi}_F \) decreases in \( \rho \), and this happens even when \( \dot{a}_F \) decreases. In other words, the declining value of the capacity-pooling benefit outweighs the increase in the value of the limited liability option as well as the impact of a lower financing cost.

### C.2.3 Technology Choice

At equilibria where the firm uses an unsecured loan with each technology, we can prove that with symmetric salvage rates and at the technology cost pair \((c_D, c_D)\), when the product markets are perfectly positively correlated \((\rho = 1)\), we have \( \dot{K}_F = 2\dot{K}_D \), and \( \dot{a}_F = \dot{a}_D \). Therefore, similar to the perfect market benchmark case, the firm is indifferent between the two technologies in imperfect capital markets. In our numerical experiments, we observe that with \( c_F = \tau_F^D(c_D) \) as function of \( \sigma \) and \( \rho \), for a given \( \rho \) there exists a demand variability threshold such that if \( \sigma \) is higher than this threshold then flexible technology preferred. Otherwise, dedicated technology is preferred. This threshold is decreasing in the demand correlation. Since the firm would be indifferent between the two technologies if they were given same financing costs, this preference is entirely determined by the ordering of the financing costs in equilibrium. For \( c_F < \tau_F^D(c_D) \), we observe that \( \dot{a}_D > \dot{a}_F \); hence a lower financing cost and the positive value of capacity pooling induces the firm to choose flexible
technology in equilibrium.

C.3 The Equilibrium Technology Choice When the Flexible Technology Has a Higher Salvage Value

In this section, we relax the symmetric salvage value of both technologies assumption and analyze the effect of a higher salvage value of the flexible technology \((\gamma_F > \gamma_D)\) on the equilibrium technology choice. For brevity, we restrict our analysis to the case in which the product markets are perfectly positively correlated, i.e. \(\rho = 1\). In perfect capital markets, as follows from Remark 2, for \(\rho = 1\) there is no capacity-pooling benefit of the flexible technology and the technology choice is determined by the unit cost threshold \(\pi_F'(c_D) = c_D \frac{1-\gamma_D}{1-\gamma_F}\). In other words, the firm strictly prefers the flexible technology only if \(c_F < c_D \frac{1-\gamma_D}{1-\gamma_F}\). Within our feasible set of technology costs \((c_F \geq c_D)\), this case is only possible if the flexible technology has a higher salvage value \((\gamma_F > \gamma_D)\).

To analyze the equilibrium technology choice in imperfect capital markets, we focus on the perfectly competitive credit market with \(U = 0\). We parameterize \(c_F'(\gamma_F) = c_D \frac{1-\gamma_D}{1-\gamma_F}\) for \(\gamma_F \geq \gamma_D\) and analyze the effect of an increase in \(\gamma_F\). In other words, we explore the equilibrium technology choice at different cost pairs \((c_D, c_F'(\gamma_F))\) as \(\gamma_F\) changes. Recall that for a given \(\gamma_F\), the firm is indifferent between the two technologies in perfect capital markets at this cost pair. We restrict our analysis to the equilibria where the firm uses a secured loan with default possibility with each technology.

**Lemma C.1** When the product markets are perfectly positively correlated \((\rho = 1)\), if both technologies are exposed to identical financing costs \((a_D = a_F = a)\), then the technology choice is determined by the unit cost threshold \(c_F'(\gamma_F) = c_D \frac{1-\gamma_D}{1-\gamma_F}\) such that when \(c_F' \leq c_D \frac{1-\gamma_D}{1-\gamma_F}\) \((c_F' > c_D \frac{1-\gamma_D}{1-\gamma_F})\) it is optimal to invest in flexible (dedicated) technology. In this case, with the technology cost pair \((c_D, c_F'(\gamma_F)) = c_D \frac{1-\gamma_D}{1-\gamma_F}\) for \(\gamma_F > \gamma_D\), the dedicated technology is strictly preferred for \(a > 0\); and the firm is indifferent between the two technologies for \(a = 0\).

**Proof of Lemma C.1:** Since the firm borrows to invest in \(K_T^1\) with technology \(T\), the optimal expected equity value of the firm with each technology \(T \in \{D, F\}\) is given by

\[
\pi_F^* = c_F K_F^1 \left(1 + \frac{a_F - \gamma_F}{b+1}\right) + B(1 + a_F) + P,
\]

\[
\pi_D^* = 2c_D K_D^1 \left(1 + \frac{a_D - \gamma_D}{b+1}\right) + B(1 + a_D) + P.
\]
For $\rho = 1$, we have $\xi_1 = \xi_2 = \xi$ and $E \left[ (\xi_{1}^{-b} + \xi_{2}^{-b})^{-\frac{1}{b}} \right] = 2^{-\frac{1}{b}} \bar{\xi}$, and it is easy to establish that with identical financing costs ($a_F = a_D = a$), the two equity values are equated at $c_F = c_D \frac{(1+a-\gamma_F)}{(1+a-\gamma_D)}$. Since the expected equity value with the flexible technology is strictly decreasing in $c_F$, this is the unique cost threshold that determines the technology choice.

We obtain $\frac{\partial c_D}{\partial a} = \frac{\gamma_D - \gamma_F}{(1+a-\gamma_F)^2} < 0$ as follows from $\gamma_D < \gamma_F$. Therefore, with the technology cost pair $(c_D, c_F^\prime(\gamma_F) = c_D \frac{1-\gamma_D}{1-\gamma_F})$, we have $c_F^\prime(\gamma_F) \geq c_D \frac{(1+a-\gamma_D)}{(1+a-\gamma_F)}$ for $a \geq 0$ with equality only holding for $a = 0$. ■

The dominance of the dedicated technology follows from the fact that compared to the perfect market benchmark case, the flexible technology is affected more from the external financing frictions at this technology cost pair. This is only a partial characterization as we do not know yet the ordering (and the magnitude) of the financing costs with each technology in equilibrium. We analyze this next.

For $\rho = 1$, we have $\xi_1 = \xi_2 = \xi$ and $E \left[ (\xi_{1}^{-b} + \xi_{2}^{-b})^{-\frac{1}{b}} \right] = 2^{-\frac{1}{b}} \bar{\xi}$, and the creditor’s expected return with technology $T \in \{D, F\}$ is given by

$$\begin{align*}
\Lambda_D(a_D) & = (2c_D K_D^1 - B) a_D - BC \Pr \left( \xi < \bar{\xi} \left( 1 + \frac{1}{b} \right) \left[ 1 - \frac{B (1 + a_D)}{2 c_D K_D^1 (1 + a_D - \gamma_D)} \right] \right), \\
\Lambda_F(a_F) & = (c_F K_F^1 - B) a_F - BC \Pr \left( \xi < \bar{\xi} \left( 1 + \frac{1}{b} \right) \left[ 1 - \frac{B (1 + a_F)}{c_F K_F^1 (1 + a_F - \gamma_F)} \right] \right). \quad (39)
\end{align*}$$

For $\gamma_F = \gamma_D$, with the technology cost pair $(c_D, c_F^\prime(\gamma_F) = c_D)$, it follows from (39) that the creditor offers identical financing costs to each technology in equilibrium, i.e. $\hat{a}_F = \hat{a}_D$. The effect of an increase in $\gamma_F$ on $\hat{a}_F$ is non-trivial. It is tempting to say that a higher salvage value increases the collateral value; decreases the default risk and hence decreases $\hat{a}_F$. However, an increase in $\gamma_F$ also alters the optimal capacity investment level (both directly for a given $c_F$ and indirectly through $c_F(\gamma_F)$). Therefore the net gain from secured lending (the first term in the creditor’s expected return) and the default risk are also affected by this change. Thus, the overall effect is indeterminate.

To determine the ordering of the equilibrium financing cost with each technology, suppose that the creditor offers identical financing costs for each technology ($a_D = a_F = a$). Using (39), for $\gamma_F > \gamma_D$ with the technology cost pair $(c_D, c_F^\prime(\gamma_F))$, we can prove that the default risk is lower with the flexible technology. This is because the cost of borrowing net of the collateral value (salvage value) of the capacity investment is lower with the flexible technology. This induces the creditor to charge a lower financing cost with the flexible technology. On the ordering of the net gain of the creditor with each technology, if $\gamma_F < 1 + (b + 1)a$, then the net gain is higher with the flexible technology. This also
induces the creditor to charge a lower financing cost with the flexible technology; hence the equilibrium financing cost is lower with the flexible technology. For \( \gamma_F > 1 + (b + 1)a \), the net gain is lower with the flexible technology and this induces the creditor to charge a higher financing cost with the flexible technology. The resulting ordering of the equilibrium financing cost cannot be proved analytically and depends on the parameter levels.

We now numerically investigate the equilibrium financing costs with the technology cost pair \( (c_D, c'_F(\gamma_F) = c_D \frac{1-\gamma_D}{1-\gamma_F}) \) for \( \gamma_F \geq \gamma_D \) using the same parameter set as before \( (c_D = 1, b = -2, P = 350 \) and \( \sigma = 2.5 \)).

Figure 28: The effect of the salvage value of the flexible technology \( (\gamma_F) \) on the equilibrium financing cost, the expected equity value and the technology choice in equilibrium at the technology cost pair \( (c_D, c'_F(\gamma_F) = c_D \frac{1-\gamma_D}{1-\gamma_F}) \) in imperfect capital markets (in a perfectly competitive credit market with \( U = 0 \)) with \( BC = 200, B = 5, \gamma_D = 0.1, \) and \( \gamma_F \in [0.1, 0.5] \) with 0.125-unit increments: An increase in \( \gamma_F \) decreases the equilibrium financing cost (Panel A). Since \( c'_F(\gamma_F) \) is increasing, despite the increase in \( \gamma_F \) and the decrease in \( \dot{\alpha}_F \), the firm’s expected equity value with flexible technology decreases (Panel B). As depicted in Panel B, for low levels of \( \gamma_F \), we have \( \dot{\pi}_F > \dot{\pi}_D \) and the flexible technology is preferred. For high levels of \( \gamma_F \), despite the lower financing cost with the flexible technology, the dedicated technology is strictly preferred as the investment cost with the flexible technology becomes significantly more expensive than for the dedicated technology.

As depicted in Panel A of Figure 28, the equilibrium financing cost is identical with
each technology for $\gamma_F = \gamma_D$. With an increase in the salvage rate $\gamma_F$, $\dot{a}_F$ decreases and the firm secures a lower financing cost with the flexible technology in equilibrium.

In summary, with the technology cost pair $\left(c_D, c_F' \left(\gamma_F \right) = c_D \frac{1 - \gamma_D}{1 - \gamma_F} \right)$, the firm is indifferent between the two technologies in perfect markets. In imperfect markets, as follows from Lemma C.1, being exposed to external financing cost favors the dedicated technology (if the financing costs are identical with each technology). However, the firm can secure a lower financing cost with the flexible technology and this favors the flexible technology. Therefore, the equilibrium technology choice is indeterminate. As depicted in Panel B of Figure 28, for sufficiently low levels of $\gamma_F$, the firm may choose the flexible technology in equilibrium. In this case, the lower equilibrium financing cost may overcome the additional value of the dedicated technology with financing costs in place and the firm prefers the flexible technology. For high levels of $\gamma_F$, the latter effect may dominate the former and the firm strictly prefers the dedicated technology in imperfect capital markets.

These results illustrate the differences in equilibrium technology choice between perfect and imperfect capital markets that arise from asymmetry in salvage values. For example, it is interesting to note here that even when the product markets are perfectly positively correlated ($\rho = 1$), flexible technology may still have value in imperfect capital markets when it does not have value in perfect capital markets. This is because the firm can secure lower financing cost with the flexible technology in equilibrium.

D References