This paper analyzes the impact of endogenous credit terms under capital market imperfections in a capacity investment setting. We model a monopolist firm that decides on its technology choice (flexible versus dedicated) and capacity level under demand uncertainty. Differing from the majority of the stochastic capacity investment literature, we assume that the firm is budget-constrained and can relax its budget constraint by borrowing from a creditor. The creditor offers technology-specific loan contracts to the firm, after which the firm makes its technology choice and subsequent decisions. Capital market imperfections impose financing frictions on the firm. We derive the unit financing costs and the firm’s decisions in equilibrium. Our analysis contributes to the capacity investment literature by extending the theory of stochastic capacity investment and flexible versus dedicated technology choice to understand the impact of capital market imperfections, and by analyzing the impact of demand uncertainty (variability and correlation) on the operational decisions and the performance of the firm under different capital market conditions. We delineate the differences between perfect and imperfect markets as a function of the firm’s loan type in equilibrium (secured with or without default possibility, unsecured) and capital market characteristics. We demonstrate that the endogenous nature of credit terms in imperfect capital markets may modify or reverse conclusions concerning capacity investment and technology choice obtained under the perfect market assumption and we explain why. The theory developed in this paper suggests some rules of thumb for the strategic management of the capacity and technology choice in imperfect capital markets.

**Key Words:** Capacity, Flexibility, Financing, Newsvendor, Limited Liability, Market Imperfection.
1 Introduction and Literature Review

Capacity investment is subject to internal or external financing frictions, especially in capital-intensive industries. If the internal capital of the firm is not sufficient to finance the desired investment level, then the firm may decide to raise external capital. External capital is more expensive because there exist capital market imperfections such as bankruptcy costs, taxes, financial distress cost or agency costs due to asymmetric information etc. (Froot et al. 1993) that create frictions in the borrowing process of the firm. However, as highlighted by Van Mieghem (2003, p. 275) “stochastic capacity models assume (often implicitly) [...] perfect capital markets, so that frictionless borrowing is possible [...].” In imperfect capital markets, the investment decision and the cost of external capital are interdependent. The objective of this paper is to increase our understanding of how capital market imperfections affect stochastic capacity investment and technology choice. A key feature of our paper is that we endogenize the cost of borrowing in a creditor-firm equilibrium.

To this end, we model a firm who produces and sells two products under demand uncertainty. The firm chooses between flexible and dedicated technologies that incur variable investment costs, and determines the capacity level and the production quantities with the chosen technology. Differing from the majority of the stochastic capacity investment literature, we assume that the firm is budget-constrained and can relax its budget constraint by borrowing from a creditor. The creditor offers technology-specific loan contracts to the firm, after which the firm makes its technology choice and subsequent decisions. We assume that the creditor incurs a fixed cost of bankruptcy if the firm defaults on the loan. In the basic model, we assume that the credit market is perfectly competitive. At the other end of the spectrum, we analyze a credit market with a monopolist creditor. In summary, the fixed bankruptcy cost and the monopolistic nature of the credit market constitute the capital market imperfections considered in this paper.

We derive the technology choice and external borrowing, capacity, and production level decisions of the firm and the creditor’s loan terms in equilibrium. We investigate how the demand uncertainty affects the capacity investment level and the performance of the firm in equilibrium for a single-product firm and a two-product firm for a given technology. We delineate the main drivers of the equilibrium technology choice and the impact of demand uncertainty on this choice. Our analysis focuses on determining the differences between perfect and imperfect capital markets; and understanding the impact of firm characteristics and different capital market conditions. We note that the objective of this paper is not to solve for the optimal capital structure of the firm (equity versus debt financing with different
contractual terms); rather we focus on technology-specific loan contracts characterized by their unit financing cost, and analyze the creditor-firm strategic interaction in that setting. Our results contribute to several streams of research, as detailed below.

The stochastic capacity investment literature analyzes the capacity-pooling value of flexible technology over dedicated technology and the impact of demand uncertainty on this value in a variety of models. We refer readers to Van Mieghem (2003) for an excellent review. As highlighted in this review paper, the operations management literature (often implicitly) assumes that capital markets are perfect, in which case operational and financial decisions decouple (Modigliani and Miller 1958). In practice, capital market imperfections exist (Harris and Raviv 1991) and impose deadweight costs of external financing, leading operational and financial decisions to interact with each other. This is because these financing costs are affected by the firm’s operational decisions, and are endogenously determined in equilibrium. There is a growing body of work in operations and finance that analyzes these interactions. Our paper’s overall contribution to this literature is to extend the theory of stochastic capacity investment and flexible versus dedicated technology choice to understand the impact of capital market imperfections. We focus on the impact of demand uncertainty on the capacity investment decision and the performance of the firm. We show that this impact takes different forms depending on the firm’s loan type in equilibrium (secured with or without default possibility, unsecured) and the capital market conditions.

In the Operations Management literature, a recent stream of papers (Lederer and Singhal 1988 and 1994, Buzacott and Zhang 2004, Xu and Birge 2004, Babich and Sobel 2004, Babich et al. 2006, Dada and Hu 2008 and Caldentey and Haugh 2009) analyze the joint financing and operating decisions of the firm and demonstrate the value of integrated decision making. All these papers focus on a single-product setting where technology choice is irrelevant. Our single-product firm analysis develops new comparative statics results under different capital market conditions, while the two-product firm analysis is entirely new to this literature. We compare our results to two of these papers in particular.

Xu and Birge (2004) analyze the effect of capital market imperfections (in particular, taxes and bankruptcy costs) on the firm’s joint financing and operating decisions in a perfectly competitive credit market. Our single-product analysis complements theirs. They numerically show that an increase in demand variability decreases the capacity investment level of the firm. We provide conditions under which this observation holds analytically and demonstrate that the opposite can be true if the firm uses an unsecured loan in equilibrium. This is because with an unsecured loan, an increase in demand variability increases the
value of the limited liability option of the firm.

Lederer and Singhal (1994) study the joint financing (optimal mix of debt and equity) and capacity investment problem in a single-product, multi-period setting under the assumption of a perfectly competitive credit market. In a numerical example, they analyze the capacity-pooling benefit of flexible technology in a multi-product firm. They show that the value of flexible technology increases with a lower demand variability, and argue that this is because the default risk of the firm decreases, which allows the firm to secure a lower financing cost in equilibrium. In our model, we demonstrate that this result is only valid at high demand correlations. At low demand correlations, the default risk of the firm is not affected by the change in demand variability because the diversification benefit of operating in two markets (which we call “financial pooling”) and capacity-pooling benefit are sufficiently large. It follows that at low demand correlations, the value of flexible technology increases in demand variability. This is because i) the value of flexible technology at a given financing cost increases in demand variability (due to capacity pooling), and ii) the equilibrium level of financing cost is insensitive to changes in demand variability (due to financial- and capacity pooling).

Several finance papers also investigate the interaction of financing and operational decisions. Dotan and Ravid (1985) and Dammon and Senbet (1988) are examples of early research that demonstrates the effect of operational investments on the financing policy of the firm in a single-period setting. We refer the reader to Childs et al. (2005) for a recent review of papers in this stream. More recently, a number of papers in the finance literature (Mauer and Triantis 1994, Mello et al. 1995, and Mello and Parsons 2000) analyze the effect of various forms of operational flexibility (e.g. shutting down the production plant) on the joint operational and financing decisions of firm in the contingent claims framework. The main focus of these papers is on financial issues; and therefore they have strong modeling assumptions concerning the firm’s operations. We demonstrate that new trade-offs arise and new insights are obtained with a more detailed formalization of the firm’s operations (the sequential nature of technology choice, capacity investment and production decisions and the impact of demand uncertainty).

MacKay (2003) empirically documents negative correlation between production flexibility and the leverage of the firm and attributes this to higher equilibrium financing cost with production flexibility as the creditor believes that production flexibility will increase the riskiness of the firm’s cash flows. Our analysis demonstrates other facets of the interaction between production flexibility (flexible technology in our case) and the equilibrium credit
terms based on a stronger formalization of the firm’s operations. We show that in imperfect capital markets, the equilibrium technology choice is determined through the interplay between financial pooling, that exists with either technology, and capacity pooling, that only exists with flexible technology. The demand variability and the demand correlation play a key role in the equilibrium technology choice through their impact on the relative effectiveness of the financial- and the capacity-pooling benefits.

The remainder of this paper is organized as follows: In §2, we describe the model and discuss the basis for our assumptions. §3 provides the equilibrium strategy for the single-product (§3.1) and the two-product firm case for a given technology (§3.2). In §4, we investigate the impact of the demand uncertainty on the firm’s capacity investment decision and performance in equilibrium. §5 investigates the equilibrium technology choice. We conclude in §6 with a discussion of the limitations of our analysis and future research including potential avenues for empirical research.

2 Model Description and Assumptions

We consider a creditor-firm interaction where borrowing terms are determined before the firm makes any decisions. The firm is a budget-constrained monopolist that makes its technology choice and capacity investment decision (potentially after borrowing from the creditor) under demand uncertainty; and produces and sells two products after the resolution of this uncertainty. The firm chooses the technology (dedicated $D$ versus flexible $F$), and the borrowing, capacity investment, and production levels so as to maximize the expected equity value. We model the firm’s decisions as a two-stage stochastic recourse problem. We focus on a stylized firm that lives for a single period and is liquidated at the end of the period. After operating profits are realized, the firm pays back its debt (if any), and default occurs if it is unable to do so. Before discussing the timeline in detail, we introduce our assumptions about the firm and the credit market.

The firm’s objective is to maximize the expected shareholder wealth by maximizing the expected value of equity. Shareholders have limited liability. The risk-free rate $r_f$ is normalized to 0.

We assume that the creditor offers a technology-specific unit financing cost $a_T$ for $T \in \{D, F\}$ to the firm. The creditor has perfect information about the firm. We assume that the firm has physical assets of value $P$ (e.g. real estate) that are pledged to the creditor as collateral. The physical assets are illiquid; they can only be liquidated with a lead time. Therefore, default can occur even if the loan is secured.

As discussed in Froot et al. (1993), outside capital is more expensive than internally
generated funds. This is because there exist transaction costs of external financing that give rise to capital market imperfections. We assume that the creditor incurs a fixed bankruptcy cost $BC$ if the firm defaults on its loan; this cost is incurred as an out-of-pocket fee. $BC$ represents the direct cost of bankruptcy to the creditor, which includes the administrative and legal fees of the bankruptcy process (Altman 1980), and is often used in the literature to represent default-related capital market imperfections (e.g., Smith and Stulz 1985). Thus, the existence of bankruptcy cost introduces a capital market imperfection in our model.

In our basic model, the credit market is perfectly competitive and the creditor makes zero expected return. This is the common assumption used in the financial economics literature (e.g., Melnik and Plaut 1986). At the other end of the spectrum, we analyze a monopolist creditor who maximizes his expected return from lending. In summary, the fixed bankruptcy cost and the monopolistic nature of the credit market constitute the capital market imperfections considered in this paper.

Returning to the timeline, before the firm makes any decisions, the creditor offers its borrowing terms $a_T \geq 0 (= r_f)$, $T \in \{D, F\}$. In stage 1, the firm determines its technology choice $T \in \{D, F\}$ (if this dominates the doing nothing option in expectation), capacity investment level and borrowing level under the corresponding financing contract $a_T$ with respect to the internal budget constraint $B$. The flexible technology $(F)$ has a single resource that is capable of producing two products and the dedicated technology $(D)$ consists of two resources that can each produce a single product. Thus, the flexible technology has a capacity-pooling benefit over the dedicated technology. Technology $T$ incurs unit capacity investment cost $c_T$. Capacity investment can be salvaged at a rate of $0 \leq \gamma_T < 1$. Since flexible capacity is typically more marketable than dedicated capacity, we assume $\gamma_F \geq \gamma_D$.

In stage 2, demand uncertainty is resolved. The firm then chooses the production quantities (equivalently, prices) to satisfy demand optimally. Price-dependent demand for each product is represented by the iso-elastic inverse-demand function $p_i(q_i; \xi_i) = \xi_i q_i^{1/b}$ for $i = 1, 2$. Here, $b \in (-\infty, -1)$ is the constant price elasticity of demand, and $p$ and $q$ denote price and quantity, respectively. $\xi_i$ represents the idiosyncratic risk component. We make specific assumptions about the distribution of $(\xi_1, \xi_2)$ throughout the text whenever necessary. For tractability, we assume that the marginal production costs of each product are 0. This is an assumption that is widely used in the literature (see Goyal and Netessine 2007 and references therein). We discuss the implications of relaxing this assumption in §6.

At termination, the firm salvages its capacity investment. If the firm is able to repay its debt from its final cash position (that consists of operating revenues and the salvage value
of capacity), it does so and, since the firm lives for a single period, terminates by liquidating its physical assets. Otherwise, default occurs and the firm goes into bankruptcy. The cash on hand and the ownership of the collateralized physical assets $P$ are transferred to the creditor. The creditor may or may not be able to retrieve the face value of the loan from the seized assets of the firm depending on whether the firm is solvent or not when the value of $P$ is taken into account. In the former case, the firm collects the remaining cash.

We use the following mathematical representation throughout the text: A realization of the random variable $\xi$ is denoted by $\tilde{\xi}$ and its expectation is denoted by $\bar{\xi}$. Bold face letters represent vectors of the required size. Vectors are column vectors and $'$ denotes the transpose operator. $x^a$ denotes the componentwise exponent $a$ of the vector $x$. $xy$ denotes the componentwise product of vectors $x$ and $y$ with identical dimensions. $Pr$ denotes probability, $E$ denotes the expectation operator and $(x)^+ \doteq \max(x, 0)$. Monotonic relations (increasing, decreasing) are used in the weak sense unless otherwise stated.

3 The Equilibrium Strategy

In this section, we characterize the equilibrium decisions of the firm and the creditor. §3.1 illustrates the single-product firm analysis. We provide the two-product firm analysis for a given technology in §3.2.

3.1 The Single-Product Firm

In the single-product setting, the firm uses a single resource and technology choice is not relevant so we eliminate the $D$ and $F$ subscripts. We also have a uni-dimensional product market uncertainty $\xi$. We assume that it has a positive support $[\xi^l, \xi^u]$ with cdf $F(.)$ and pdf $f(.)$. The proofs for this section are provided in §A of the Technical Appendix.

3.1.1 Analysis of the Firm’s Problem for a Given Financing Cost

In this section, we describe the optimal solution for the firm’s capacity investment, external borrowing and production decisions using backward induction starting from stage 2.

Stage 2, Production Decision: In stage 1, the firm with budget $B$ borrowed $e$, invested in capacity level $K$ and placed $B + e - cK$ into the cash account (at the risk-free rate). In this stage, the firm observes the demand realization $\tilde{\xi}$ and determines the production quantity $q$ within the existing capacity limit $K$ to maximize the stage-two equity value $\Pi(q; K, e, B, \tilde{\xi})$.

To derive $\Pi$, note that two outcomes are possible: If the firm’s final cash position (consisting of stage-two operating profits, cash account holdings and the salvage value of capacity) is sufficient to cover the face value of the loan, i.e. if

$$ qp(q; \tilde{\xi}) + (B + e - cK) + \gamma cK \geq e(1 + a), $$

then the firm does not default. Otherwise, it defaults and its assets (including the ownership
of physical assets $P$) are transferred to the creditor. The bankruptcy cost $BC$ is borne by the creditor as an out-of-pocket expenditure. The firm receives the remaining cash (if any) after the face value of the loan is deducted from its seized assets. With the limited liability assumption, we can therefore write

$$\Pi\left(q; K, e, B, \tilde{\xi}\right) = qp(q; \tilde{\xi}) + (B + e - cK) + \gamma cK + P - e(1 + a).$$

(1)

Maximizing the stage-two equity value is equivalent to maximizing the operating profit. We find $q^* = \arg \max_{0 \leq q \leq K} qp(q; \tilde{\xi}) = K$, in other words, the firm optimally utilizes all of its available capacity. Then $\Pi^* (K, e, B, \tilde{\xi}) = h\tilde{\xi}K^{(1 + \frac{1}{\beta})} + (B + e - cK) + \gamma cK + P - e(1 + a)$. 

Stage 1, Capacity Choice and External Financing: In this stage, the firm has an internal budget $B \geq 0$ and determines the optimal capacity investment level $K^*$ and the optimal external borrowing level $e^*$ so as to maximize its expected equity value, $\pi(K, e; B) = \mathbb{E}\left[\Pi^* (K, e, B, \tilde{\xi})\right]$. It is easy to show that at optimality, $e = (cK - B)^+$ is satisfied, that is, the firm exactly borrows what it needs to cover its capacity investment. Thus, the optimal expected equity value of the firm, $\pi^*(B)$, can be found as follows:

$$\pi^*(B) = \max_{K \geq 0} \mathbb{E}\left[\tilde{\xi}K^{(1 + \frac{1}{\beta})} + (B - cK)^+ + \gamma cK + P - (cK - B)^+(1 + a)\right]^+. \quad (2)$$

For a given capacity investment level $K$, if the firm has borrowed, it does not default when demand is such that $\tilde{\xi} \geq b(K) \doteq \left((1 + a - \gamma)cK^{-\frac{1}{\beta}} - \frac{B(1 + a)}{K^{(1 + \frac{1}{\beta})}}\right)$, while it defaults, but is able to pay back the loan after its collateralized assets are liquidated when $\tilde{\xi} \geq l(K) \doteq b(K) - \frac{P}{K^{(1 + \frac{1}{\beta})}}$. We call $b(K)$ and $l(K)$ the bankruptcy threshold and the limited liability threshold, respectively, for investment level $K$. If the firm does not have any physical assets to collateralize ($P = 0$), then the bankruptcy threshold equals the limited liability threshold.

It is easy to establish that $l(K)$ is strictly increasing in $K$. We define $K^l$ as the (unique) solution to $l(K^l) = \xi^l$, where $\xi^l$ is the lower bound on demand. For $K \leq K^l$, the bracketed expression in (2) is non-negative for any demand realization $\tilde{\xi}$, i.e. the loan is secured. For $K > K^l$, for some $\tilde{\xi}$, the bracketed expression is negative, i.e. the loan is unsecured. The objective function in (2) is strictly concave for $K \in [0, K^l]$, but is not necessarily globally concave in $K$. Note that as the bracketed expression in (2) becomes more negative, not being liable for negative cash flows becomes more valuable. In this case, we say that the value of the limited liability option of the firm increases.

**Proposition 1** For the firm with an internal budget $B \geq B^h \doteq cK\left[1 - \frac{\xi^l}{\xi^l(1 + \frac{1}{\beta})}\right] \left[1 - \frac{\gamma}{1 + a}\right]$
\[
\frac{P}{1+a} \quad \text{where} \quad \hat{K} = \left( \frac{\xi u(1+\frac{1}{\gamma})}{(1+\frac{1}{\gamma})c} \right)^{-b}, \quad \text{the unique} \quad K^* \quad \text{is given by}
\]

\[
K^* = \begin{cases} 
K^0 = \left( \frac{\xi u(1+\frac{1}{\gamma})}{(1+\frac{1}{\gamma})c} \right)^{-b} & \text{if} \quad B \geq cK^0 \\
\frac{B}{c} & \text{if} \quad cK^1 \leq B < cK^0 \\
K^1 = \left( \frac{\xi u(1+\frac{1}{\gamma})}{(1+\frac{1}{\gamma})c} \right)^{-b} & \text{if} \quad B < cK^1
\end{cases}
\]

The condition \( B \geq B^h \) ensures that the objective function is globally concave and the optimal investment level can be found in closed form.\(^1\) Here, \( K^0 \) is the budget-unconstrained capacity level and \( K^1 \) is the capacity investment level with borrowing if the loan needed to make this investment is secured. If the budget realization is high enough to cover the corresponding cost \( cK^0 \), then \( K^* = K^0 \) with no borrowing. For \( cK^1 \leq B < cK^0 \), the budget is insufficient to cover \( cK^0 \), and the marginal revenue of capacity is lower than its marginal cost with borrowing. Therefore, the firm invests in the capacity level \( B \) that fully utilizes its internal budget. For \( B < cK^1 \), the firm borrows to invest in capacity. In this case, since the internal budget level is sufficiently high \( (B \geq B^h) \), the loan is secured.

**Proposition 2** For the firm with \( B < B^l \) \( \hat{K} = cK^1 \left[ 1 - \frac{\xi u}{\xi u(1+\beta)} (1+\frac{1}{\gamma})c \right] \) decrease in \( K \) for \( K > K^l \). 

The optimal solution is not necessarily unique, as neither global concavity nor unimodality can be guaranteed. It can be shown that in this budget range the firm would use an unsecured loan. The optimal solution is not necessarily unique, as neither global concavity nor unimodality can be guaranteed. It can be shown that in this budget range the firm would use an unsecured loan to invest in \( K^1 \). Therefore, the firm optimally takes more investment risk with borrowing, and increases \( K^* \) beyond \( K^1 \).

For the firm with \( B \in [B^l, B^h) \), we cannot explicitly characterize the optimal capacity investment level for a general distribution of \( \xi \). To guarantee the unimodality of the objective function and thus, the uniqueness of the solution, we make the following assumption:

**Assumption 1** Let \( b \geq -2 \) and \( \int_{(1-F(\xi))/f(\xi)}^{\xi u} (1-F(\xi)) d\xi \) decrease in \( K \) for \( K > K^l \).

The distributional assumption is satisfied for the normal\(^2\), uniform, exponential and triangular distributions, for example. We note here that in the literature, the commonly used

\(^1\)For \( P \geq \hat{P} = \xi u(1+\beta) \), \( c^h(1+\frac{1}{\gamma})^{-b} \) \( (1-\frac{\xi u}{\xi u(1+\beta)}) (1-\gamma) \) \( a \geq 0 \), we have \( B^h < 0 \) for any \( a \geq 0 \). In other words, Proposition 1 characterizes the optimal solution for any \( B \geq 0 \) and \( a \geq 0 \) when \( P \) is large enough.

\(^2\)Since we focus on the normal distribution to analyze the effect of demand uncertainty in §4, we prove the unimodality result in Lemma A.2 of the Technical Appendix.
assumption to guarantee unimodality is to focus on distributions with increasing generalized failure rates (see for example, Buzacott and Zhang 2004). This assumption is relevant for the price-taker newsvendor problem but is not useful for our price-setting newsvendor model. The restriction on the price elasticity of demand \( (b \geq -2) \) is a sufficient condition required for all the distributions listed above, but can be relaxed for some of these distributions individually.\(^3\) The specified range includes \( b = -2 \), which is the main choice for numerical examples used in the literature (see for example, Chod et al. 2009).

**Proposition 3** If Assumption 1 is satisfied, then the unique \( K^* \) is given by

\[
K^* = \begin{cases} 
K^0 & \text{if } B \geq cK^0 \\
\frac{B}{c} & \text{if } cK^1 \leq B < cK^0 \\
K^1 & \text{if } cK^1 \left[ 1 - \frac{1}{\xi(1+\xi)} \right] \left[ 1 - \frac{\gamma}{1+a} \right] - \frac{P}{1+a} \leq B < cK^1 \\
\overline{K} & \text{if } 0 \leq B < cK^1 \left[ 1 - \frac{1}{\xi(1+\xi)} \right] \left[ 1 - \frac{\gamma}{1+a} \right] - \frac{P}{1+a} 
\end{cases}
\]  

(3)

where \( \overline{K} \in (K^1, \hat{K}) \) is the unique solution to \( MP(\overline{K}) = 0 \).

The intuition of the first two cases in (5) is similar to Proposition 1. If the budget is large enough such that the firm can invest in \( K^1 \) using a secured loan, then the optimal capacity level with borrowing is \( K^1 \). If the budget is sufficiently low such that the firm would use an unsecured loan to invest in \( K^1 \), then the firm optimally takes more investment risk and the optimal capacity investment level with borrowing is \( \overline{K} > K^1 \). Using (5), the optimal expected equity value of the firm, \( \pi^* \), can be obtained, and is given in the proof of Proposition 3. It can be shown that \( \pi^* \) decreases in \( a \) (Lemma A.1), in other words, the firm benefits from a lower financing cost.

Throughout the single-product analysis, either we assume \( P \geq \overline{P} \) and use the characterization given in Proposition 1, or we assume an arbitrary \( P \) and use Assumption 1 and the characterization given in Proposition 3.

### 3.1.2 Characterization of the Creditor’s Expected Return

Let \( \Lambda(a) \) denote the creditor’s expected return for a given unit financing cost \( a \). When the firm borrows for a given \( a \geq 0 \), \( \Lambda(a) \) is given by

\[
\Lambda(a) = (cK^*(a) - B) a - F(b(K^*(a))) BC - \int_{\xi_1}^{\max\{0, (K^*(a))\}} \left[ (cK^*(a) - B)(1 + a) - \xi (K^*(a))^{(1+\frac{1}{\xi})} - \gamma cK^*(a) - P \right] f(\xi) d\xi,
\]

(4)

\(^3\)For example, with the uniform distribution, we can show that \( b \geq -3 \) is a sufficient condition.
where $K^*(a)$ denotes the optimal capacity level of the firm for a given $a$. The first term is the creditor’s net gain from lending if the loan is secured by the collateralized assets of the firm, the second term denotes the expected default cost (payable when the firm goes to bankruptcy), and the third term is the expected loss due to the unsecured part of the loan.

**Proposition 4** If Assumption 1 is satisfied, then the creditor’s expected return $\Lambda(a)$ is characterized by

\[
\begin{align*}
&i) \quad (cK^3(a) - B)a \\
&ii) \quad \left\{ \begin{array}{ll}
(cK^3(a) - B)a - F(b(K^3(a)))BC & \text{for } 0 \leq a < a^d \text{ if } cK^0 (1 - \gamma) \frac{(\xi(1+\frac{a}{\xi}) - \xi^d)}{\xi(1+\frac{a}{\xi})} \leq B, \\
(cK^3(a) - B)a & \text{for } a^d \leq a < a^m \text{ if } cK^0 (1 - \gamma) \frac{(\xi(1+\frac{a}{\xi}) - \xi^d)}{\xi(1+\frac{a}{\xi})} - P \leq B
\end{array} \right.
\end{align*}
\]

where $a^m = \left[ \left( \frac{cK^0}{B} \right)^{\frac{1}{\gamma}} - 1 \right] (1 - \gamma), L(K) = \int_{\xi(K)}^{l(K)} \left[ (cK - B) (1 + a) - \xi (1+\frac{a}{\xi}) - \gamma cK - P \right] dF(\xi)$, $a^d$, the unsecured loan threshold, is the unique solution to $B = cK^0 (1 - \gamma) - b \frac{(\xi(1+\frac{a}{\xi}) - \xi^d)}{\xi(1+\frac{a}{\xi})} \frac{(1+a^d - \gamma)(b+1)}{(1+a^d)}$. $a^d$ and $a^m$, the secured loan with default possibility threshold, is the unique solution to $B = cK^0 (1 - \gamma) - b \frac{(\xi(1+\frac{a}{\xi}) - \xi^d)}{\xi(1+\frac{a}{\xi})} \frac{(1+a^d - \gamma)(b+1)}{(1+a^d)}$.

If $B \geq cK^0$, the firm does not borrow for any $a \geq 0$, and the creditor does not have any returns (this is omitted from the statement of the proposition). Otherwise, the firm borrows if the financing cost is not very high, i.e. for $a \in [0, a^m]$.

If the internal budget level is sufficiently high (case (i)), the firm borrows to invest in $K^3(a)$ but never defaults for any $a \in [0, a^m]$. This case can only occur if there is a positive lower bound on demand or a positive salvage value of capacity. If the internal budget level is moderate (case (ii)), for a small $a$, the firm borrows to invest in $K^3(a)$ and may default on the loan, but the creditor can always retrieve the face value of the loan through the collateralized assets. For large $a$, the firm borrows less to invest in $K^3(a)$ and does not default. In summary, in case (ii), the firm may default but the borrowing is always secured. If the internal budget level is sufficiently low (case (iii)), the firm uses an unsecured loan to invest in $K(a)$ for small $a$. For moderate $a$, the firm borrows less to invest in $K^3(a)$, may default, but the loan is secured. For large $a$, the firm borrows even less and does not default.

If $P$ is sufficiently large, i.e. $P \geq \overline{P}$, only cases (i) and (ii) of Proposition 4 are relevant and Assumption 1 is not required.

\footnote{\textit{a} < \textit{a}^m \text{ is equivalent to } \textit{B} < \textit{cK}^3(\textit{a}) \text{ in Proposition 3.}}
3.1.3 Equilibrium Characterization

We now turn to the characterization of the equilibrium. We use the \( \dot{x} \) notation to denote equilibrium quantities: \( \dot{a} \) is the equilibrium unit financing cost and \( \dot{K} = K^*(\dot{a}) \), \( \dot{\pi} = \pi^*(\dot{a}) \) are the equilibrium capacity investment level and expected equity value of the firm respectively. When there are multiple \( a \)'s that satisfy the objective of the creditor, we set \( \dot{a} \) to the smallest such value. \( \dot{a} \) is Pareto-optimal for the firm because its optimal expected equity value increases as \( a \) decreases. Let \( a^N \) be the unique solution to \( (1 + a - \gamma)(1 + a - \gamma + ab) = \frac{B}{cK_0(1-\gamma)^{1/2}} \) which maximizes the net gain from secured lending.

**Proposition 5** For the perfectly competitive credit market, if Assumption 1 is satisfied or if \( P \geq P \), \( \dot{a} = 0 \) for case (i) of Proposition 4 and \( \dot{a} \in (0,a^d) \) otherwise. In the monopolistic credit market, \( \dot{a} \) can take any value in \( (0,a^{\max}) \). In particular, if Assumption 1 is satisfied or if \( P \geq P \), for cases (i) and (ii), \( \dot{a} = a^N \) if \( a^N \geq a^d \) and \( \dot{a} \in (a^N,a^d) \) otherwise.

If the firm does not default for any feasible \( a \), then the creditor’s expected return is always non-negative. In this case, in the perfectly competitive credit market, \( \Lambda(a) = 0 \) is achieved at \( \dot{a} = 0 \). If the firm may default for some \( a \), then \( \dot{a} < a^d \), i.e. the equilibrium financing cost in a perfectly competitive credit market is such that the firm has a positive default probability. In this case, either of the two equilibria, \( 0 < \dot{a} < a^l \) or \( a^l \leq \dot{a} < a^d \), may arise.

In contrast to the perfectly competitive credit market case, the monopolist creditor uses the marginal expected profit, i.e. \( \frac{\partial}{\partial a} \Lambda(a) \), and not the expected profit to determine the unit financing cost in equilibrium. When the loan is secured for any \( a \) (cases (i) and (ii)), \( \dot{a} \) can be characterized analytically as a solution to \( \frac{\partial}{\partial a} \Lambda(a) = 0 \). Our numerical experiments show that the creditor’s expected return may not be unimodal in \( a \) if the firm uses an unsecured loan (case (iii)). In particular, there may be two different local maxima, one in the unsecured lending region, and the other one in the secured lending region. Consequently, in the monopolist creditor case, all three equilibria \( (0 < \dot{a} < a^l, a^l \leq \dot{a} < a^d, a^d \leq \dot{a}) \) may arise depending on the parameter specifications.

In summary, depending on the credit market, firm and product market characteristics, any of the following three types of equilibria can be observed: an equilibrium where the firm uses a secured loan without default possibility, i.e. \( a^d \leq \dot{a} < a^{\max} \), an equilibrium where the firm uses a secured loan with default possibility, i.e. \( a^l \leq \dot{a} < a^d \), and an equilibrium where the firm uses an unsecured loan, i.e. \( \dot{a} < a^l \). When the firm uses a secured loan it invests in \( K^1(\dot{a}) \) whereas when the firm uses an unsecured loan, it invests in \( K(\dot{a}) \). For \( P \geq P \) the firm always uses a secured loan in equilibrium and only the first two equilibria are observed.
3.2 The Two-Product Firm For A Given Technology

The analysis is similar to the single-product firm analysis of §3.1 with minor modifications. For brevity, we only provide a synopsis of the analysis. Since some of the parameters are technology specific, we reintroduce subscripts \( D \) and \( F \). We assume that \( \xi' = (\xi_1, \xi_2) \) has a symmetric bivariate normal distribution (with \( \xi_1 = \xi_2 = \bar{x} \) and covariance matrix \( \Sigma \), where \( \Sigma_{ii} = \sigma^2 \) and \( \Sigma_{ij} = \rho \sigma^2 \) for \( i \neq j \)) because this is the natural setting to study the impact of demand correlation \( \rho \). Whenever necessary, we use the convention that the support of the marginal distribution of \( \xi_i \) is characterized by \([\xi_i^l, \xi_i^u]\) where \( \xi_i^l = \bar{x} - 3\sigma \) and \( \xi_i^u = \bar{x} + 3\sigma \) as almost all of the probability mass is located in this range.

We first analyze the firm’s decision problem for a given financing cost \( a_T \). With the dedicated technology, since \( \xi \) has a symmetric distribution, the firm optimally invests in identical capacity levels for both resources. Therefore, we can use a single resource level \( K_D \) to characterize the firm’s optimal capacity investment portfolio. The analysis of the two-product firm is almost identical to a single-product firm with product market uncertainty \( \xi_1 + \xi_2 \) and capacity investment cost \( 2c_D \). \( \xi_1 + \xi_2 \) has a normal distribution with mean \( \bar{\mu} = 2\bar{x} \) and standard deviation \( \sigma = \sigma \sqrt{2(1 + \rho)} \). Therefore, with \( b \geq -2 \), Assumption 1 is satisfied and \( K_D^* \) is characterized by using an analogue of Proposition 3:

**Proposition 6** If \( b \geq -2 \) and \( \xi' = (\xi_1, \xi_2) \) has a symmetric bivariate normal distribution, then the unique investment level \( K_D^* \) for each resource is given by

\[
K_D^* = \begin{cases} 
K_D^0 = \left( \frac{\bar{x}(1+\frac{1}{b})}{(1-\gamma_D)c_D} \right)^{-b} & \text{if } B \geq 2c_D K_D^0 \\
\frac{B}{2c_D} & \text{if } 2c_D K_D^1 \leq B < 2c_D K_D^0 \\
\frac{B}{2c_D} & \text{if } 2c_D K_D^1 \leq B < 2c_D K_D^0 \\
\frac{1}{(1+a_D-\gamma_D)c_D} & \text{if } 0 \leq B < 2c_D K_D^1 \left[ 1 - \frac{\xi^l}{\xi^u(1+\frac{1}{b})} \right] \left[ 1 - \frac{\gamma_D}{\gamma^u(1+\frac{1}{b})} \right] - \frac{P}{1+\alpha_D} \leq -1 \\
\frac{1}{(1+a_D-\gamma_D)c_D} & \text{if } 0 \leq B < 2c_D K_D^1 \left[ 1 - \frac{\xi^l}{\xi^u(1+\frac{1}{b})} \right] \left[ 1 - \frac{\gamma_D}{\gamma^u(1+\frac{1}{b})} \right] - \frac{P}{1+\alpha_D} \\
\end{cases}
\]

\( K_D \) is the unique solution to \( MP_D(K_D) = 0 \) with \( MP_D(K_D) \) characterized by

\[
[1 - \Phi\left(\frac{l_D(K_D) - \bar{\mu}}{\bar{\sigma}}\right)] \left[ 1 + \frac{1}{b} \right] (1 + a_D - \gamma_D) c_D + \left( 1 + \frac{1}{b} \right) \sigma K_D^\left(\frac{1}{b}\right) \phi\left(\frac{l_D(K_D) - \bar{\mu}}{\bar{\sigma}}\right)
\]

where \( l_D(K_D) = \bar{K}_D^{-1} \left[ 2(1 + a_D - \gamma_D) c_D - K_D \left[ 1 - \frac{1}{\gamma_D} \right] [B (1 + a_D) + P] \right] \) is the limited liability threshold with the dedicated technology, \( \bar{\mu} = 2\bar{x}, \bar{\sigma} = \sigma \sqrt{2(1 + \rho)} \) and \( \Phi(), \phi() \) are cdf and pdf of the standard normal random variable respectively.

The proof is very similar to Proposition 3 of the single-product case and thus, is omitted. For a sufficiently large \( P \), i.e. \( P \geq \bar{T}_D \equiv 2c_D^{b+1} \left[ \xi^u (1+\frac{1}{b}) \right]^{-b} \left[ 1 - \frac{\xi^l}{\xi^u (1+\frac{1}{b})} \right] \left[ 1 - \gamma_D \right]^{b+1}, \)
the loan is secured, and only the first three forms of $K_D^*$ in Proposition 6 are relevant, paralleling Proposition 1.

With the flexible technology, the analysis of the two-product firm is identical to a single-product firm with product market uncertainty $\left(\xi_1^b + \xi_2^b\right)^{-\frac{1}{b}}$ and capacity investment cost $c_F$. For a sufficiently large $P$, i.e. $P \geq \mathcal{P}_F = 2\mathcal{F}_F^{b+1} \left[\xi_u^a (1 + \frac{1}{b})\right]^{-b} \left[1 - \frac{\xi_l^f}{\xi_u^a (1 + \frac{1}{b})}\right] \left[1 - \gamma_F\right]^{b+1}$, the optimal capacity investment level is the same as in Proposition 1 except for one modification: In $K_F^\alpha$ (and $K_F^\beta$), the term $\xi$ is replaced by $\mathbb{E}\left[\left(\xi_1^b + \xi_2^b\right)^{-\frac{1}{b}}\right]$. This new term captures the capacity-pooling benefit of the flexible technology. For an arbitrary value of $P$, Assumption 1 is not satisfied for the random variable $\left(\xi_1^b + \xi_2^b\right)^{-\frac{1}{b}}$, therefore an analogue of Proposition 3 cannot be used. Nevertheless, there exists a $B_F^h$ such that for $B \geq B_F^h$, the optimal capacity investment level is characterized by the analogue of Proposition 1, and a $B_F^b$ such that for $B < B_F^b$, the optimal capacity investment level $\overline{K}_F$ is a solution to $MP_F(\overline{K}_F) = 0$, where

$$MP_F(K_F) = \int_\mathcal{Y}_F(K_F) \left[\left(1 + \frac{1}{b}\right) \left(\xi_1^b + \xi_2^b\right)^{-\frac{1}{b}} \mathcal{K}_F^{\xi_0} \mathcal{R}_F \left(1 + a_F - \gamma_F\right) c_F \right] f(\xi_1, \xi_2) d\xi_1 d\xi_2,$$

with $\mathcal{Y}_F(K_F) \triangleq \left\{\xi : \xi' \geq \xi'' \geq (\xi_1^l, \xi_1^t); l_F(K_F) \leq \left(\xi_1^b + \xi_2^b\right)^{-\frac{1}{b}} \leq 2^{-\frac{1}{b}} \xi_u^a\right\}$, and $l_F(K_F) \triangleq \overline{K}_F^\alpha (1 + a_F - \gamma_F) c_F - K_F \left(1 - \frac{1}{b}\right)[B (1 + a_F) + P]$ is the limited liability threshold with the flexible technology, $f(\xi_1, \xi_2)$ is the joint pdf of $\xi$. For $B \in [B_F^l, B_F^b)$, there is no analytical characterization of $K_F^*$; thus we resort to numerical experiments.

For the creditor’s problem, if we assume a sufficiently large $P$ for each technology, i.e. $P > \mathcal{P}_T$ for $T \in \{D, F\}$, then the loan is always secured. The creditor’s expected return with each technology as a function of $a_T$ can be characterized in a similar fashion to Proposition 4 where only cases (i) and (ii) are relevant. With the dedicated technology, for an arbitrary value of $P$, Assumption 1 is satisfied and the creditor’s expected return can be characterized in a similar fashion to Proposition 4 where all three cases are relevant. With the flexible technology, Assumption 1 is not satisfied. As result, for an arbitrary value of $P$, we cannot guarantee the existence and uniqueness of the $a_F$ thresholds $a_F^l$ and $a_F^d$ paralleling those in Proposition 4. Thus, the structure of the creditor’s expected return given in case (iii) does not need to hold. In this case, we resort to numerical experiments to characterize the creditor’s expected return.

The characterization of the unique Pareto-optimal equilibrium $\hat{a}_T$ for each technology under the different capital market conditions is similar to the single-product case. For $P \geq \mathcal{P}_T$, there exist two different equilibria with each technology: An equilibrium where
the firm uses a secured loan without default possibility ($\hat{a}_T \geq a_T^d$) and an equilibrium where
the firm uses a secured loan with default possibility ($\hat{a}_T \in (0, a_T^d)$). The characterization
is similar to Proposition 5. For an arbitrary value of $P$, we observe the three different
equilibria paralleling the single-product case.

4 The Impact of Demand Uncertainty

4.1 The Single-Product Firm

The goal of this paper is to analyze the impact of endogenous credit terms under capital
market imperfections in a capacity investment setting. To this end, we first identify the
perfect capital market equilibrium and present comparative statics results with respect to
demand variability in the single-product firm case.

If the capital markets are perfect, there is no fixed bankruptcy cost ($BC = 0$). In this
case, the firm’s capacity investment decision is independent of financing decision:

**Remark 1** In the perfect capital market equilibrium, for any firm with $B \geq 0$, we have $\hat{K} = K^0 = \left(\frac{\bar{\xi}(1 + \frac{1}{\gamma})}{(1 - \gamma)c}\right)^{-b}$ and the expected equity value of the firm is given by $\hat{\pi} = B + P + \frac{cK^0(1 - \gamma)}{-(b + 1)}$. $\hat{K}$ and $\hat{\pi}$ are independent of the demand variability.

The equilibrium investment level is the budget-unconstrained investment level $K^0$ for the
firm with any internal budget level as in traditional stochastic capacity models: The firm
simply chooses the optimal investment level without regard to the budget limit, and im-
plements it by borrowing if necessary. This replicates the well-known result about the
decoupling of operational and financial decisions in perfect markets (Modigliani and Miller
1958); but we do it to have the benchmark specific to our model. We now show that
the impact of the demand variability shown in Remark 1 is modified once capital market
imperfections and endogenous credit terms are taken into account, and we explain why.

Our goal is not to undertake a complete characterization of the equilibrium, but to show
the existence of certain effects that arise from capital market imperfections. For this reason,
we focus on a specific demand distribution that satisfies Assumption 1, and assume that
$\xi$ has a normal distribution with mean $\bar{\xi}$ and standard deviation $\sigma$. Results for $P \geq \bar{P}$
where Assumption 1 is not needed are discussed later. For brevity, we also normalize the
salvage rate $\gamma$ to zero. The numerical experiments used in this section use the following
data set: $b = -2$, $c \in \{1, 2\}$, $\bar{\xi} \in \{20, 30, 40\}$, $P \in \{0, 100, 200\}$, $BC \in \{100, 250, 500\}$ and
$B \in \{5, 20, 45\}$. We investigate 162 numerical instances for both of the imperfect capital
market models that we study. To analyze the impact of demand variability, we use the
mean-preserving spread of the normal distribution. For a given mean $\bar{\xi}$, a higher $\sigma$ leads
to a higher demand variability. In our numerical experiments, to deal with the issue of non-negativity of demand, we assume that the coefficient of variation is not very large, and the effect of negative values is negligible. Thus, we use $\sigma \in [10\%, 30\%]$ of $\tilde{\xi}$ with 2%-unit increments. For convenience, we summarize the results of this section in Table 1, where the boxed comparative statics results are proven analytically and the rest are existence results observed in numerical experiments. The supporting technical statements of this section, denoted by B.x, are provided in §B of the Technical Appendix.

### Table 1: Differences between perfect and imperfect markets concerning the impact of a (local) decrease in demand variability in single-product investments with normal ($\tilde{\xi}, \sigma$) demand uncertainty where $\sigma \leq 0.3\tilde{\xi}$. The boxed comparative statics results are proven analytically.

<table>
<thead>
<tr>
<th>Perfect Market</th>
<th>Imperfect Market</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Perfectly Competitive Credit Market</strong></td>
<td><strong>Monopolist Creditor</strong></td>
</tr>
<tr>
<td>Form of $\dot{K}$</td>
<td>Effect</td>
</tr>
<tr>
<td>$K^0$</td>
<td>$\dot{K} - \Pi -$</td>
</tr>
<tr>
<td>$K^0$</td>
<td>$\dot{K} - \Pi -$</td>
</tr>
<tr>
<td>$K^1(\dot{a})$</td>
<td>secured w/o default possibility</td>
</tr>
<tr>
<td>$K^1(\dot{a})$</td>
<td>secured w/o default possibility</td>
</tr>
<tr>
<td>$\dot{K}(\dot{a})$</td>
<td>unsecured</td>
</tr>
<tr>
<td>$\dot{K}(\dot{a})$</td>
<td>unsecured</td>
</tr>
</tbody>
</table>

As follows from Remark 1, in perfect capital markets, the firm’s equilibrium capacity decision and expected equity value do not depend on the demand variability $\sigma$. In imperfect capital markets, this result continues to hold at equilibria where the firm uses a secured loan without default possibility (Row 1 of the “Imperfect Market” column of Table 1). In the other two cases, $\dot{K}$ and $\dot{\pi}$ depend on the demand variability.

**Proposition 7** (Row 2 of Table 1) At equilibria where the firm uses a secured loan with default possibility, a lower $\sigma$ locally increases $\dot{\pi}$ and $\dot{K}$ through a decrease in $\dot{a}$ in a perfectly competitive credit market. For the monopolist creditor, a lower $\sigma$ locally decreases (decreases) $\dot{K}$ and $\dot{\pi}$ through a decrease (increase) in $\dot{a}$ if $\sigma < \tilde{\xi} - b(K^1(\dot{a}))$ ($\sigma > \tilde{\xi} - b(K^1(\dot{a}))$) where $b(K^1(\dot{a}))$ is the bankruptcy threshold.

From the firm’s perspective, for an arbitrary $a$, the demand variability does not alter the
optimal capacity investment level or the equity value (as the firm uses a secured loan). Therefore, the impact of \( \sigma \) is only determined by its effect on \( \hat{a} \). In a perfectly competitive credit market, a decrease in \( \sigma \) decreases the downside risk, and in turn, the default risk of the firm. This decreases the expected default cost and increases the expected return of the creditor for an arbitrary \( a \). Therefore, the rate \( \hat{a} \) decreases. For the monopolist creditor, it can be shown that if \( b \geq -2 \) and \( \frac{\xi}{\sigma} \leq 0.5 \), we have \( \sigma < \bar{\xi} - b(K^*(\hat{a})) \). This condition is satisfied in our setting as \( b \geq -2 \) by Assumption 1, and we assume \( \sigma \leq 0.3\bar{\xi} \) to avoid negative realizations of \( \xi \). Therefore, a decrease in \( \sigma \) decreases the marginal default risk of the firm. This decreases the marginal expected default cost and \( \hat{a} \) decreases. In summary, since \( \hat{a} \) decreases in both capital market models considered, \( \hat{K} \) and \( \hat{\pi} \) increase in both cases.

At equilibria where the firm uses an unsecured loan (Row 3 of Table 1), the impact of demand variability is determined through the interplay between two effects: i) the value of the limited liability option of the firm and ii) the equilibrium financing cost. For a given \( a \), a decrease in \( \sigma \) decreases \( K^*(a) = \bar{K}(a) \) and \( \pi^*(a) \) (Lemma B.1). This is because as the likelihood of low demand states decreases, the value of the limited liability option of the firm decreases. Moreover, for a given \( \sigma \), a higher unit financing cost decreases \( \bar{K}(a) \) (Lemma B.2) and \( \pi^*(a) \) (Lemma A.1). If a decrease in \( \sigma \) increases \( \hat{a} \), then we conclude that \( \hat{K} \) and \( \hat{\pi} \) decrease since both effects work in the same direction; otherwise, the result depends on which effect dominates. We now analyze the impact of \( \sigma \) on \( \hat{a} \).

In a perfectly competitive credit market, from the creditor’s perspective, decreasing \( \sigma \) has three distinct effects corresponding to the three components of \( \Lambda(a) \). As we show in Lemma B.3, the default risk\(^5\) and the expected loss due to the unsecured part of the loan decrease (as for a fixed \( \bar{K} \), the downside risk of the firm’s operating cash flows decreases, and the firm borrows less as \( \bar{K} \) decreases); and the creditor’s net gain decreases (the firm borrows less as \( \bar{K} \) decreases). The first two effects work to decrease \( \hat{a} \), whereas the third effect works to increase it. In all of our numerical experiments, we observe that the first two effects dominate and \( \hat{a} \) decreases with a decrease in \( \sigma \). To determine whether the decrease in \( \hat{a} \) or the decrease in the value of limited liability option of the firm dominates, we resort to numerical experiments. For \( \hat{\pi} \), we observe that the former dominates and \( \hat{\pi} \) increases with a decrease in \( \sigma \). For \( \hat{K} \), we observe that with a decrease in \( \sigma \), the former effect dominates at high \( \sigma \) levels and \( \hat{K} \) decreases, then with a further decrease in \( \sigma \), the latter effect dominates and \( \hat{K} \) increases as \( \sigma \) decreases (and the equilibrium shifts to a secured loan with a further reduction in \( \sigma \)).\(^6\) The turning point may not always exist (which is the case in the majority

\(^5\)Lemma B.3 proves this with an additional condition which is satisfied in our numerical experiments.

\(^6\)In our numerical experiments for both of the capital market conditions considered, we observe that there
of our numerical experiments). Nevertheless, there exist instances in which \( \dot{K} \) increases with a decrease in \( \sigma \).

For the monopolist creditor, we observe in a few instances that \( \dot{a} \) may increase with a decrease in \( \sigma \) when the creditor’s expected return is bimodal. The decrease in \( \sigma \) may induce the creditor to switch from one local maximizer (in the unsecured lending region) to the other local maximizer (in the secured lending region). This leads to a discontinuous increase in \( \dot{a} \) such that \( \dot{K} \) and \( \dot{\pi} \) decrease. Otherwise, \( \dot{a} \) decreases with a decrease in \( \sigma \), and the impact on \( \dot{K} \) and \( \dot{\pi} \) is similar to the perfectly competitive credit market case.

Finally, we discuss the impact of demand variability on firms with a sufficiently large value of \( P \), where Assumption 1 is not required. In this case, the equilibrium where the firm uses an unsecured loan does not exist. Since the loan is secured, the impact of \( \sigma \) is solely determined by its impact on \( \dot{a} \). As in Table 1, \( \dot{K} \) and \( \dot{\pi} \) continue to be independent of \( \sigma \) at equilibria where the firm uses a secured loan without default possibility because \( \dot{a} \) does not depend on \( \sigma \). Otherwise, the results depend on the distribution of \( \xi \). In a perfectly competitive credit market, as follows from (4), the sign of the impact of \( \sigma \) on \( \dot{a} \) is determined by the sign of its impact on the default probability \( F(b(K^1)) \), i.e. on the cdf of \( \xi \). For the monopolist creditor, the impact of \( \sigma \) on \( \dot{a} \) is determined by its impact on the marginal default probability \( f(b(K^1)) \), i.e. on the pdf of \( \xi \). When \( \xi \) is normally distributed, and \( b \geq -2 \) does not hold, all the results of Table 1 at equilibria where the firm uses a secured loan continue to hold with one modification: For the monopolist creditor, a decrease in \( \sigma \) may decrease \( \dot{K} \) and \( \dot{\pi} \) if \( \sigma \) is sufficiently low.

In summary, the impact of the demand variability is determined through the interplay between the value of the limited liability option of the firm (only with an unsecured loan) and the equilibrium financing cost. We find that:

1. The value of the limited liability option of the firm decreases with a decrease in \( \sigma \).
2. The impact of \( \sigma \) on \( \dot{a} \) is through its impact on the default risk with a secured loan. This impact is multi-dimensional with an unsecured loan, as the capacity investment level is affected by the demand variability due to limited liability.
3. A decrease in \( \sigma \) decreases \( \dot{a} \) with a secured loan, and, in the majority of the cases, with an unsecured loan. Therefore, the impact of a decrease in \( \sigma \) is determined by the trade-off between lowering \( \dot{a} \) and lowering the value of the limited liability option:

| Exist two threshold values of \( \sigma \): One below which the firm uses a secured loan without default possibility and one above which the firm uses an unsecured loan in equilibrium. As \( \sigma \) increases, the equilibrium shifts from a secured loan without default possibility to one with default possibility and then to an unsecured loan. The second \( \sigma \) threshold could be sufficiently high such that we only observe secured loans in equilibrium. |
a. For $\hat{\pi}$, the equilibrium financing cost is the main determinant, a decrease in $\sigma$ increases $\hat{\pi}$.

b. For $\hat{K}$, the equilibrium financing cost is the main determinant with a secured loan or with an unsecured loan when $\sigma$ is sufficiently high: A decrease in $\sigma$ increases $\hat{K}$. The value of the limited liability option of the firm is the main determinant when $\sigma$ is moderate: A decrease in $\sigma$ decreases $\hat{K}$. This is because $\hat{a}$ is not very sensitive to changes in $\sigma$ in this range. At lower $\sigma$ values, there is no default possibility and $\hat{K}$ does not change in $\sigma$.

4. Interestingly, the firm may be worse off with a decrease in $\sigma$ in a monopolistic credit market, and $\hat{K}$ and $\hat{\pi}$ decrease. This is observed with a secured loan if $\sigma$ is sufficiently low and $b < -2$; and with an unsecured loan if the creditor’s expected return is bimodal.

4.2 The Two-Product Firm

In this section, we investigate the effect of demand variability ($\sigma$) and correlation ($\rho$) in the two-product firm setting. As discussed in §3.2, we assume that $\xi$ has a symmetric bivariate normal distribution (with $\xi_1 = \xi_2 = \bar{\xi}$ and covariance matrix $\Sigma$, where $\Sigma_{ii} = \sigma^2$ and $\Sigma_{ij} = \rho \sigma^2$ for $i \neq j$). For the numerical experiments in this section, we use the same data set with the single-product firm analysis except $\bar{\xi} \in \{10, 15, 20\}$ and $\rho \in \{-0.9995, -0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75, 0.9995\}$. Our main results are summarized in Table 2 where the boxed comparative statics results are proven analytically and the rest are existence results observed in numerical experiments.

4.2.1 Dedicated Technology

In perfect capital markets, paralleling Remark 1, the firm’s equilibrium capacity level for each resource and the expected equity value with the dedicated technology are given by

$$\hat{K}_D = K_D^0 = \left(\frac{\bar{\xi}(1+\frac{1}{2})}{1-\gamma_D c_D}\right)^{-b} \quad \text{and} \quad \hat{\pi}_D = B + P + \frac{2c_D K_D^0 (1-\gamma_D)}{-b+1}$$

respectively. $\hat{K}_D$ and $\hat{\pi}_D$ depend only on the mean demand.

In imperfect capital markets, demand uncertainty, i.e. the correlation coefficient $\rho$ and the demand variability $\sigma$, may matter. As discussed in §3.2, the two-product firm analysis with the dedicated technology is very similar to the single-product firm analysis with product market uncertainty $\xi_1 + \xi_2$ and capacity investment cost $2c_D$. For a bivariate normal $\xi$, $\xi_1 + \xi_2$ has a normal distribution with mean $\bar{\xi} = 2\xi$ and standard deviation $\sigma = \sqrt{2(1+\rho)}$.

A decrease in $\sigma$ or $\rho$ leads to a decrease in $\sigma$, and thus, the impact of these two changes are identical to the impact of a decrease in $\sigma$ in a single-product firm. Therefore, results summarized in Table 1 as well as the results for a sufficiently large value of $P$, where Assumption 1 is not required, for the single-product firm continue to hold with a decrease in $\sigma$ or $\rho$ in the two-product firm with the dedicated technology. We note that the impact of
Table 2: Differences between perfect and imperfect markets in two-product investments with bivariate normal demand uncertainty for a perfectly competitive credit market. The boxed results are proven analytically. For the monopolist creditor, the impact of a lower $\sigma$ or $\rho$ with dedicated technology is identical to the impact of a lower $\sigma$ in the single-product case, i.e. the results in the “Monopolist Creditor” column of Table 1 hold. With flexible technology, the results are identical to Table 2 except that the impact of $\rho$ and $\sigma$ at equilibria where the firm uses a secured loan without default possibility is not proven analytically.

Demand correlation $\rho$ follows from a financial-pooling argument: Operating in two markets creates a diversification benefit for the firm, i.e. the variability of total demand, $\sigma^2$, is lower than the sum of the variability of the individual demands, $2\sigma^2$. As $\rho$ increases, the financial-pooling benefit decreases as the firm generates similar revenues from both markets.

In our numerical experiments, we observe an interesting interaction between the demand correlation and the demand variability:

1. At low $\sigma$ levels, the firm uses a secured loan without default possibility and the impact of $\sigma$ or $\rho$ on $\dot{K}_D$ and $\dot{\Pi}_D$ is zero; and this is observed at all $\rho$ levels.
2. The interaction between $\sigma$ and $\rho$ is more subtle at higher $\sigma$ levels:
   a. In a perfectly competitive credit market, for a given $\rho$, there exists a threshold value of $\sigma$, $\bar{\sigma}_D(\rho)$, below which $\dot{a}_D \approx 0$ and is insensitive to $\rho$ and $\sigma$. Consequently, the insensitivity...
of $\hat{K}$ or $\hat{\pi}$ to $\sigma$, which is observed at secured loan equilibria without default possibility, is now observed at secured loan equilibria with default possibility and even unsecured loan equilibria. This is due to the financial pooling phenomenon that makes $\bar{\sigma}$ small relative to $\sigma$ for sufficiently low values of $\rho$ such that the default probability becomes negligible. As $\rho$ increases, the financial-pooling benefit decreases and $\bar{\sigma}_D(\rho)$ decreases.

b. In a monopolistic credit market, the impact of the financial pooling phenomenon is different from the perfectly competitive credit market case and $\hat{a}_D$ is sensitive to $\rho$ and $\sigma$. This is because, as we discussed in §4, the marginal default probability, i.e. the pdf of $\xi$, is relevant. The impact of $\sigma$ or $\rho$ on the marginal default probability is not negligible when the default probability is negligible. Therefore, $\hat{a}$ and in turn, $\hat{K}$ and $\hat{\pi}$ are affected by $\sigma$ at secured loan equilibria with default possibility or unsecured loan equilibria.

4.2.2 Flexible Technology

In perfect capital markets, the firm’s equilibrium capacity level and the expected equity value with the flexible technology are given by $\hat{K}_F = K_F^0 = \left( \frac{M_F(1+\epsilon)}{1-\gamma_F} \right)^{-b}$ and $\hat{\pi}_F = B + P + \frac{c_F K_F^0 (1-\gamma_F)}{(b+1)}$ respectively, where $M_F = E \left[ \left( \xi_{1}^{-b} + \xi_{2}^{-b} \right)^{-\frac{1}{b}} \right]$. $\hat{K}_F$ and $\hat{\pi}_F$ depend on the demand variability ($\sigma$) and the demand correlation ($\rho$) through the term $M_F$. This term captures the capacity-pooling feature of flexible technology. Unfortunately, it is not possible to derive analytically the effect of $\rho$ and $\sigma$ on $M_F$ for bivariate normal $\xi$. To derive the analytical results for flexible technology in Table 2, we make the following assumption:

Assumption 2 $M_F = E \left[ \left( \xi_{1}^{-b} + \xi_{2}^{-b} \right)^{-\frac{1}{b}} \right]$ is decreasing in $\rho$ and increasing in $\sigma$.

This assumption is in line with the traditional argument on flexible technology investment: Its value increases in demand variability and decreases in demand correlation. In our numerical experiments, consistent with this assumption, we observe that $M_F$ decreases with a lower $\sigma$ or a higher $\rho$. Therefore, $\hat{K}_F$ and $\hat{\pi}_F$ decrease in perfect capital markets.

In imperfect capital markets, the impact of the demand variability ($\sigma$) and the demand correlation ($\rho$) with the flexible technology is determined through the interplay among the value of capacity pooling, the value of the limited liability option of the firm (only with an unsecured loan) and the equilibrium financing cost. The results in Table 2 can be explained as follows:

1. An increase in $\rho$ or a decrease in $\sigma$ decreases the value of capacity pooling for a given $a_F$ under Assumption 2. A decrease in $\rho$ or $\sigma$ decreases the value of the limited liability option of the firm for a given $a_F$ as the likelihood of low demand states decreases.
2. The impact of $\sigma$ and $\rho$ on $\dot{a}_F$ is multi-dimensional. With a secured loan, this is because the capacity investment level is affected by $\sigma$ and $\rho$ due to capacity pooling, and alters the default risk and net gain from secured lending. With an unsecured loan, the change in the capacity investment level is due to capacity pooling and limited liability. In our numerical experiments, we observe that $\dot{a}_F$ decreases with a decrease in $\sigma$ or $\rho$ in a perfectly competitive credit market. In a monopolistic credit market, we observe the same for the majority of the cases analyzed, but there exist instances in which $\dot{a}_F$ increases with a decrease in $\sigma$ or $\rho$.

3. For the impact of $\rho$ on $\dot{K}_F$ and $\dot{\pi}_F$, the value of capacity pooling is the main determinant (both with a secured or an unsecured loan and in both of the capital market conditions analyzed): A decrease in $\rho$ increases $\dot{K}_F$ and $\dot{\pi}_F$.

4. For the impact of $\sigma$ on the same, we find that:
   a. At low $\sigma$ values, the firm uses a secured loan without default possibility and the value of capacity pooling is the main determinant such that a decrease in $\sigma$ decreases $\dot{K}_F$ and $\dot{\pi}_F$ (and this is observed at all $\rho$ levels). In a perfectly competitive credit market, this is because $\dot{a}_F = 0$ and does not depend on $\sigma$. For the monopolist creditor, $\dot{a}_F > 0$ and depends on $\sigma$ due to the capacity-pooling feature (through the term $M_F$). It can be proven that a lower $\sigma$ decreases $\dot{a}_F$. In our numerical experiments, the decreasing value of capacity pooling outweighs the decrease in $\dot{a}_F$ such that $\dot{K}_F$ and $\dot{\pi}_F$ decrease.
   b. At higher $\sigma$ levels, the impact of $\sigma$ is more subtle and depends on capital market conditions and $\rho$:
      - In a perfectly competitive credit market, we observe that for a given $\rho$, there exists a threshold value of $\sigma$, $\bar{\sigma}_F(\rho)$, below which $\dot{a}_F \approx 0$ and is insensitive to $\sigma$. This is due to the financial-pooling and the capacity-pooling benefits that make the default probability negligible. As $\rho$ increases, these benefits decrease and $\bar{\sigma}_F(\rho)$ decreases. When $(\sigma, \rho)$ are such that $\sigma \leq \bar{\sigma}_F(\rho)$, the sole determinant is capacity pooling such that $\dot{K}_F$ and $\dot{\pi}_F$ decrease with a decrease in $\sigma$. When $\sigma > \bar{\sigma}_F(\rho)$, we have $\dot{a}_F > 0$ and the impact of $\sigma$ on $\dot{K}_F$ and $\dot{\pi}_F$ depends on $\rho$. If $\rho$ is not high, the value of capacity pooling and the value of the limited liability option (only with an unsecured loan) are the main determinants such that $\dot{K}_F$ and $\dot{\pi}_F$ decrease with a decrease in $\sigma$. This is because $\dot{a}_F$ is not sensitive to $\sigma$ due to the financial-pooling and the capacity-pooling benefits. At high $\rho$ levels, the equilibrium financing cost starts dominating such that $\dot{K}_F$ and $\dot{\pi}_F$ increase with a decrease in $\sigma$. This is because $\dot{a}_F$ is very sensitive to $\sigma$ as the financial-pooling and capacity-pooling benefits are low, and the decrease in $\dot{a}_F$. 

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overcomes the decrease in the already low values of capacity pooling and the value of the limited liability option of the firm.

- In a monopolist credit market, we do not observe the insensitivity of \( \dot{a}_F \) to \( \sigma \). This is because the impact of \( \sigma \) on the marginal default probability is not negligible when the default probability is negligible. Since the impact of \( \sigma \) on the marginal default probability does not have a clear pattern, there is no clear pattern of dependency between \( \sigma \) and \( \rho \) either. When a decrease in \( \sigma \) increases \( \dot{a}_F \), all three effects, i) capacity pooling, ii) limited liability (only with an unsecured loan) and iii) the equilibrium financing cost, work in the same direction and \( \dot{K}_F \) and \( \dot{\pi}_F \) decrease. When a decrease in \( \sigma \) decreases \( \dot{a}_F \), in the majority of the cases analyzed the capacity pooling is the main determinant such that \( \dot{K}_F \) and \( \dot{\pi}_F \) decrease. However, there are instances in which the decrease in \( \dot{a}_F \) may outweigh the other two effects and \( \dot{K}_F \) and \( \dot{\pi}_F \) decrease with a decrease in \( \sigma \); this is observed at sufficiently high \( \rho \) levels.

In summary, the differences between the flexible and the dedicated technologies in Table 2 are due to the capacity-pooling feature of the flexible technology. Besides its value for a given financing cost \( a_F \), which is similar to the perfect capital market benchmark case, capacity pooling has a strategic value in imperfect capital markets as it is one of the main determinants of \( \dot{a}_F \). This strategic value has interesting implications for the equilibrium technology choice in imperfect capital markets as we illustrate in the next section.

5 Technology Choice

In §4.2, we investigated the impact of demand uncertainty (\( \sigma \) and \( \rho \)) for a given technology. In this section, we investigate the equilibrium technology choice in imperfect markets and how this choice is affected by \( \sigma \) or \( \rho \) compared to the technology choice in perfect capital markets. Since there is no fixed cost and the firm always invests in a positive level of capacity with each technology, investing in either technology dominates not making any technology investment. We first characterize the equilibrium technology choice in perfect capital markets.

Remark 2 If the capital markets are perfect, there exists a unique variable cost threshold \( \tau_D^P(c_F) \) such that when \( c_D \leq \tau_D^P(c_F) \) \( (c_D > \tau_D^P(c_F)) \), it is optimal to invest in dedicated
(flexible) technology. This threshold is given by

\[
\bar{c}_D(c_F) = c_F \left( \frac{1 - \gamma_F}{1 - \gamma_D} \right) \left[ \frac{2^{-\frac{1}{\tau}} \xi}{E \left[ (\xi_1^{-1} + \xi_2^{-1})^{-\frac{1}{\tau}} \right]} \right]^{\frac{1}{\tau+1}} \leq c_F,
\]

where the last inequality holds at equality only if i) the salvage values are symmetric (\(\gamma_F = \gamma_D\)) and ii) the demands are deterministic (\(\sigma = 0\)) or perfectly positively correlated (\(\rho = 1\)).

The threshold \(\bar{c}_D(c_F)\) is a variant of the flexibility premium of Chod et al. (2009). Since flexible capacity has a higher salvage value and has a capacity-pooling benefit, we have \(\bar{c}_D(c_F) \leq c_F\). As we discussed in §4.2, the term \(E \left[ (\xi_1^{-b} + \xi_2^{-b})^{-\frac{1}{\tau}} \right]\) captures the capacity-pooling benefit of the flexible technology. Under Assumption 2, this threshold decreases with an increase in demand variability (\(\sigma\)) and a decrease in demand correlation (\(\rho\)) due to the increasing capacity-pooling benefit. With symmetric salvage rates, Remark 2 shows that there is no capacity-pooling benefit (\(\bar{c}_D(c_F) = c_F\)) only if the demands are deterministic or are random but perfectly positively correlated.

We now investigate the equilibrium technology choice in imperfect capital markets. We restrict our attention to firms with a sufficiently large \(P\), i.e. \(P \geq P_T\), such that the firm uses a secured loan in equilibrium. This assumption is made for analytical convenience, as we cannot analytically characterize the equilibrium behavior with the flexible technology when the firm uses an unsecured loan. We also assume symmetric salvage rates (\(\gamma_F = \gamma_D\)).\(^7\) With this assumption, the equilibrium technology choice in imperfect capital markets is driven by two factors: the capacity-pooling value of the flexible technology and the relative magnitude of the equilibrium financing cost with each technology. For our numerical experiments, we use the following data set: \(b = -2\), \(\gamma_F = \gamma_D = 0.1\), \(P = 650\), \(BC = 200\), \(c_F \in \{0.5, 1, 1.5, 2, 2.5\}\), \(\sigma \in \{10\%, 15\%, 20\%, 25\%, 30\%\}\) of \(\bar{\xi}\) and \(\rho \in \{-0.9995, -0.75 - 0.5, -0.25, 0, 0.25, 0.5, 0.75, 0.9995\}\) and \(\bar{\xi} = 15, B = 15\) or \(\bar{\xi} = 20, B = 20\). Our main results are summarized in Table 3. Here, when the flexible (dedicated) technology is chosen at a larger set of technology cost pairs \((c_D, c_F)\) in the imperfect capital market compared to the perfect market case, we say “Flexible (Dedicated) technology is favored.” When the technology choice in imperfect capital market is identical to the perfect market case with all

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\(^7\)Additional interesting observations can be made that result from the asymmetry in the salvage value of capacity with each technology. For example, we can show that when the product markets are perfectly positively correlated (\(\rho = 1\)), and when the flexible technology is not strictly preferred in perfect capital markets, it can be strictly preferred in imperfect capital markets. This is because the firm can secure a lower financing cost with the flexible technology in equilibrium due to a higher salvage rate.
the technology cost pairs, we say “Neither technology is favored.” The supporting technical statements of this section, denoted by C.x, are provided in §C of the Technical Appendix.

<table>
<thead>
<tr>
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<th>Perfectly Competitive Credit Market</th>
<th>Monopolist Creditor</th>
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<tr>
<td>Low</td>
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<td>High</td>
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Table 3: Equilibrium technology choice in imperfect capital markets with respect to demand variability ($\sigma$) and demand correlation ($\rho$) for firms with a sufficiently large $P$ and bivariate normal demand where $\sigma \in [10\%, 30\%]$ of $\bar{\xi}$ and $\rho \in [-0.9995, 0.9995]$. $F$ ($D$) refers to “Flexible (Dedicated) technology is favored” compared to perfect market benchmark. “N” stands for “Neither technology is favored.”

To delineate the intuition behind Table 3, we will focus on a technology cost pair that is insightful to discuss, $(\bar{\tau}_D^I(c_F), c_F)$, where the firm is indifferent between the two technologies in perfect capital markets. There are two main reasons why we choose to analyze this particular technology cost pair. First, if there exists a unique cost threshold $\bar{\tau}_D^I(c_F)$ in imperfect capital markets, which is observed in all of our numerical experiments, then the equilibrium technology choice in imperfect capital markets at this technology cost pair tells us which technology is favored in imperfect capital markets: If flexible (dedicated) technology is chosen, this implies $\bar{\tau}_D^I(c_F) < \bar{\tau}_D^P(c_F)$ ($\bar{\tau}_D^I(c_F) > \bar{\tau}_D^P(c_F)$), and flexible (dedicated) technology is favored in imperfect capital markets. If the firm is indifferent, this implies $\bar{\tau}_D^I(c_F) = \bar{\tau}_D^P(c_F)$ and neither technology is favored in imperfect capital markets. Second, we can show that at this technology cost pair, flexible (dedicated) technology is favored if and only if $\hat{a}_F < \hat{a}_D$ ($\hat{a}_F > \hat{a}_D$), so we can focus on the ordering of $\hat{a}_F$ and $\hat{a}_D$ for our discussion. We next analyze the equilibrium technology choice at this cost pair, and the impact of the demand variability and the demand correlation on this choice.

At low $\sigma$ values, as discussed in §4.2, the firm uses a secured loan without default possibility with each technology in equilibrium. At this equilibrium, it can be shown that $\hat{a}_D = \hat{a}_F$ and the firm is indifferent between the two technologies at the technology cost pair $(\bar{\tau}_D^P(c_F), c_F)$. Therefore, neither technology is favored in imperfect capital markets. As demand variability increases, the firm uses a secured loan with default possibility with each
technology, and the equilibrium technology choice can be different from the perfect capital market benchmark case. This difference depends on the demand correlation as well as the capital market conditions as we analyze next.

In a perfectly competitive credit market, neither technology is favored at low demand correlation levels or moderate \((\sigma, \rho)\) levels, otherwise the flexible technology is favored. As discussed in \S 4.2, for a given \(\rho\), there exists a threshold with each technology, \(\bar{\sigma}_T(\rho)\), below which \(\hat{a}_T \approx 0\). In our numerical experiments, we observe \(\bar{\sigma}_F(\rho) > \bar{\sigma}_D(\rho)\). This is because the financial-pooling benefit of the dedicated technology is not sufficient to avoid default in equilibrium, whereas the financial- and the capacity-pooling benefits of the flexible technology are sufficient. At \((\sigma, \rho)\) levels such that \(\sigma \leq \bar{\sigma}_D(\rho)\), we have \(\hat{a}_F = \hat{a}_D \approx 0\) and neither technology is favored. In our numerical experiments, this is observed at low \(\rho\) levels and moderate \((\sigma, \rho)\) levels. At \((\sigma, \rho)\) levels such that \(\bar{\sigma}_D(\rho) < \sigma \leq \bar{\sigma}_F(\rho)\), we have \(\hat{a}_D > \hat{a}_F = 0\). Therefore, flexible technology is strictly preferred with the technology cost pair \((\bar{\sigma}_D(c_F), c_F)\) and flexible technology is favored. At \((\sigma, \rho)\) levels such that \(\sigma > \bar{\sigma}_F(\rho)\) we have \(\hat{a}_F\) and \(\hat{a}_D\) both positive. We show that the ordering of \(\hat{a}_D\) and \(\hat{a}_F\) is determined by the ordering of the default risk with identical financing costs, which in turn is determined by the trade-off between the capacity-pooling benefit of the flexible technology and the higher total capacity investment made under the dedicated technology. In our numerical experiments, we observe that the former effect dominates and flexible technology is strictly preferred at the technology cost pair \((\bar{\sigma}_D(c_F), c_F)\). Therefore, flexible technology is favored.

In a monopolistic credit market, the dedicated technology may be favored if the demand correlation and the demand variability are high, otherwise the flexible technology is favored. When \(\sigma\) is not low, at the technology cost pair \((\bar{\sigma}_D(c_F), c_F)\), the technology with the higher marginal default probability is chosen in equilibrium. We observe that unless \(\sigma\) and \(\rho\) are high, \(\hat{a}_F < \hat{a}_D\) and flexible technology is strictly preferred at this technology cost pair. Therefore, flexible technology is favored. At high \((\sigma, \rho)\) levels there are some instances with \(\hat{a}_F > \hat{a}_D\). When this happens, dedicated technology is strictly preferred at the technology cost pair \((\bar{\sigma}_D(c_F), c_F)\), and dedicated technology is favored.\(^8\)

In summary, the equilibrium technology choice in imperfect capital markets, and the impact of demand variability and demand correlation on this choice are determined through

\(^8\)Interestingly, at very high \(\rho\) levels, \(\rho = 0.9995\) in our setting, dedicated technology may be chosen in a monopolistic credit market equilibrium even with identical technology costs \((c_F = c_D)\). This is because the negative impact of the higher \(\hat{a}_F\) is sufficient to outweigh the already low value of capacity pooling with the flexible technology.
the interplay between the financial-pooling benefit, that exists with either technology, and the capacity-pooling benefit, that only exists with flexible technology.

1. At low $\sigma$ levels, the firm uses a secured loan without default possibility with each technology in equilibrium. The equilibrium technology choice in imperfect capital markets is identical to the perfect market benchmark case.

2. At low $\rho$ levels and moderate $(\sigma, \rho)$ levels, the firm uses a secured loan with default possibility with each technology. In a perfectly competitive credit market, the financial-pooling benefit is significantly high with either technology and the default probability is negligible in equilibrium. Therefore, the equilibrium technology choice is identical to the perfect market benchmark case. In a monopolistic credit market, the marginal default probability is not negligible, and is higher with the flexible technology. Therefore, flexible technology is favored.

3. For higher $\sigma$ and $\rho$ levels, the equilibrium technology choice deviates from the perfect market benchmark due to the impact of capacity pooling on the equilibrium financing cost: Unless $\sigma$ and $\rho$ are high, the capacity-pooling feature of the flexible technology induces the firm to secure a lower financing cost in equilibrium and flexible technology is favored. At high $(\sigma, \rho)$ levels, flexible technology is favored in a perfectly competitive credit market whereas either technology can be favored in a monopolistic credit market.

We close this section with a remark. In the finance literature, it is argued that operational flexibility decreases the firm’s default risk by generating higher returns due to its option value (see, for example, MacKay 2003). Our analysis shows that this argument is not obvious with a stronger formalization of the firm’s operations. Since the argument in MacKay (2003) implicitly assumes away the incremental cost of operational flexibility (flexible technology in our model), we will focus on the case $c_F = c_D$. Anticipating the option value of operational flexibility (capacity-pooling benefit of the flexible technology in our case), the firm optimally adjusts the other operational decisions (the capacity investment level). Therefore, the default risk in equilibrium changes. In a perfectly competitive market, when $\hat{a}_F > \hat{a}_D$, the default risk is lower with the dedicated technology in equilibrium (Lemma C.1). When $\hat{a}_F < \hat{a}_D$, the lower financing cost with the flexible technology works to decrease the default risk, and the default risk comparison is not analytically provable. In our numerical experiments, we observe $\hat{a}_F \leq \hat{a}_D$ in a perfectly competitive credit market and the default risk is lower with the flexible technology, paralleling the argument in the finance literature. With the monopolist creditor, we observe $\hat{a}_F \geq \hat{a}_D$ in the majority of the instances; but in all the cases analyzed, the default risk is lower with the flexible technology,
again paralleling the argument in the finance literature.

6 Conclusion

This paper contributes to the stochastic capacity investment literature by relaxing the (often implicit) perfect capital market assumption and analyzing the impact of endogenous credit terms under capital market imperfections. A joint operational and financial perspective is adopted to develop theory and insights into capacity management and technology choice in imperfect capital markets. In a single-product setting, we analyze the impact of the demand variability on the capacity investment decision and the performance of the firm in equilibrium. In a two-product setting, we analyze the impact of demand uncertainty (variability and correlation) on the same, as well as the choice between flexible and dedicated technology in equilibrium. Our single-product analysis contributes to the literature by providing new comparative statics results under different capital market conditions. Except for a numerical analysis in Lederer and Singhal (1994), there is no formal treatment of the two-product case in the literature, therefore, the two-product analysis is a distinct contribution of our research.

In a single-product setting, the impact of the demand variability in equilibrium is determined through the interplay between the value of the limited liability option of the firm (only with an unsecured loan) and the equilibrium financing cost. In a two-product setting, the same argument applies for the impact of demand variability and correlation with the dedicated technology. With the flexible technology, there is a third facet in this interplay, the capacity-pooling benefit. In a two-product firm, the impact of demand variability and correlation are interdependent as the default probability, and in turn, the equilibrium financing cost depend on both. This is due to financial pooling, i.e. the diversification benefit from operating in two markets, with dedicated technology and due to both financial and capacity pooling with flexible technology. The interplay between the financial-pooling and capacity-pooling benefits drives the equilibrium technology choice in imperfect capital markets.

Our results are summarized in Tables 1, 2 and 3 and show how comparative statics results in imperfect capital markets depend on the firm’s loan type in equilibrium (unsecured versus secured, with or without default possibility) and the different capital market conditions studied. Our summaries at the end of each section suggest some rules of thumb for the strategic management of the capacity investment portfolio and technology choice and provide the basis for potential empirical research in this domain. While a formal development of empirical hypotheses is beyond the scope of this paper, the following predictions
of our model would be interesting to explore empirically. From the single-product analysis, we have:

1. With a secured loan, the higher the demand variability, the lower will be the performance of the firm, regardless of the competitiveness of the credit market. The same holds true with an unsecured loan if the credit market is highly competitive.

2. With a secured loan, the higher the demand variability, the lower will be the capacity investment regardless of the competitiveness of the credit market.

From the two-product analysis, we can hypothesize the following:

1. The higher the demand variability or demand correlation, the lower will be the performance of firms using dedicated technology when the credit market is highly competitive.

2. The higher the demand variability, the lower (higher) will be the capacity investment and performance of firms using flexible technology when the demand correlation is high (low) and the loan is secured in a highly competitive credit market.

3. The higher the demand correlation, the lower will be the capacity investment of firms using flexible technology regardless of the competitiveness of the credit market.

4. The higher the demand correlation, the lower will be the performance of firms regardless of the firm’s technology choice if the credit market is highly competitive.

5. With a secured loan, unless product demands are highly negatively correlated, the higher the demand variability, the more prevalent flexible technology choice will be when the credit market is highly competitive.

6. With a secured loan, the higher the competitiveness of the credit market, the more prevalent flexible technology choice will be.

This paper brings constructs and assumptions motivated by the finance literature into a classical operations management problem and develops new insights. In turn, by a stronger formalization of operational decisions than in the finance literature (the sequential nature of technology choice, capacity investment and production decisions and the impact of demand uncertainty), we provide novel insights on issues discussed in this literature. For example, Melnik and Plaut (1986) derive several relations among the parameters of loan contracts based on the assumption that the borrowing level is independent of the unit financing cost and that the default probability increases in the unit financing cost. Our analysis demonstrates that these assumptions may not be valid with a more formal representation of operational decisions: The firm optimally adjusts its capacity investment level; thus the borrowing level may decrease and the default probability increase in the unit financing cost. As argued in MacKay (2003), firms with higher operational flexibility are
assumed to have a lower default risk due to the option value of operational flexibility. Our analysis shows that this argument acquires new dimensions with a stronger formalization of the firm’s operations, but still holds true: Anticipating the option value of operational flexibility (capacity-pooling benefit of the flexible technology), the firm optimally adjusts the other operational decisions (capacity investment level), and the default risk in equilibrium changes. Nevertheless, the equilibrium default risk continues to be lower with the flexible technology due to its capacity-pooling benefit.

There are a number of limitations to the present study that lead to open research questions. First, we focus on a particular type of financing contract and two types of capital market imperfections. The firm can also issue equity or raise external capital by other forms of loan contracts, or may be exposed to other capital market imperfections such as taxes, agency costs etc. As the different capital market imperfections examined here show, the operational implications are expected to be model-specific.

Among these other market imperfections, agency costs arising from asymmetric information between the creditor and the firm are worth discussing. In our model, we assume that the creditor has perfect information about the firm. In reality, the creditor may not have perfect information about the risk profile of the operational investments, nor be able to monitor the firm after the loan is taken, nor have the same valuation of the collateralized assets as the firm. Each of these would create agency costs and impose additional financing frictions. Our analysis provides partial answers for this case. For example, if there is asymmetric information, and if there is no signalling by the firm or screening by the creditor, the creditor would offer identical financing costs for each technology. In this case, as we discussed in §5, the technology choice in imperfect capital markets is identical to the technology choice in perfect capital markets. With a proper modeling of the interaction between the creditor and the firm under asymmetric information, new trade-offs and new implications will arise as discussed in, for example, Brunet and Babich (2009).

Relaxing the assumptions we made on the production environment gives rise to a number of interesting possibilities, both in the theory of capacity management and integrated risk management. For example, we assume that second stage production is costless. With a positive production cost, the optimal production decision is limited by the cash availability of the firm (financial capacity constraint) in addition to the physical capacity constraint. This brings an additional facet to the problem: The allocation of the financial capacity between the two stages; and between the products in the second stage in a two-product setting. We also assume that the internal budget of the firm is deterministic. This bud-
get may depend on some economic factors and can be random. Moreover, if the internal budget depends on a tradable asset, then the firm can engage in financial risk management to engineer the budget as discussed in Froot et al (1993). The optimal technology choice (flexible versus dedicated) together with the decision of engaging in financial risk management form the optimal integrated risk management portfolio of the firm. Boyabatlı et al. (2010) analyze the effect of budget variability and financial risk management on the stochastic capacity investment problem with a more detailed model of the firm’s production environment (that includes positive production cost and engaging in financial risk management) with hard financial constraints (no borrowing). It would be interesting to analyze the impact of endogenous credit terms on the integrated risk management portfolio of the firm (that consists of financial risk management and flexible versus dedicated technology choice).

We assume a stylized firm that is liquidated at the end of a single period. Li et al. (2003) model the dynamic capital structure choice of the firm without bankruptcy. Extending our analysis to a multi-period setting is certainly a non-trivial task and requires further research. Besides the usual operational dynamics of multi-period models (capacity and inventory carryover), there also exist additional dynamics coming from capital carryover: The firm may decide to hold some of the earlier loans as internal capital for future expenditures. In addition, the indirect costs of bankruptcy should be incorporated because the financing cost in equilibrium depends on the earlier loan performance of the firm.

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Technical Appendix

§A contains the proofs for the equilibrium strategy in the single-product firm. §B provides the proofs for the impact of demand uncertainty in the single-product firm. §C includes the analysis of the equilibrium technology choice. We present the proofs for selected technical statements that we develop in this Appendix in §D. We use the following identities for the standard normal random variable with cdf $\Phi(.)$ and pdf $\phi(.)$ throughout the Appendix: $\phi'(z) = -z\phi(z)$, $\int_{-\infty}^{v} z\phi(z)dz = -\phi(v)$ and $1 > \left[ \frac{\phi(v)}{1-\Phi(v)} \right]^2 - \frac{v\phi(v)}{1-\Phi(v)} > 0$, where the last two inequalities are proven in Sampford (1953).

A Equilibrium Strategy in The Single-Product Firm

A.1 Analysis of The Firm’s Problem

Proof of Proposition 1: If the firm does not have the limited liability option, then $\pi(K)$ is strictly concave in $K$ and the unique optimal capacity investment level $K^*$ is given by

$$K^* = \begin{cases} 
K^0 = \left( \frac{\xi(1+\frac{1}{B})}{(1-\gamma)c} \right)^{-b} & \text{if } B \geq cK^0 \\
\frac{B}{c} & \text{if } cK^1 \leq B < cK^0 \\
K^1 = \left( \frac{\xi(1+\frac{1}{B})}{(1+a-\gamma)c} \right)^{-b} & \text{if } B < cK^1 
\end{cases}$$

(5)
With limited liability, when the firm borrows \((K > \frac{B}{c})\), we have \(l(K) = K^{\frac{1}{\gamma}} (1 + a - \gamma) c - K^{(-1 - \frac{1}{\gamma})} [B (1 + a) + P]\) such that the firm is able to pay back the face value of the loan if and only if \(\tilde{\xi}\) is no less than \(l(K)\). For \(\tilde{\xi} > l(K)\), the optimal equity value \(\Pi^* > 0\), and for \(\tilde{\xi} \leq l(K)\), \(\Pi^* = 0\). We obtain

\[
\frac{\partial l(K)}{\partial K} = -\frac{1}{b} K^{(\frac{1}{\gamma} - 1)} (1 + a - \gamma) c + \left(1 + \frac{1}{b}\right) K^{(-2 - \frac{1}{\gamma})} [B (1 + a) + P] > 0.
\]

Therefore, we can identify the unique \(K^l < K^u\) such that \(l(K^l) = \xi^l\) and \(l(K^u) = \xi^u\). Since \(l(K)\) is strictly increasing in \(K\), we have \(l(K) \geq \xi^u\) for \(K \geq K^u\); hence \(\Pi^* = 0\) at each \(\tilde{\xi}\) and \(\pi^* = 0\) for \(K \in [K^u, \infty)\). Therefore, it is sufficient to analyze the problem for \(K \in [0, K^u]\).

We have three separate cases to consider:

**Case 1:** For \(K \in \left[0, \frac{B}{c}\right]\), similar to the no limited liability case, the firm does not borrow, and the expected equity value of the firm is \(\pi^* = \max_K \xi K (1 + \frac{1}{\gamma}) + B + P - c (1 - \gamma) K\).

**Case 2:** For \(K \in \left[\frac{B}{c}, K^l\right]\), similar to the no limited liability case, the firm optimally borrows, and is always able to pay back the face value of the loan.\(^1\) The expected equity value of the firm is \(\pi^* = \max_K \xi K (1 + \frac{1}{\gamma}) + B (1 + a) + P - c (1 + a - \gamma) K\).

**Case 3:** For \(K \in (K^l, K^u)\) the firm always borrows, and for some demand realization, is not able to pay back the face value of the loan; hence the expected equity value of the firm is \(\pi^* = \max_K \int_{l(K)}^{\xi^u} \left[\xi K (1 + \frac{1}{\gamma}) + B (1 + a) + P - c (1 + a - \gamma) K\right] f(\xi) d\xi\).

Let \(g(K)\) denote the objective function in the overall optimization problem and \(g^i(K)\) denote the objective function in case \(i\). It is easy to establish that \(g(K)\) is continuous at the boundaries \(K = \frac{B}{c}\) and \(K = K^l\); and hence \(g(K)\) is continuous in \(K\). It follows from (5) that \(g(K)\) is strictly concave in \(K\) for \(K \in \left[0, K^l\right]\) and has a kink at \(K = \frac{B}{c}\). We obtain

\[
\frac{\partial g^3(K)}{\partial K} = \int_{l(K)}^{\xi^u} \left[\left(1 + \frac{1}{b}\right) \xi K (\frac{1}{\gamma}) - (1 + a - \gamma) c\right] f(\xi) d\xi.
\]

It is easy to verify that \(\frac{\partial g^3(K)}{\partial K}_{|K=0} = \frac{\partial g^3(K)}{\partial K}_{|K=\infty}\); hence \(g(K)\) does not have a kink at \(K = K^l\). Define \(G(K, \xi) = (1 + \frac{1}{\gamma}) \xi K (\frac{1}{\gamma}) - (1 + a - \gamma) c\) as the integrand of (7) (without the density function). Note that \(G(K, \xi)\) is increasing in \(\xi\), and decreasing in \(K\). The second order condition of \(g^3(K)\) is given by

\[
\frac{\partial^2 g^3(K)}{\partial K^2} = \int_{l(K)}^{\xi^u} \left[\frac{1}{b} \left(1 + \frac{1}{b}\right) \xi K^{(\frac{1}{\gamma} - 1)}\right] f(\xi) d\xi - \frac{\partial l(K)}{\partial K} G(K, l(K)) f(l(K)).
\]

Note that the first term is negative and we obtain \(G(K, l(K)) = -\frac{B (1 + a) + P}{K} < 0\); hence the second term is positive. Therefore, the concavity of \(g^3(K)\) is not obvious. We define

\(^1\)It can be shown that for \(\xi^l \geq 0\) and \(\gamma \geq 0\), \(K^l \geq \frac{B}{c}\), where the equality only holds if \(\xi^l = 0\) and \(\gamma = 0\).
\( \hat{K} = \left( \xi u(1 + \frac{1}{2}) \right)^{-b} \). We have \( l(\hat{K}) = (1 + \frac{1}{2}) \xi^u \left[ 1 - \frac{b(1+a) + b}{K(1+a-\gamma)c} \right] < \xi^u \), thus \( \hat{K} < K^u \) and is in the feasible region of \( K \). Note that \( G(\hat{K}, \xi^u) = 0 \) and \( G(\hat{K}, \xi) < 0 \) for \( \xi \in \left[ l(\hat{K}), \xi^u \right] \) (as \( G(K, \xi) \) is strictly increasing in \( \xi \)). Therefore \( \frac{\partial g^3(K)}{\partial K} \bigg|_{\hat{K}} < 0 \). Since \( G(K, \xi) \) is strictly decreasing in \( K \), \( \frac{\partial g^3(K)}{\partial K} \bigg|_{\hat{K}} < 0 \) for \( K \in \left[ \hat{K}, K^u \right] \).

In summary, \( g(K) \) is strictly concave in \( K \) for \( K \in [0, K^l] \) (with a kink at \( K = \frac{B}{c} \)), and is strictly decreasing in \( K \) for \( K \in \left[ \hat{K}, K^u \right] \). It follows that \( g(K) \) will be unimodal if \( K^l > \hat{K} \). Since \( \frac{\partial g(K)}{\partial K} > 0 \) (from (6)), this is equivalent to \( l(\hat{K}) \leq \xi^i \), which gives us \( B > B^h \).

In this case, \( K^* \) is in the strictly concave part and is unique. \( K^* \) is identical to (5).

**Proof of Proposition 2:** In the proof of Proposition 1, we already established that the stage-1 objective function \( g(K) \) is strictly concave in \( K \) for \( K \in [0, K^l] \) and strictly decreasing in \( K \) for \( K \in \left[ \hat{K}, K^u \right] \). We obtain 
\[
\frac{\partial g(K)}{\partial K} \bigg|_{K^l} = \frac{(1 + \frac{1}{2}) \xi^u}{(K^l)^{-1/2}} \left[ 1 - (K^l)^{-1/2} \right]
\]
where \( K^l = \left( \frac{\xi u(1 + \frac{1}{2})}{(1+a-\gamma)c} \right)^{-b} \). It follows that \( \frac{\partial g(K)}{\partial K} \bigg|_{K^l} > 0 \) if and only if \( K^l < K^1 \). In this case \( g(K) \) is strictly increasing for \( K \in [0, K^l] \) and strictly decreasing in \( K \) for \( K \in \left[ \hat{K}, K^u \right] \). Since \( g(K) \) is continuous in \( K \), there exists at least one \( K^* \in (K^l, K^u) \) such that \( \frac{\partial g(K)}{\partial K} \bigg|_{K^*} = 0 \).

Since \( \frac{\partial g(K)}{\partial K} > 0 \) (from (6)), \( K^l < K^1 \) is equivalent to \( l(K^1) > \xi^i \), which gives us \( B > B^l \).

To prove that \( K^* \in (K^1, K^u) \), it is sufficient to show that \( \frac{\partial g(K)}{\partial K} > 0 \) for \( K \in (K^l, K^1] \). For \( K > K^l \), as follows from (7), we have 
\[
\frac{\partial g(K)}{\partial K} = (1 + \frac{1}{2}) \frac{K^l}{b} \int_{l(K)}^{\xi^u} \left[ \xi - \frac{\xi}{(K^K)^{-1/2}} \right] f(\xi) d\xi.
\]
Let \( H(K) \equiv \int_{l(K)}^{\xi^u} \left[ \xi - \frac{\xi}{(K^K)^{-1/2}} \right] f(\xi) d\xi \). Note that, for \( K > K^l \), \( H(K) \) and \( \frac{\partial g(K)}{\partial K} \) have the same sign, so we can use \( H(K) \) to characterize the sign of \( \frac{\partial g(K)}{\partial K} \). Define \( M(K, \xi) = \xi - \frac{\xi}{(K^K)^{-1/2}} \) as the integrand in \( H(K) \) (without the density function). We obtain 
\[
M(K, l(K)) = K^l \left[ (1+a-\gamma)c - \frac{b(1+a) + b}{K} \right] < 0
\]
since \( b < -1 \). As \( M(K, \xi) \) is strictly increasing in \( \xi \), \( M(K, \xi) < 0 \) for \( \xi \in [\xi^l, l(K)] \). Therefore, we have 
\[
H(K) > \int_{\xi^u}^{\xi^l} \left[ \xi - \frac{\xi}{(K^K)^{-1/2}} \right] f(\xi) d\xi = \xi \left[ 1 - (K^K)^{-1/2} \right].
\]
For \( K \leq K^1 \), we have \( \xi \left[ 1 - (K^K)^{-1/2} \right] \geq 0 \); and hence \( H(K) > 0 \) for \( K \in (K^l, K^1] \).

**Proof of Proposition 3:** Throughout the proof we will focus on the cases that we demonstrated in the proof of Proposition 1 with the same notation. In the proof of Proposition 1, we already established that the stage-1 objective function \( g(K) \) is strictly concave in \( K \) for \( K \in [0, K^l] \) and strictly decreasing in \( K \) for \( K \in \left[ \hat{K}, K^u \right] \). Also, as discussed in the proof of Proposition 2; for \( K \geq K^l \), we have \( sgn \left( \frac{\partial g^3(K)}{\partial K} \right) = sgn(H(K)) \) where 
\[
H(K) = \int_{l(K)}^{\xi^u} \left[ \xi - \frac{\xi}{(K^K)^{-1/2}} \right] f(\xi) d\xi.
\]
Therefore we will focus on \( H(K) \) to prove the uni-
modality of \( g(K) \). From integration by parts, we obtain
\[
H(K) = \int_{l(K)}^{\xi_u} \bar{F}(\xi) d\xi - \bar{F}(l(K)) \left[ K^{-\frac{1}{b}} \left( \frac{(1 + a - \gamma) c}{-(b + 1)} + \frac{B(1 + a) + P}{K} \right) \right] .
\]

Define \( \Delta(K) \doteq K^{-\frac{1}{b}} \left( \frac{(1 + a - \gamma) c}{-(b + 1)} + \frac{B(1 + a) + P}{K} \right) \). We obtain
\[
\frac{\partial \Delta(K)}{\partial K} = \left( 1 + \frac{1}{b} \right) K^{-1} \left( l(K) + \frac{-b(b + 2)}{(b + 1)^2} (1 + a - \gamma) c \right) .
\]

Note that for \( K > K^l, l(K) > \xi^l \geq 0 \); hence for \( b \geq -2 \) the second term is positive and \( \frac{\partial \Delta(K)}{\partial K} > 0 \) for \( K > K^l \). We obtain \( H(K) = \bar{F}(l(K)) \left[ \frac{\int_{l(K)}^{\xi_u} \bar{F}(\xi) d\xi}{\bar{F}(l(K))} - \Delta(K) \right] \). As \( \Delta(K) \) is increasing in \( K \), and since \( \frac{\int_{l(K)}^{\xi_u} \bar{F}(\xi) d\xi}{\bar{F}(l(K))} \) is decreasing in \( K \) (which follows from Assumption 1), then for \( K > K^l \), \( H(K) \) can only change sign once, which is from positive to negative.

In summary, \( g(K) \) is i) strictly concave in \( K \) for \( K \in [0, K^l] \) with a kink at \( K = \frac{B}{c} \), ii) unimodal in \( K \) for \( K \geq K^l \), and is not kinked at \( K = K^l \), iii) strictly decreasing in \( K \) for \( K \in [K^l, K^u] \). It follows that

1. If \( \frac{\partial^2 g(K)}{\partial K^2} \bigg|_{K^l} \leq 0 \), then \( \frac{\partial g(K)}{\partial K} = \frac{\partial^2 g(K)}{\partial K^2} < 0 \) for \( K \geq K^l \), and \( g(K) \) is unimodal. \( K^* \) is unique and is characterized by the strictly concave part (similar to Proposition 1).

2. If \( \frac{\partial^2 g(K)}{\partial K^2} \bigg|_{K^l} > 0 \), then for \( K > K^l \), \( \frac{\partial g(K)}{\partial K} = \frac{\partial^2 g(K)}{\partial K^2} \) is first positive and then changes sign once and then is negative, therefore \( g(K) \) is unimodal in \( K \). \( K^* \) is unique and is characterized by \( \frac{\partial^2 g(K^*)}{\partial K^2} = 0 \) (or \( MP(K^*) = 0 \), as defined in \( MP(K) = \int_{l(K)}^{\xi_u} \frac{(1 + \frac{1}{b}) K}{K} - (1 + a - \gamma) c f(\xi) d\xi \)). Let \( \bar{K} \) denote the optimal solution in this case.

From Proposition 2, we have \( K \geq K^l \).

As it follows from the proof of Proposition 2, \( \frac{\partial^2 g(K)}{\partial K^2} \bigg|_{K^l} > 0 \) is equivalent to \( B < B^l \), and the optimal solution \( K^* \) is as given in the Proposition. The optimal expected equity value of the firm with a given internal budget level \( B \), \( \pi^*(B) \), follows directly:

\[
\left\{ \begin{array}{ll}
\frac{cK^0(1-\gamma)}{-(b+1)} + B + P & \text{if } B \geq cK^0 \\
\left( \frac{B}{c} \right)^{1+b} + \gamma B + P & \text{if } cK^1 \leq B < cK^0 \\
\left( \frac{cK^1(1+a-\gamma)}{-(b+1)} + B(1+a) + P \right) & \text{if } cK^1 \left[ 1 - \frac{\xi^l}{\xi^{1+b}} \right] \left[ 1 - \frac{\gamma}{1+a} \right] - \frac{P}{1+a} \leq B < cK^1 \\
\mathbb{E} \left[ \xi K^{\left( 1 + \frac{1}{b} \right)} - (1 + a - \gamma) c\bar{K} + B(1+a) + P \right]^{+} & \text{if } 0 \leq B < cK^1 \left[ 1 - \frac{\xi^l}{\xi^{1+b}} \right] \left[ 1 - \frac{\gamma}{1+a} \right] - \frac{P}{1+a}
\end{array} \right.
\]

**Lemma A.1** \( \pi^* \) decreases in \( a \).

**Lemma A.2** If \( \xi \) is normally distributed, then \( \frac{\int_{l(K)}^{\xi_u} \bar{F}(\xi) d\xi}{\bar{F}(l(K))} \) decreases in \( K \).
A.2 Characterization of the Creditor’s Expected Return

Proof of Proposition 4: We define \( S^1(a) = cK^0 (1 - \gamma)^{-b} \left[ 1 - \frac{\xi^l}{\xi (1 + \frac{b}{\xi})} (1 + a - \gamma)^{(b+1)} (1 + a) \right] \) such that for a given \( a \), for \( B \geq S^1(a) \), the firm uses a secured loan (and invests in \( K^*(a) = K^1(a) \)) without default possibility. \( B \geq S^1(a) \) is equivalent to \( b(K^1) \leq \xi^l \). Hence, both the default cost and the expected loss due to the unsecured part of the loan are 0 in (4). We define \( S^2(a) = S^1(a) - \frac{P}{(1 + a)} \) such that for \( S^1(a) - B \geq S^2(a) \), the firm uses a secured loan (and invests in \( K^*(a) = K^1(a) \)) with default possibility. \( B \geq S^2(a) \) is equivalent to \( l(K^1) \leq \xi^l \). Hence, the default cost is strictly positive but the expected loss due to the unsecured part of the loan is 0 in (4). For \( B < S^2(a) \), the firm optimally borrows to invest in \( K^*(a) = K(a) \). In this case, the firm uses an unsecured loan and both the default cost and the expected loss due to the unsecured part of the loan are strictly positive in (4).

In summary, for any given \( a \), the ordering of \( B \) and thresholds \( S^1(a) \) and \( S^2(a) \) determine the optimal borrowing level of the firm, and hence the form of \( \Lambda(a) \). We obtain \( \frac{\partial S^1(a)}{\partial a} < 0 \) for \( a \in [0, a^{max}] \) thus, we can analyze the problem in two cases.

Case 1: \( B \geq S^1(0) = cK^0(1 - \gamma) \left[ 1 - \frac{\xi^l}{\xi (1 + \frac{1}{\xi})} \right] \). As \( S^1(a) \) is strictly decreasing, we have \( B \geq S^1(a) \) (and hence \( B > S^2(a) \) \( \forall a \in [0, a^{max}] \)). Therefore, we have \( \Lambda(a) = (cK^1(a) - B) a \) for \( 0 \leq a < a^{max} \).

Case 2: \( B < S^1(0) = cK^0(1 - \gamma) \left[ 1 - \frac{\xi^l}{\xi (1 + \frac{1}{\xi})} \right] \). In this case, the ordering of \( B \) and \( S^2(a) \) is important in characterizing \( \Lambda(a) \). We have \( S^2(0) = cK^0(1 - \gamma) \left[ 1 - \frac{\xi^l}{\xi (1 + \frac{1}{\xi})} \right] - P \) and we obtain
\[
\frac{\partial S^2(a)}{\partial a} = \frac{1}{(1 + a)^2} \left[ P - cK^0(1 - \gamma)^{-b} \left( 1 - \frac{\xi^l}{\xi (1 + \frac{1}{\xi})} \right) (1 + a - \gamma)^{b+1} \right] (1 + a - \gamma)^b [-b(1 + a) - \gamma]
\]
Notice that \( S^2(0) \) is positive (negative) if \( P \) is less (greater) than \( cK^0(1 - \gamma) \left[ 1 - \frac{\xi^l}{\xi (1 + \frac{1}{\xi})} \right] \).

Since \( (1 + a - \gamma)^b [-b(1 + a) - \gamma] \) is strictly decreasing in \( a \), for \( P \geq cK^0(-b-\gamma) \left[ 1 - \frac{\xi^l}{\xi (1 + \frac{1}{\xi})} \right] \), we have \( \frac{\partial S^2(a)}{\partial a} \geq 0 \) for \( a \geq 0 \). For \( P < cK^0(1 - b - \gamma) \left[ 1 - \frac{\xi^l}{\xi (1 + \frac{1}{\xi})} \right] \), there exists a unique \( a \) such that \( \frac{\partial S^2(a)}{\partial a} \leq 0 \) for \( a \leq a \) and \( \frac{\partial S^2(a)}{\partial a} > 0 \) for \( a > a \). Since the signs of \( S^2(0) \) and \( \frac{\partial S^2(a)}{\partial a} \) depend on \( P \), we have three subcases. Before analyzing them, we first present a Lemma that we will use throughout the rest of the proof.

Lemma A.3 We have \( B \geq S^1(a^{max}) > S^2(a^{max}) \), \( \forall B \geq 0 \).

Subcase 2.1: \( P \geq cK^0 \left[ 1 - \frac{\xi^l}{\xi (1 + \frac{1}{\xi})} \right] (-b - \gamma) \).

In this case, we have \( S^2(0) < 0 \) and \( \frac{\partial S^2(a)}{\partial a} \geq 0, \forall a \). For \( a^{max} = \left[ \left( \frac{cK^0}{P} \right)^{-\frac{1}{b}} - 1 \right] (1 - \gamma) \), we
obtain $S^2(a^{max}) < 0$. Hence, for $a \in [0, a^{max})$, we have $S^2(a) < 0 < B$. It follows that the firm always uses a secured loan (and invests in $K^1(a)$). For $B < S^1(0)$ (which follows from the definition of Case 2), since $S^1(a)$ is strictly decreasing in $a$ and $B \geq S^1(a^{max})$ (from Lemma A.3), it follows that there exists a unique $a^d$, as defined by $S^1(a^d) = B$ (where the superscript $d$ refers to “default”). We have $B < S^1(a)$ for $a < a^d$, and the firm uses a secured loan with default possibility, and $B \geq S^1(a)$ for $a \geq a^d$, the firm uses a secured loan without default possibility. Therefore $\Lambda(a)$ is characterized by

$$\Lambda(a) = \begin{cases} cK^1(a) - Ba - F(b(K^1(a)))BC & \text{if } 0 \leq a < a^d \\ cK^1(a) - B & \text{if } a^d \leq a < a^{max}. \end{cases}$$

**Subcase 2.2:** $cK^0 \left[ 1 - \frac{\xi}{\xi(1+\xi)} \right] (1 - \gamma) \leq P < cK^0 \left[ 1 - \frac{\xi}{\xi(1+\xi)} \right] (-b - \gamma)$.

We have $S^2(0) \leq 0$, and $S^2(a)$ is first strictly decreasing, and then strictly increasing in $a$. We obtain $S^2(a^{max}) < 0$; hence $S^2(a) < 0$ for $a \in [0, a^{max})$ in this case. Therefore $\Lambda(a)$ is identical to subcase 2.1.

**Subcase 2.3:** $cK^0 \left[ 1 - \frac{\xi}{\xi(1+\xi)} \right] (1 - \gamma) > P$

We have $S^2(0) > 0$, and $S^2(a)$ is first strictly decreasing, and then strictly increasing in $a$. If $B \geq S^2(0)$ (and $B < S^1(0)$ by definition of Case 2), since $B \geq S^1(a^{max}) > S^2(a^{max})$ (from Lemma A.3), $\Lambda(a)$ is characterized in a similar fashion to the other two subcases. If $B < S^2(0)$, as $S^2(a)$ is first strictly decreasing, and then strictly increasing in $a$ and $B \geq S^1(a^{max}) > S^2(a^{max})$ (from Lemma A.3), there exists a unique $a^l \in [0, a^{max})$, as defined in $S^2(a^l) = B$ (where the superscript $l$ refers to “limited liability”). We have $B < S^2(a)$ for $a < a^l$ and $B \geq S^2(a)$ for $a \geq a^l$. Since $S^2(a) = S^1(a) - \frac{P}{1+a}$, it follows that $a^l \leq a^d$, with equality only holding for $P = 0$. Therefore, we have the following three regions: For $a < a^l(< a^d)$, we have $B < S^2(a)$ (and $B < S^1(a)$), the firm uses an unsecured loan; for $a^l \leq a < a^d$, we have $S^2(a) \leq B < S^1(a)$, and the firm uses a secured loan with default possibility; and for $a \geq a^d$, we have $S^2(a) < S^1(a) \leq B$, and the firm uses a secured loan without default possibility. 

**A.3 Equilibrium Characterization**

**Proof of Proposition 5:**

**The Perfectly Competitive Credit Market.** Note that $\Lambda(a) > 0$ for $a^d \leq a < a^{max}$, $\Lambda(0) < 0$ when $a^d > 0$ and $\Lambda(a)$ is continuous in $a$. Therefore, there exists at least one $a$ such that $\Lambda(a) = 0$ and since $\pi^*$ can be proven to decrease in $a$ (Lemma A.1), the equilibrium is the minimum of such $a$’s. If $a^d = 0$ then $\dot{a} = 0$, otherwise $\dot{a} \in (0, a^d)$.

**The Monopolist Creditor.** Since $\Lambda(a) > 0$ for $a \in [a^d, a^{max})$, a loan contract is always
offered in equilibrium. Since \( \Lambda(0) \leq 0 \), we have \( \dot{a} \in (0, a^{\text{max}}) \).

We first analyze the firms that never default (case i of Proposition 4). The creditor’s problem is given by \( N^* = \max_{a \in [0, a^{\text{max}}]} \Lambda(a) = (cK^0(1 - \gamma)^{-b}(1 + a - \gamma)b - B) \) a where “N” stands for the net gain from secured lending. Let \( \dot{a}^M \) denote the optimal solution. We obtain \( \partial \Lambda(a) \partial a_a = cK^0(1 - \gamma)^{-b} \left[ J(a) - \frac{B}{cK^0(1 - \gamma)^{-b}} \right] \) where \( J(a) = (1 + a - \gamma)b^{-1}(1 + a - \gamma + ab) \). Note that \( \frac{\partial \Lambda(a)}{\partial a} \) has the same sign as \( \frac{\partial J(a)}{\partial a} \). We now analyze \( J(a) \) to characterize the solution for \( \Lambda(a) \). We obtain \( J(0) = (1 - \gamma)b > \frac{B}{cK^0(1 - \gamma)^{-b}} \)

and \( J(a^{\text{max}}) = (1 - \gamma)b \frac{B}{cK^0} \left[ 1 + \frac{b_{\text{max}}}{cK^0(1 - \gamma)^{-b}} \right] < \frac{B}{cK^0(1 - \gamma)^{-b}} \). Therefore, \( \frac{\partial \Lambda(a)}{\partial a} \mid _{a^{\text{max}}} > 0 \) and \( \frac{\partial J(a)}{\partial a} \mid _{a_{\text{max}}} < 0 \). We obtain \( \frac{\partial J(a)}{\partial a} = -b(1 + a - \gamma)(b - 2)[-a(b + 1) - 2(1 - \gamma)] \). \( \frac{\partial J(a)}{\partial a} \) has the same sign as \( [-a(b + 1) - 2(1 - \gamma)] \). In other words, for \( a > \frac{2(1 - \gamma)}{b + 1} \), we have \( \frac{\partial J(a)}{\partial a} > 0 \) and \( \Lambda(a) \) is strictly convex. For \( a < \frac{2(1 - \gamma)}{b - 1} \), \( \frac{\partial J(a)}{\partial a} < 0 \) and \( \Lambda(a) \) is strictly concave. As the feasible region is \( a \in [0, a^{\text{max}}] \), we have two cases to consider.

**Case 1:** \( a^{\text{max}} > \frac{2(1 - \gamma)}{b + 1} \), which is equivalent to \( B < \frac{cK^0}{1 + \frac{2}{b+1}} \).

We have that \( J(a) \) is decreasing for \( 0 < a < \frac{2(1 - \gamma)}{b + 1} \) and increasing for \( \frac{2(1 - \gamma)}{b + 1} \leq a < a^{\text{max}} \).

We also have \( J(0) > \frac{B}{cK^0(1 - \gamma)^{-b}} \) and \( J(a^{\text{max}}) < \frac{B}{cK^0(1 - \gamma)^{-b}} \). As a result, there exist a unique \( a^N < \frac{2(1 - \gamma)}{b + 1} \), as defined by \( J(a^N) \mid _{a^N} = \frac{B}{cK^0(1 - \gamma)^{-b}} \) such that for \( 0 \leq a < a^N \), \( J(a) > \frac{B}{cK^0(1 - \gamma)^{-b}} \) (and hence \( \frac{\partial \Lambda(a)}{\partial a} > 0 \)) and for \( a^N \leq a < a^{\text{max}} \), \( J(a) < \frac{B}{cK^0(1 - \gamma)^{-b}} \) (and hence \( \frac{\partial \Lambda(a)}{\partial a} < 0 \)).

In summary, \( \Lambda(a) \) is characterized as follows: For \( 0 \leq a < a^N \left( < \frac{2(1 - \gamma)}{b + 1} \right) \), \( \Lambda(a) \) is concave increasing; for \( a^N \leq a < a^{\text{max}} \), \( \Lambda(a) \) is concave decreasing; and for \( \frac{2(1 - \gamma)}{b + 1} < a < a^{\text{max}} \), \( \Lambda(a) \) is convex decreasing. Hence, \( \dot{a}^M \) is unique and is given by \( \dot{a}^M = a^N \).

**Case 2:** \( a^{\text{max}} \leq \frac{2(1 - \gamma)}{b - 1} \), which is equivalent to \( B \geq \frac{cK^0}{1 + \frac{2}{b-1}} \).

We have that \( J(a) \) is decreasing for \( 0 < a < a^{\text{max}} \). We have \( J(0) > \frac{B}{cK^0(1 - \gamma)^{-b}} \) and \( J(a^{\text{max}}) < \frac{B}{cK^0(1 - \gamma)^{-b}} \); therefore, similar to Case 1, there also exists a unique \( a^N < a^{\text{max}} \), as defined by \( J(a^N) \mid _{a^N} = \frac{B}{cK^0(1 - \gamma)^{-b}} \) such that for \( 0 \leq a < a^N \), \( J(a) > \frac{B}{cK^0(1 - \gamma)^{-b}} \) (and hence \( \frac{\partial \Lambda(a)}{\partial a} > 0 \)) and for \( a^N \leq a < a^{\text{max}} \), \( J(a) < \frac{B}{cK^0(1 - \gamma)^{-b}} \) (and hence \( \frac{\partial \Lambda(a)}{\partial a} < 0 \)). In summary, \( \Lambda(a) \) is characterized as follows: For \( 0 \leq a < a^N \left( < \frac{2(1 - \gamma)}{b - 1} \right) \), \( \Lambda(a) \) is concave increasing; and for \( a^N \leq a < a^{\text{max}} \), \( \Lambda(a) \) is concave decreasing. Hence, unique \( \dot{a}^M \) is given by \( \dot{a}^M = a^N \).

We now analyze the firms that may default but use a secured loan (Case ii of Proposition 4). We obtain \( \frac{\partial F(b(K^1(a)))}{\partial a} = -f(b(K^1(a)))\xi(1 + \frac{1}{b})\frac{B}{cK^0(1 - \gamma)^{-b}} \left[ 1 + \frac{a + 1)(a + 1)}{(1 + a - \gamma)^{-b}(b - 1)} \right] < 0 \) where \( f(.) \) denotes the pdf of \( \xi \). Thus, for \( 0 \leq a < a^d \), the default risk is strictly decreasing in \( a \). On the other hand, the creditor’s net gain from secured lending is strictly increasing in \( a \) for \( 0 \leq a < a^N \) and strictly decreasing in \( a \) for \( a > a^N \). For \( a^N \geq a^d \), the creditor’s expected
return is increasing in \( a \) for \( a < a^N \), thus we have \( \hat{a}^M = a^N \). For \( a^N < a^d \), the creditor’s expected return is increasing in \( a \) for \( a < a^N \) and decreasing in \( a \) for \( a > a^d \). Therefore, there exists at least one maximizer in \((a^N, a^d)\) and is a solution to \( cK^0(1-\gamma)^{-b}1 + a - \gamma b^{-1}(1 + a - \gamma + ab) - B - BC\frac{\partial F(b(K^1(a)))}{\partial a} = 0 \).

**B  The Impact of Demand Uncertainty**

**Proof of Proposition 7:** We first provide the proof for the perfectly competitive credit market case. Since this equilibrium is relevant for firms that may default but use a secured loan (Case ii of Proposition 4) and firms that may use an unsecured loan (Case iii of Proposition 4); we will analyze these two cases separately.

For firms that may default but use a secured loan (Case ii of Proposition 4), for \( a \in [0, a^d] \), we obtain \( \frac{\partial \Lambda(a)}{\partial a} = B C \phi \left( \frac{b(K^1(a)) - \bar{\xi}}{\sigma} \right) \left( \frac{b(K^1(a)) - \bar{\xi}}{\sigma^2} \right) < 0 \) since \( b(K^1(a)) = \bar{\xi}(1 + \frac{1}{b}) \left[ 1 - \frac{b}{cK^1(a)} \right] < \bar{\xi} \). From the Pareto-optimality of the equilibrium, i.e. \( \hat{a} \) is the minimum \( a \) that satisfies \( \Lambda(a) = 0 \), it follows that with a decrease in \( \sigma \), \( \hat{a} \) decreases.

For firms that may use an unsecured loan, since \( \hat{a} \in [a^l, a^d] \), it follows from above that \( \frac{\partial \Lambda(a)}{\partial a}|_{\hat{a}} < 0 \). With a decrease in \( \sigma \) from \( \sigma_0 \) to \( \sigma_1 \), it follows that \( \Lambda(\hat{a}(\sigma_0) ; \sigma_1) > 0 \). From the Pareto-optimality of the equilibrium, we have \( \hat{a}(\sigma_1) < \hat{a}(\sigma_0) \).

Since for a given \( a, \pi^* \) is independent of \( \sigma \), we have \( \frac{\partial \pi^*}{\partial a} = \frac{\partial \pi^*}{\partial a} \bigg|_{\hat{a}} \frac{\partial \sigma}{\partial a} \). From Lemma A.1, we have \( \frac{\partial \pi^*}{\partial a} < 0 \), hence \( \frac{\partial \pi^*}{\partial a} > 0 \). Similarly, \( K = K^1(\hat{a}) \) is independent of \( \sigma \); hence we have \( \frac{\partial K}{\partial a} = \frac{\partial K^1(\hat{a})}{\partial a} \bigg|_{\hat{a}} \frac{\partial a}{\partial a} \). Since \( K^1(\hat{a}) \) decreases in \( a \), the result follows.

We now provide the proof for the monopolist creditor case. Let \( \hat{a}^M \) denote the equilibrium financing cost for the monopolist creditor. \( \hat{a}^M \) satisfies \( FOC(\hat{a}^M) = 0 \) where

\[
\frac{\partial \Lambda(a)}{\partial a} = FOC(a) = cK^0(1 + a)^{b-1}(1 + a + ab) - B + BC \phi \left( \frac{b(K^1(a)) - \bar{\xi}}{\sigma} \right) \bar{\xi}(1 + \frac{1}{b}) \frac{B}{cK^0} \left( \frac{-b}{(1 + a)^{(b+1)}} \right).
\]

For firms that may default but use a secured loan (Case ii of Proposition 4), Proposition 5 shows that there exists a global maximizer \( \hat{a}^M \in (a^N, a^d) \) where \( a^N \) is given in Proposition 5 and \( a^d \) is given in Proposition 4. We have two subcases to consider.

If the creditor’s expected return \( \Lambda(a) \) is unimodal in \([0, a^d]\), \( \hat{a}^M \) denotes the unique maximizer. In this case, from the implicit function theorem, we have \( \frac{\partial \hat{a}^M}{\partial a} = -\frac{\partial FOC(a)/\partial a}{\partial FOC(a)/\partial a} \bigg|_{\hat{a}^M} \).

Since \( \hat{a}^M \) is the maximizer, we have \( \text{sgn} \left( \frac{\partial FOC(a)}{\partial a} \right) = \text{sgn} \left( \frac{\partial FOC(a)}{\partial a} \bigg|_{\hat{a}^M} \right) \). We obtain

\[
\frac{\partial FOC(a)}{\partial a} = BC \phi \left( \frac{b(K^1(a)) - \bar{\xi}}{\sigma} \right) \bar{\xi}(1 + \frac{1}{b}) \frac{B}{cK^0} \left( \frac{-b}{(1 + a)^{(b+1)}} \right) \left[ \left( \frac{b(K^1(a)) - \bar{\xi}}{\sigma} \right)^2 - 1 \right]. \tag{8}
\]

It follows that \( \text{sgn} \left( \frac{\partial FOC(a)}{\partial a} \bigg|_{\hat{a}^M} \right) > (\leqslant) 0 \) if \( \sigma < (>) \bar{\xi} - b(K^1(\hat{a}^M)) \). Using \( b(K^1(a)) = \bar{\xi}(1 + \frac{1}{b}) \left[ 1 - \frac{b}{cK^1(a)} \right] < \bar{\xi} \), \( \sigma < \bar{\xi} - b(K^1(\hat{a}^M)) \) is equivalent to \( \frac{\bar{\xi}}{1 + \frac{1}{b}} \frac{B}{cK^0} < (1 + \hat{a}^M)^{-b} \). Since
\(\dot{a}^M \geq 0\), it is sufficient to show that \(\frac{a}{\xi} < \frac{B(1+\frac{1}{b})}{cK^0} + \frac{1}{b}\). For \(b \geq -2\), this is satisfied for \(\frac{a}{\xi} < 0.5\). Therefore, \(\dot{a}^M\) is increasing in \(\sigma\).

If the creditor’s expected return \(\Lambda(a)\) is not unimodal in \([0, a^d]\), since \(\Lambda(a)\) is increasing in \(a\) for \(a < a^N\) (as follows from the proof of Proposition 5), there exist at least two local maximizers. From the Pareto optimality of \(\dot{a}^M(\sigma)\), since \(\left|\frac{\partial \Lambda(a)}{\partial \sigma}\right|\) is finite, for a sufficiently small change in \(\sigma\) the global maximizer does not shift from one local maximizer to the another one. Therefore, the result above continues to hold for local changes in \(\sigma\).

For firms that may use an unsecured loan (Case iii of Proposition 4), we have \(\dot{a}^M(\sigma) \in [\underline{a}, a^d]\). Similar to the case above, since \(\left|\frac{\partial \Lambda(a)}{\partial \sigma}\right|\) is finite, for a sufficiently small change in \(\sigma\) the global maximizer does not change. Therefore, the result above continues to hold for local changes in \(\sigma\).

If \(\dot{a}\) decreases (increases) in \(\sigma\), then \(K\) and \(\pi\) increase (decrease).

**Lemma B.1** If \(\xi\) is normally distributed with \((\bar{\xi}, \sigma)\) then for a given financing cost \(a\), for the firm that uses an unsecured loan, \(K^*\) and \(\pi^*\) increase in \(\sigma\).

**Lemma B.2** If \(\xi\) is normally distributed, \(K^* = \overline{K}\) is decreasing in \(a\).

**Lemma B.3** If \(\xi\) is normally distributed with \((\bar{\xi}, \sigma)\) when the firm uses an unsecured loan, the creditor’s net gain from secured lending and its expected loss due to the unsecured part of the loan increase in \(\sigma\). Its expected default cost increases in \(\sigma\) if \(b(\overline{K}(a)) \leq \bar{\xi}\).

## C Technology Choice

**Proof of Remark 2** The form of \(\sigma_D^P(c_F)\) follows from a direct comparison of \(\dot{\pi}_D\) and \(\dot{\pi}_F\) in perfect capital markets. Since \(\gamma_F \geq \gamma_D\) by assumption, to prove \(\sigma_D^P(c_F) \leq c_F\) it is sufficient to show \(\mathbb{E}^{-b} \left(\xi_1^{-b} + \xi_2^{-b}\right)^{-\frac{1}{b}} \geq 2\xi^{-b}\). From Hardy et. al (1988, p.146), if \(d \in (0,1)\) and \(X, Y\) are non-negative random variables then the following is true: \(\mathbb{E}^{1/d} [X^d + Y^d] \geq \mathbb{E}^{1/d} [X^d] + \mathbb{E}^{1/d} [Y^d]\) where equality only holds when \(X\) and \(Y\) are effectively proportional, i.e. \(X = \lambda Y\). In \(\sigma_D^P(c_F)\), we have \(d = -\frac{1}{b} \in (0,1)\) and \(\xi \geq \xi_1 \geq 0\), replacing \(X\) with \(\xi_1^{-b}\) and \(Y\) with \(\xi_2^{-b}\) gives the desired result. Notice that \(\sigma_D^P(c_F) = c_F\) only if \(\xi_1 = \xi_2\) (since we focus on the symmetric bivariate distribution) and \(\gamma_F = \gamma_D\). \(\xi_1 = \xi_2\) is only possible if either \(\xi\) is deterministic or \(\rho = 1\).

**Lemma C.1** In a perfectly competitive credit market, for firms that have a sufficiently large value of \(P\), with the technology cost pair \((c_D, c_F)\), where \(c_D > \sigma_D^P(c_F)\), \((c_D < \sigma_D^P(c_F))\) if
0 < \dot{a}_D < \dot{a}_F (0 < \dot{a}_F < \dot{a}_D), then the default risk is lower with the dedicated (flexible) technology in equilibrium.

D Selected Proofs for Supporting Lemmas

Proof of Lemma A.2: By using standard normal random variable, the derivative of the expression with respect to \( K \) is given by

\[
\frac{-1}{1 - \Phi \left( \frac{l(K) - \xi}{\sigma} \right)} \partial l(K) \partial K \left[ 1 - \Phi \left( \frac{l(K) - \xi}{\sigma} \right) \right]^2 - \phi \left( \frac{l(K) - \xi}{\sigma} \right) \int_{\frac{1}{l(K) - \xi}}^{\infty} (1 - \Phi(z)) dz \tag{9}
\]

where \( \Phi(.) \) and \( \phi(.) \) are the cdf and pdf of the standard normal random variable respectively. Since \( \frac{\partial l(K)}{\partial K} > 0 \), it is sufficient to show that the last term in parenthesis is positive. Let \( v = \frac{l(K) - \xi}{\sigma} \). Using integration by parts, we obtain \( \int_{v}^{\infty} (1 - \Phi(z)) dz = \phi(v) - v(1 - \Phi(v)) \). Substituting this in (9), it is sufficient to show that \( 1 > \left[ \frac{\phi(v)}{1 - \Phi(v)} \right]^2 - \frac{\phi(v)}{1 - \Phi(v)} \), which directly follows from Sampford (1953).

Proof of Lemma B.1: For a given \( a \), the optimal expected equity value is given by

\[
\pi^* = \left[ 1 - \Phi \left( \frac{l(K) - \xi}{\sigma} \right) \right] \left[ \frac{z}{\xi K^{1 + \frac{1}{b}}} + B(1 + a) + P - (1 + a)cK \right] + \sigma K^{1 + \frac{1}{b}} \phi \left( \frac{l(K) - \xi}{\sigma} \right). 
\]

We have \( \frac{\partial \pi^*}{\partial \sigma} = \frac{\partial \pi}{\partial K} \frac{\partial K}{\partial \sigma} + \frac{\partial \pi}{\partial \sigma} \frac{\partial K}{\partial \sigma} \), where the first term is zero from the optimality of \( K \). We obtain \( \frac{\partial \pi}{\partial \sigma} |_K = \frac{\pi}{K^{1 + \frac{1}{b}}} \phi \left( \frac{l(K) - \xi}{\sigma} \right) > 0. \)

For \( K^* = K \), since \( K \) is the unique maximizer, we have \( \text{sgn} \left( \frac{\partial \pi}{\partial \sigma} \right) \bigg|_K = \text{sgn} \left( \frac{\partial M P(K)}{\partial \sigma} \right) \bigg|_K \).

Using the optimality condition

\[
\left[ 1 - \Phi \left( \frac{l(K) - \xi}{\sigma} \right) \right] \left[ (1 + \frac{1}{b}) \xi K^{\frac{1}{b}} - (1 + a)c \right] = -\sigma K^{\frac{1}{b}} \phi \left( \frac{l(K) - \xi}{\sigma} \right), \tag{10}
\]

we obtain \( \frac{\partial M P(K)}{\partial \sigma} \bigg|_K = (1 + \frac{1}{b}) \xi K^{\frac{1}{b}} \phi \left( \frac{l(K) - \xi}{\sigma} \right) \left[ \left( \frac{l(K) - \xi}{\sigma} \right)^2 + 1 - \frac{\phi \left( \frac{l(K) - \xi}{\sigma} \right) \left( \frac{l(K) - \xi}{\sigma} \right)}{1 - \Phi \left( \frac{l(K) - \xi}{\sigma} \right)} \right] \).

Let \( z = \frac{l(K) - \xi}{\sigma} \). We need to show that \( 1 > z \left[ \frac{\phi(z)}{1 - \Phi(z)} - z \right] \). It follows from Sampford (1953) that \( \left[ \frac{\phi(z)}{1 - \Phi(z)} - z \right] < 1 - \Phi(z) / \phi(z) \); therefore it is sufficient to show \( 1 > z \left[ 1 - \Phi(z) / \phi(z) \right] \), which also follows from Sampford (1953).

Proof of Lemma B.2: Since \( K \) is the unique maximizer, we have \( \text{sgn} \left( \frac{\partial \pi}{\partial a} \right) = \text{sgn} \left( \frac{\partial M P(K)}{\partial a} \right) \bigg|_K \).

Using the optimality condition in (10), we obtain \( \frac{\partial M P(K)}{\partial a} \bigg|_K = \)

\[
\left[ 1 - \Phi \left( \frac{l(K) - \xi}{\sigma} \right) \right] \left[ -c + (1 + \frac{1}{b}) \xi K^{\frac{1}{b}} \frac{\partial l(K)}{\partial a} \left[ \left( \frac{\phi \left( \frac{l(K) - \xi}{\sigma} \right)}{1 - \Phi \left( \frac{l(K) - \xi}{\sigma} \right)} \right)^2 - \frac{\phi \left( \frac{l(K) - \xi}{\sigma} \right) \left( \frac{l(K) - \xi}{\sigma} \right)}{1 - \Phi \left( \frac{l(K) - \xi}{\sigma} \right)} \right] \right].
\]
Denoting $Y$ as the last expression in brackets and using 
\[
\frac{\partial l(K)}{\partial a} = K^{-1} \left[ c - \frac{B}{K} \right], 
\]
the desired result follows because 
\[-c + (1 + \frac{1}{b}) \left[ c - \frac{B}{K} \right] Y < 0 \text{ as } Y < 1 \text{ from Sampford (1953).} \]

**Proof of Lemma B.3:** We only provide the proof for the expected loss due to the unsecured part of the loan. The proofs for the default risk and the net gain from secured lending can be obtained in a similar fashion, and are omitted. With the normal distribution assumption, it is given by
\[
\Phi \left( \frac{l(K) - \bar{z}}{\sigma} \right) \left[ K(1 + a)c - \bar{z}K^{(1 + \frac{1}{b})} - B(1 + a) - P \right] + \sigma \phi \left( \frac{l(K) - \bar{z}}{\sigma} \right) K^{(1 + \frac{1}{b})}.
\]
Taking the derivative with respect to $\sigma$, and using the optimality condition in (10), the derivative with respect to $\sigma$ is given by 
\[
\phi \left( \frac{l(K) - \bar{z}}{\sigma} \right) K^{(1 + \frac{1}{b})} + \frac{\partial K}{\partial \sigma} \left[ (1 + a)e - \bar{z}K^{\frac{1}{b}} \right].
\]
This term is positive because 
\[
\frac{\partial K}{\partial \sigma} > 0 \text{ from Lemma B.1 and the last expression is positive from (10).}
\]

**Proof of Lemma C.1:** We only provide the proof for $c_D > c_P(c_F)$. The proof for $c_D < c_P(c_F)$ can be obtained in a similar fashion, and is omitted. If both technologies are exposed to identical financing costs, the net gain from secured lending is lower with the dedicated technology. Since we have $\Lambda_T(\hat{a}_T) = 0$ in equilibrium, default risk comparison can be obtained by comparing the net gain from secured lending terms. As follows from the proof of Proposition 5, the net gain from secured lending is first increasing then decreasing in $a_T$. Since the default risk is decreasing in $a_T$, due to the Pareto-optimality of the equilibrium, the equilibrium exists on the increasing part of the net gain term. Therefore, if $\hat{a}_F \geq \hat{a}_D$, then net gain from secured lending with dedicated technology is even lower. This concludes the proof. 

**E References**
