Optimal Organizational Design in a Dichotomous-Choice Project Selection Model

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ABSTRACT

This paper studies various aspects of the optimal design of economic organizations in the context of a project selection model, where decisions are made by fallible managers regarding the adoption or rejection of investment projects. I analyze the role of marginal decision costs, and establish that the sequential decision architecture is a pair of probability thresholds, and a corresponding pair of majority rules that vary with the stage of project evaluation. The paper also analyzes the adjustment in the minimum organizational size and the decision architecture as changes occur in the quality of the investment environment and managerial expertise.

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1. Introduction

In most economic organizations, matters of strategic importance and which involve significant risk are often decided by a team of decision-makers. The proverbial saying that "plans fail for lack of counsel, but with many advisers, they succeed" recognizes that individuals may not always have access to all the relevant information or possess the right expertise to make correct decisions. As a result, collective decision-making can potentially improve the quality of decisions as well as increase the chances of success of new organizational initiatives.

This paper looks at the optimal design of economic organizations in the context of a project selection model, where decisions are made by fallible managers regarding the adoption or rejection of investment projects. As is well-known, fallibility in decision-making may arise because individuals are limited in their training, ability or experience, and thus may not always make the correct decisions. Errors in judgment occur even if all the pertinent information necessary for the decision are available, decision-makers are sincere, have no vested interests to act differently and information is transmitted accurately. Within the fallibility framework, this paper analyzes the various aspects of the optimal organizational design: the minimum organizational size, the optimal sequential decision rules, and the impact of changes in the quality of the investment environment and the managerial expertise on the optimal architecture.

While earlier studies by Ben-Yashar and Nitzan (2001b), and Koh (1992a, 1992b, 1994b) and (Sah and Stiglitz (1986, 1988) considered specific sequential architectures in project evaluation and analyze their comparative properties, this paper considers the role that marginal decision costs play in the design of the organizational structure and establish the optimal sequential decision architecture. The motivation for the focus on marginal decision costs is the observation that most organizations operate at full managerial capacity, in the sense all the available managerial resources are fully deployed in making production and investment decisions at any point in time. With a fixed pool of managerial expertise within the organization, managerial tasks often have to be prioritized and for each task taken up by a group of managers, another task will only be attended to later. In order to utilize managerial time and expertise optimally, the allocation of managerial expertise at each point in time should recognize the marginal benefit of further deliberation on a decision versus the opportunity costs to the organization of doing so. The opportunity costs include the impact of potential delay in managerial attention on other projects, but also potential monetary loss to
the organization, as in the case when first-mover advantage matters in making investment
decisions – if the project under review is adopted by competitors – as well as the additional
resources that would be deployed to continuing the evaluation before a decision is made on
acceptance or rejection.

A contribution of this paper to the literature is to characterize the optimal sequential
decision architecture in the context of a project selection model, when the marginal decision
costs are present. When such marginal decision costs are absent, as may be the case when
managerial expertise are not fully deployed or when the organization can access additional
resources at no cost, the optimal decision rule takes the form of an optimal majority rule with
full participation by all the decision-makers in the decision process. This is the situation
studied in the literature in the context of committee decision-making, and the results for
optimal decision-making in fixed-size committees have been generalized and unified by Ben-
Yashar and Nitzan (1997). Intuitively, when the decision process is sequential and marginal
decision costs are positive, the optimal decision architecture includes the option to make the
decision earlier, as the expected benefits of further deliberation may be out-weighed by the
opportunity costs involved. We provide a characterization of the optimal decision rule.

The analysis in this paper is primarily concerned with sequential decision
architectures that arise due to the presence of marginal decision costs. There are other reasons
that may give rise to sequentiality in decision-making; for instance when managers are not
identical in abilities or hold different portfolios of responsibilities, a pre-ordering of the
decision-makers in the process is central to decision-making efficiency. In Section 6 of this
paper, I shall comment briefly on the issue of heterogeneous managerial ability and its impact
on the optimal sequential architecture.

Related Literature

There is an established literature on optimal group decision-making that face
dichotomous choices. Some of the earlier important work include Nitzan and Paroush (1982,
The recent literature emphasizing managerial fallibility that is related to this paper include
Sah and Stiglitz (1985, 1986, 1988). The recent research has studied two specific sequential
architectures – the hierarchy and polyarchy. In a strict hierarchical review process, a project is
rejected and the evaluation ends if one manager rejects the project. A project is only accepted
if the manager at the top of the hierarchy accepts it. By contrast, in a polyarchical review process, a project will be given further chances within the organization if it is turned down, and will be accepted once one manager accepts it. Another decision structure that has been the subject of much research is the committee with an optimally derived majority decision rule. The committee architecture has been studied in Koh (1994a), Ben-Yashar and Nitzan (1997) and Sah and Stiglitz (1985, 1988).

Within the framework of the dichotomous-choice model, the literature has studied the comparative properties of these organizational architectures, as well as the conditions under which a particular architecture dominates the others in terms of the implications on the welfare of the organizations and the costs to the organization. Ben-Yashar and Nitzan (2001b) and Koh (1992b) noted that the optimality of the hierarchical or polyarchial structures hinges on stringent conditions regarding the quality of the investment environment and the evaluation expertise of the managers. Koh (1993) discussed the impact of first-mover advantage and market competition on the choice of decision architecture.

The techniques used in this paper to characterize the optimal sequential architecture for economic organizations are familiar methods employed in the statistical decision literature in the determination of optimal stopping rules (see, for instance, Astrom (1970) and Degroot (1970)) and in design of sequential probability ratio testing (see, for instance, Bertsekas (1987)). In the terminology of the statistical decision literature, the design of the sequential decision architecture we study in this paper is akin to determining the optimal stopping rule for a sequential sampling process when a choice has to be made between two hypotheses. For the project selection problem at hand, there are two courses of actions to choose from when evaluation ends, and the types of potential errors that can occur are; Type I error when good projects are rejected, and Type II errors, when bad projects are accepted. The analysis in this paper thus provides another example of the application of statistical decision theory to the study of the economics of organization.

Organization of Paper

The paper is organized as follows. Section 2 introduces the project selection model. Section 3 derives the optimal majority rule for an accept-reject decision, which forms part of the optimal sequential decision rule. When marginal decision costs are zero, this optimal majority rule is also the optimal decision rule for the organization. Section 4 discusses the properties of the optimal organizational structure and analyzes the impact on the
organizational structure as the quality of the investment environment and the level of managerial expertise varies. Section 5 derives the optimal sequential decision architecture, when marginal decision costs are positive, and discuss its implications. Section 6 provides some concluding remarks.

Summary of main results

We present a summary of the results of the paper. In the project selection model presented in Section 2, a project that is either good or bad is evaluated sequentially by managers who then vote to accept or reject the project. Each additional evaluation incurs a constant marginal decision cost to the organization. At each stage of the project evaluation process, the organization has to decide if the evaluation process should be completed by either accepting or rejecting the project or to proceed for further evaluation and incurring further organizational costs. The objective of the organization is to determine an optimal sequential decision architecture that maximizes the expected payoff net of total decision costs.

In Section 3, we analyze the decision to accept or reject the project when it has undergone a series of evaluation and show that this decision will be based on an optimal majority rule (given in Proposition 1) that varies with the stage of evaluation, the quality of the investment environment and the expertise of the managers. This optimal majority rule forms part of the optimal sequential decision architecture, as by definition, the organization makes an “accept” or “reject” decision when the evaluation process were to reach maximum possible number of reviews, as set by the size of the management organization.

In Section 4, I discuss the quality of the organizational decision-making process. When the optimal organizational decision rule in Proposition 1 is implemented, the hierarchical or polyarchical decision architectures can be shown to be feasible architectures under particular conditions of the investment environment and managerial expertise (Propositions 2). The optimality of these architectures is discussed, and it is noted that in situations when the organizational size is constrained, the hierarchy or polyarchy architectures are general suboptimal structures, and will be dominated by a trivial decision to either always reject or always accept.

Furthermore, in Proposition 3, I note that the optimal decision architecture is not strictly sequential in the sense that the evaluation process would begin with a single manager, and a decision is then made if the project should proceed for further evaluation. Even if
marginal decision costs are zero, the investment environment and the quality of managerial expertise generally dictates a minimum organizational organization, denoted $M_{\min}$, which is greater than one. The optimal decision architecture is to begin the decision process with an initial review by the team of $M_{\min}$ managers. The intuition for this result is that generally, the quality of investment environment is not neutral, in the sense that there is often a natural bias to make only “accept” or only “reject” decisions if no evaluation is undertaken. Thus, for project evaluation to be informative, a minimum number of evaluations must be undertaken if the decision process is not dominated by a naïve strategy of either always accepting projects or not always rejecting projects.

Beginning with the initial review by the team of $M_{\min}$ managers in a committee setting, there are now three possible decisions at each evaluation stage: (1) accept the project and end the review process, (2) reject the project and end the review process, or (3) request for an additional evaluation. In Section 5, I establish, in Proposition 5 that the optimal sequential decision architecture is characterized by two probability thresholds at each stage of evaluation: if the conditional probability that the project is good exceeds the upper probability threshold, the project will be accepted without further evaluation; similarly, if the said conditional probability is below the lower probability threshold, the project will be rejected and evaluation ends. Otherwise, an additional review is desirable. Corresponding to the pair of probability thresholds is a pair of sequential majority rules, one for acceptance and another for rejection. When marginal decision costs are positive, the minimum size of the organization, as presented in Proposition 2, will be larger, as the presence of positive marginal decision costs makes the naïve strategies of always accepting or always rejecting more attractive. In Proposition 6, I show that it is possible to recursively derive the optimal sequential majority rules, which define the range within which an additional project review is desirable.

2. The Model

Consider an economic organization whose objective is to maximize the net expected payoffs from selecting and implementing investment projects, and faces the following investment environment: good projects yield a fixed payoff of $\pi_{GA}$ if correctly chosen and $\pi_{GR}$ if incorrectly rejected; bad projects yield a payoff of $\pi_{BR}$ if correctly rejected and $\pi_{BA}$ if incorrectly adopted. Let the proportion of good projects be $\alpha$, so that the proportion of bad
projects is \((1 - \alpha)\).\(^1\) We can think of \(\alpha\) and \((1 - \alpha)\) as, respectively, the a-priori probabilities that a project is good or bad. We require \(\pi_{GA} > \pi_{GR}\) and \(\pi_{BR} > \pi_{BA}\). Without loss of generality, we assume that \(\pi_{BR} > \pi_{GR}\).\(^2\)

Project evaluation is to be carried out managers who can differentiate good projects from bad ones, but only imperfectly. They make independent decisions to approve or reject projects but are otherwise identical in their expertise. Let the probability that a manager will approve a good project be \(g\), and the probability that he will approve a bad project be \(b\). Expertise is modeled by requiring \(g > b\). Evaluation of the project is to take place sequentially and each evaluation incurs a constant marginal decision cost of \(C\). For the analysis later in the paper, define \(\delta_a \equiv g/b\) measure the ability to discriminate good projects; correspondingly, \(\delta_r \equiv (1 - b)/(1 - g)\) measure the ability to discriminate bad projects. Since \(g > b\), it follows that \(\delta_a > 1\) and \(\delta_r > 1\), so that the product \(\delta_a \delta_r = \delta > 1\).

Let \(m\) be the number of evaluations to date. Furthermore, denote \(z_i, i = 1, \ldots, m\), as an independent Bernoulli variable representing the outcome of the \(i\)th review: \(z_i = 1\) represents a positive vote and \(z_i = 0\) represents a negative vote. Let \(P(m, n)\) denote the posterior probability that a prospect is good after \(m\) inspections with \(n\) favorable reviews, where \(n = \sum_{i=1}^{m} z_i\). The Bayesian updating of \(P(m, n)\) is represented by:

\[
P(m + 1, n + z_{m+1}) = \frac{P(m, n)\Gamma(z_{m+1} | G)}{\Gamma(1|G) + (1 - P(m, n))\Gamma(z_{m+1} | B)}
\]  

(1)

where \(\Gamma(1|G) = g\), \(\Gamma(0|G) = (1 - g)\), \(\Gamma(1|B) = b\), \(\Gamma(0|B) = (1 - b)\). We can simplify the expression in (1), using recursion to yield

\[
P(m, n) = \frac{\alpha g^n (1 - g)^{m-n}}{\alpha g^n (1 - g)^{m-n} + (1 - \alpha) b^n (1 - b)^{m-n}}
\]  

(2)

where \(P(0, 0) = \alpha\). The formula for \(P(m, n)\) indicates that the decision at stage \(m\) – whether to accept or reject the project, or proceed for an additional evaluation – is based on the degree of agreement about the project’s quality, and not on the history of the evaluation and the particular order in which opinions were formed. This is due to the assumption of identical,

\(^1\) The formulation of the model and the notation used in this paper can be recast straightforwardly into an equivalent setting shown in Ben-Yashar and Nitzan (1997).

\(^2\) Our analysis is equally applicable in both situations: (1) \(\pi_{GA} > \pi_{BR} \geq \pi_{GR} > \pi_{BA}\); (2) \(\pi_{GA} > \pi_{GR} \geq \pi_{BR} > \pi_{BA}\).
independent managerial expertise, so that \( P(m, n) \) is a sufficient statistic for the history of evaluation. When managers are not identical in their abilities, the ordering of the managers and the resultant evaluation history would be important for the decision on each project. Since \( g > b \), it is a routine matter to note the following relationships hold: (i) \( P(m, n+1) > P(m, n) \), (ii) \( P(m+1, n) < P(m, n) \), and (iii) \( P(m+1, n+1) > P(m, n) \).

Examples of dichotomous choice problems within economic organizations occur in the selection of projects for R&D funding or the determination of startup companies for venture investment. In the model presented here, we can think of the investment environment – represented by the proportion of good projects (\( \alpha \)) and the set of payoffs \( \{ \pi_{GA}, \pi_{BR}, \pi_{GR}, \pi_{BA} \} \) facing an organization as either the same for all firms within the industry, or, perhaps, more realistically, specific to each firm. Using the venture capital industry as an illustration, top-tier venture firms typically receive a higher proportion of the better projects or ideas (with also higher expected payoffs if successful), while lesser-known firms receive a relatively larger proportion of weaker projects. We may further generalize the setting to one where the a-prior probability of a project’s quality (\( \alpha \)) is dependent on the originator of the project or the idea, based on the past track record of the originator. This interpretation of the project-selection setting is particularly applicable to the problem internal resource allocation. Within organizations, business divisions routinely present business proposals or projects for budgetary approval; this is typically accompanied by a risk-return study of the potential payoffs and downside risks. Optimal decision-making in such a situation can be modeled as a project selection problem by the management team tasked to evaluate and decide on the merits of these competing proposals.

3. The Decision to Accept or Reject

Let \( M \) denote the organizational size, which sets the maximum possible number of evaluations \(^3\). We begin by analyzing the decision to accept or reject the project at stage \( m (= 1, \ldots, M) \) of the evaluation process. Define \( Z_a(P(m, n)) \) and \( Z_r(P(m, n)) \) to be, respectively, the expected payoff from accepting and rejecting the project after \( m \) evaluations, where

\[
Z_a(P(m, n)) = P(m, n)\pi_{GA} + (1 - P(m, n))\pi_{BA}
\]

\[
Z_r(P(m, n)) = P(m, n)\pi_{GR} + (1 - P(m, n))\pi_{BR}
\]

\(^3\) The fixed costs of hiring the \( M \) managers are incurred upfront, and so would not affect the determination of the optimal decision architecture.
Given our assumption of $\pi_{GA} \geq \pi_{BR} > \pi_{GR} > \pi_{BA}$, $Z_a(p)$ is linear and increasing in $p$ and $Z_r(p)$ is linear and decreasing in $p$. (See Figure 3 for an illustration.)

**Proposition 1**: There exists a majority decision rule $N(m)$ such that at stage $m$ of the evaluation process, the project should be considered for adoption if the number of approvals exceeds $N(m)$; otherwise, the project should be considered for rejection.

The derivation of $N(m)$ is straightforward. For the decision to accept (reject) at stage $m$ to be the optimal course of action, we show that $Z_a(P(m, n)) > (<) Z_r(P(m, n))$, which is equivalent to the condition that $P(m, n) > (<) Q^c$ where

$$Q^c \equiv \frac{\pi_{BR} - \pi_{BA}}{\pi_{GA} - \pi_{GR} + \pi_{BR} - \pi_{BA}}$$

(4)

Define $\gamma(m)$ as a function such that $P(m, \gamma(m)) = Q^c$. Using the definition of $P(m, n)$ in (2), we can solve for an explicit solution of $\gamma(m)$ to yield:

$$\gamma(m) = \frac{1}{\ln \delta} (-\ln \beta + m \ln \delta_r)$$

(5)

where

$$\beta = \frac{\alpha(\pi_{GA} - \pi_{GR})}{(1-\alpha)(\pi_{BR} - \pi_{BA})}$$

(6)

and $\delta = \delta_s \delta$, as defined earlier. Next, we note that depending on the parameters, $\beta$, $\delta_s$, and $\delta_r$, $\gamma(m)$ may be less than zero or greater than $m$. (A full characterization of $\gamma(m)$ is provided in the Appendix). The optimal majority rule $N(m)$ is therefore defined as follows:

$$N(m) \equiv \text{Int}(\min\{m, \max(0, \gamma(m))\})$$

(7)

where $\text{Int}(x)$ denotes the smallest integer greater than or equal to $x$.

In Section 5, I will show that if the marginal decision cost is constant at each stage, the optimal decision architecture is characterized by a probability range $(q'(m), q''(m))$, and an equivalent pair of majority rules $\{N'(m), N''(m)\}$, where $N'(m) < N(m) < N''(m)$, for $m = 1, \ldots, M-1$. If the number of approvals exceeds $N'(m)$, the project will be accepted, and if the number of approvals is fewer than $N'(m)$, the project will be rejected. Otherwise, an additional review is beneficial in terms of a higher expected net project payoff. Since the maximum number of evaluations is set by $M$, it follows that $N'(M) = N(M) = N''(M)$. Therefore, $N(M)$ provides the decision rule for the case if the evaluation process were to reach stage $M$, the last possible evaluation stage. Furthermore, if managerial expertise is not fully deployed within the organization, so that marginal decision costs are zero, every project should be reviewed by all the $M$ managers, so that $N(M)$ is the optimal decision rule for the organization.
Figure 1 illustrates the different possible variations for the optimal decision rule $N(m)$, for different organizational sizes, when there are no variable costs.

**FIGURE 1 ABOUT HERE**

4. The Quality of Organizational Decision-Making

This section analyzes the optimal adjustment of the decision rule $N(m)$ when changes occur in the quality of the investment environment and managerial expertise. As will be shown in the next section, under the optimal sequential architecture, the pair of majority rules \( \{N_r(m), N_a(m)\} \) bounds and tracks $N(m)$, and converges to $N(m)$ at $m = M$.\(^4\) It follows that as the organization carry out adjustments to \( \{N_r(m), N_a(m)\} \) in response to changes in the investment environment and managerial expertise, to maintain the optimality of the decision architecture, corresponding adjustments occur in $N(m)$ as well. Hence, an analysis of $N(m)$ allows us to understand the qualitative aspects of the optimal adjustments of the optimal sequential architecture \( \{N(m), N'(m)\} \).

We begin by noting that there are two indicators of the quality of the investment environment: $\beta$ (defined in (6)) and $Q_c$ (defined in (4)). Firstly, $\beta$ is the ratio of the expected payoff gains in the two states of the investment environment when the right decisions are made for each type of project. It is straightforward to show that $\beta > 1$ corresponds to a situation when no evaluation is to be undertaken, always accepting projects is preferred. Similarly, when $\beta < 1$, this corresponds to a situation where if no evaluation is to be undertaken, always rejecting projects is preferred. In this sense, $\beta > 1$ describes a favorable investment environment while $\beta < 1$ describes a mediocre investment environment. Lastly, $\beta = 1$ describes a neutral environment, in which the organization, without further information on a project, is indifferent towards pursuing every project (always accept) or not getting into business (always reject). The objective of an economic organization, when faced with these different investment environments, is to decide if it should invest and acquire the ability to make informed decisions, rather than adopt naïve

\(^4\) This will be shown in Proposition 6 and illustrated in Figures 5, 6 and 7.
strategies of pursuing every project (always accept) or not getting involved in the business at all (i.e. always reject).

\( Q^c \) is another indicator of the quality of the investment environment; it is increasing in \((\pi_{BR} - \pi_{BA})\) and decreasing in \((\pi_{GA} - \pi_{GR})\). Since a project will be considered for acceptance if the conditional probability \( P(m, n) \) is greater than \( Q^c \), it can be thought of as defining the threshold conditional quality for a project to be considered for acceptance. Since \( P(m, \gamma(m)) = Q^c \), it follows that \( \gamma(m) \) is increasing in \( Q^c \), holding \( m \) constant; therefore, \( N(m) \) is non-decreasing in \( Q^c \). The economic interpretation of \( Q^c \) is that it measures the importance of making the right decision for each type of project. As the payoff differential of bad projects increases, making it relatively more important for the organization not to accept bad projects, the quality of the organizational decision-making, as represented by \( N(m) \), becomes more stringent; i.e. the bar for acceptance will be raised. Similarly, as the payoff differential of good projects widens, the decision rule \( N(m) \), will be adjusted so that the likelihood of acceptance is improved.

Clearly, changes in the investment environment and managerial expertise, affects \( N(m) \), and in turn, will impact the choice of the optimal organizational size \( M \). From Figure 1, we note that if \( \beta \neq 1 \), the decision rule \( N(m) \) is trivially 0 or \( m \) when the organization is below a certain size. For instance, (A) when \( \beta > 1 \) and \( M < \text{Int} \left( \frac{\ln \beta}{\ln \delta_r} \right) \), \( N(m) = 0 \ \forall \ m \leq M \) and (B) when \( \beta < 1 \) and \( M < \text{Int} \left( -\frac{\ln \beta}{\ln \delta_u} \right) \), \( N(m) = m \ \forall \ m \leq M \). If the size of the management organization is set below the minimum level, project evaluation is clearly uninformative in the sense that under scenario (A), the decision rule is dominated by accepting all projects without evaluation and not incurring and any evaluation costs, while in scenario (B), rejecting all projects and not getting into business is the optimal course of action. When there are no marginal decision costs, we can state the following:

**Proposition 2:** (a) When \( \beta > 1 \), the minimum size of the organization is \( M_{\text{min}} = \text{Int} \left( \frac{\ln \beta}{\ln \delta_r} \right) \), with an optimal decision rule \( N(M_{\text{min}}) = 1 \); (b) When \( \beta < 1 \), the minimum size of the organization is \( M_{\text{min}} = \text{Int} \left( -\frac{\ln \beta}{\ln \delta_u} \right) \), with an optimal decision rule \( N(M_{\text{min}}) = M_{\text{min}} \).
The intuition for this result is that the investment environment and the quality of the managerial expertise creates a natural bias, in the absence of any evaluation, either to accept all projects when $\beta > 1$, or reject all projects – i.e. not to enter into business at all – when $\beta < 1$. Hence, under either scenario of the investment environment, the value of the initial evaluation is to generate information of sufficient value – in the sense that neither the “accept” nor “reject” decision is the dominant choice – to overcome this natural environment bias. The minimum organizational size simply indicates the smallest initial management team needed to produce informative evaluation to overcome the initial environmental bias.

It is straightforward to see from Figure 1 that if the size of the management organization is fixed at the minimum size described in Proposition 2, then the basic management organization described in Proposition 2a is a polyarchy while the basic management organization described in Proposition 2b is a hierarchy\(^5\), in the sense defined in the introductory section of this paper. It is straightforward to see that the optimality of both the hierarchy and the polyarchy architectures are sensitive to slight variations in the organizational size, the investment environment as well as managerial expertise. Thus, unless the optimal size of the organization happens also to be the minimum organizational size described in Proposition 2, the hierarchy as well as the polyarchy is sub-optimal decision architectures. Furthermore, if budgetary constraints force the organization to reduce its size below the minimum level required for informative evaluation in the sense discussed earlier, the hierarchy is dominated by not considering any investment, and the polyarchy is dominated by simply accepting all projects. Therefore, the robustness of the hierarchy and the polyarchy as optimal decision architectures is weak, and the conditions for their optimality are very stringent. While the literature has studied the comparative merits of these two sequential architectures, the analysis here illustrates that these architectures are generally non-optimal architectures if externally imposed on the organization\(^6\).

The minimum organizational size stated in Propositions 2a and 2b applies strictly to the situation when marginal decision costs are zero. In situations where marginal decision costs are positive, the minimum organizational size will be larger when the optimal sequential decision architecture is implemented, as will be shown in Section 5. Figure 6 illustrates that when $\beta < 1$, the minimum organizational size for a hierarchy will increase to $M_{\min}^\beta$, and each

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\(^5\) Generally, the hierarchy and the polyarchy are feasible organizational structures for an organizational size of $M_{\min}$, and possibly for $M_{\min} + 1$.

\(^6\) The results presented here complement those in Ben-Yashar and Nitzan (2001b), which examined the robustness of the polyarchy and the hierarchy as optimal architectures.
project now requires at least $M'_{\min}$ approvals before it is accepted. Similarly, Figure 7 shows that when $\beta > 1$, the minimum organizational size for a polyarchy is also larger, given by $M'_{\min}$, and each project now requires at least $M'_{\min}$ rejections before evaluation is terminated. The reason for the increase in the minimum organizational size for these two architectures follows from our earlier comment that the investment environment and the quality of managerial expertise creates a natural bias in favour of a naïve strategy of either accepting all or rejecting all projects. If marginal decision costs are positive, the attractiveness of the naïve strategies becomes greater ex-ante, which translates into a larger minimum organizational size for informative evaluation to dominate the naïve strategies.

From the above discussion, it should be apparent that once the minimum organizational size is implemented, it does not make a difference if the project is initially reviewed in a sequential process or simultaneously, as in a committee setting, by the initial team of $M_{\min}$ managers. Indeed, if speed in decision-making is desirable, due to, for instance, the existence of first-mover advantage (as discussed in Koh (1993)), it would be preferable for the initial review to be carried out simultaneously. The corollary, within the context of our model, is that re-organizing an existing strict sequential decision architecture into one that begins with an initial review carried out simultaneously by a team of the $M_{\min}$ managers would potentially improve organizational performance. We can state the following:

**Proposition 3:** In the project selection model, the initial review of the project will be carried out simultaneously by a team of $M_{\min}$ managers, where $M_{\min} = \min \left( \frac{\ln \beta}{\ln \delta_a}, \frac{\ln \beta}{\ln \delta_r} \right)$. Subsequent evaluation, if desired, will be carried out sequentially.

Next, we analyze how the decision rule $N(m)$ varies with the investment environment and the quality of managerial expertise. This is carried out by obtaining the comparative statics for $\gamma(m)$, defined in (5). For the purpose of our analysis, and without loss to generality to the results, we shall consider $N(m)$ as continuous, and conduct the analysis with $\gamma(m)$. We make the further assumption that $\gamma(m)$ is twice continuously differentiable. We provide the following derivatives for easy reference:

\[
\frac{d\gamma(m)}{dm} = \frac{\ln \delta_r}{\ln \delta} \in (0,1) \quad (8)
\]

\[
\frac{d\gamma(m)}{d\beta} = -\frac{1}{\beta} \quad \frac{d^2\gamma(m)}{d\beta^2} = \frac{1}{\beta^2} \quad (9)
\]
\[
d\gamma(m) = \frac{\ln \delta_r}{\delta_r (\ln \delta)^2} \quad \frac{d^2 \gamma(m)}{dmd\delta_r} = -\frac{\ln \delta_r}{\delta_r (\ln \delta)^2}
\]
\[
\frac{d\gamma(m)}{d\delta_r} = \frac{\ln \beta}{\delta_r (\ln \delta)^2} \quad \frac{d^2 \gamma(m)}{d\delta_r^2} = -\frac{\ln \beta}{(\delta_r)^2 (\ln \delta)^3} \{2 + \ln \delta\}
\]
\[
\frac{d\gamma(m)}{d\delta_a} = -\frac{1}{\delta_a (\ln \delta)^2} \{\ln \beta + m \ln \delta_a\} = -\frac{1}{\delta_a \ln \delta} \gamma(m)
\]
\[
\frac{d^2 \gamma(m)}{d\delta_a^2} = \frac{1}{(\delta_a)^2 (\ln \delta)^2} \{\ln \beta + m \ln \delta_a\} \{2 + \ln \delta\} = \frac{2 + \ln \delta}{(\delta_a)^2 \ln \delta} \gamma(m)
\]

The derivative in (8) provides the marginal decision rule; i.e. for an increase in size of the economic organization, the corresponding optimal adjustment in the decision rule \(N(m)\).

It is straightforward to see that the marginal decision rule will be a marginal majority rule, i.e. \(d\gamma(m)/dm = 0.5\), only if the probability of a manager accepting good or bad projects are equal, in i.e. \(g = b = 0.5\). Since \(g \geq b\), Figure 2 shows the different combinations of \(g\) and \(b\) that result in \(d\gamma(m)/dm > 0.5\).

FIGURE 2 ABOUT HERE

When marginal decision costs are zero, it is well-known that the simple (50%) majority rule is the optimal decision rule when \(\beta = 1\) and \(\delta_a = \delta_r\) (which requires that \(g = b = 0.5\)) regardless of the size of the organization. When managerial quality is unknown (or untested), it is reasonable to model such situations by assuming \(g = b = 0.5\). Thus, the marginal majority rule in this case is 0.5, so that \(N(M)/M \rightarrow 0.5\) as \(M \rightarrow \infty\), regardless of the quality of the investment environment. In other words, as the size of the organization expands, the simple majority rule is approximately optimal regardless of the investment environment. The corollary of this well-known result is that if the abilities of the decision-makers are known or if the abilities of the managers improve – either in making “accept” or “reject” decisions or both – a simple majority rule is invariably sub-optimal. The organization would improve the quality of its decision process, and thereby enhance its profitability, if it takes into account these asymmetries and optimally utilize them in setting its decision rule.
Quality of investment environment

As shown in (9), a better investment environment, as represented by a higher $\beta$, will lead the organization to optimally lower the majority required for accepting projects, but at a decelerating pace, ceteris paribus. More interestingly, we note an improvement in $\beta$ increases the minimum organizational size if $\beta > 1$, and reduces the minimum organizational size if $\beta > 1$. The reasoning here is that when $\beta > 1$, an increase in $\beta$ increases the desirability of accepting every project. By raising the minimum organizational size required for informative evaluation, it leads potentially to a situation that unless the organization can obtain additional managerial resources to conduct the evaluation of the investment projects, the optimal strategy here may be to simply accept all projects, as the environmental bias makes the initial evaluation less desirable.

In the case where $\beta < 1$, when the quality of the investment environment improves – as represented by an increase in $\beta$ – a smaller team is now required to conduct the initial evaluation of the project, thereby making it more attractive for new businesses of smaller minimum size to be established, if previously, it was uneconomical to do so. Conversely, if the investment environment deteriorates, and firms differ in terms of their decision-making abilities, we should also see an exit of weaker, less profitable firms from the industry; firms that remain are those who have better decision-making ability. Similarly, if firms do not face the same investment environment, a general decline in the quality of the investment environment will negatively impact those firms who face investment environments below the industry average, while firms who face above-average investment environments specific to themselves will survive.

One observation that is relevant to the preceding discussion is that in the wake of the bursting of the dotcom bubbles, many small venture capital firms, established in the last few years, have exited the industry; the venture capital firms that continue to do well are those that are recognized for their expertise and who have the pick of the best investment opportunities in the market. Of course, other factors are at play as well, as these established top-tier firms have had a strong investment track record, and therefore have an advantage in terms of their access to investment funds, which is critical to continued operations.

Quality of Managerial expertise

Improvements in managerial expertise are captured by improvements in $\delta_a$ and $\delta_r$, which define the comparative skills in making “accept” and “reject” decisions. The partial
derivatives in (10) indicate that the marginal decision rule will be tightened if the expertise in making “reject” decisions improves (as represented by an increase in $\delta_r$), but will be made less stringent, if the expertise in making “accept” decisions improves (as represented by an increase in $\delta_a$). Thus, the organization optimally adjusts the marginal decision rule to balance the organizational expertise in both the “accept” and “reject” decisions, when managerial expertise in either area improves. An increase in $\delta_a$ or $\delta_r$ may be due to a rise in the probability of accepting good projects, $g$, or a decrease in the probability of accepting bad projects, $b$, or both. The results in (11) and (12) can be explained similarly. When the managerial ability to discriminate bad projects and make “reject” decisions improves, the organization will optimally respond by setting tighter standards for approving projects, in the form of requiring a higher majority for acceptance. Similarly, an improvement in the ability to discriminate and accept good projects leads to a less stringent decision rule.

Furthermore, improvements in either $\delta_a$ or $\delta_r$ also affect the minimum organizational size. Suppose marginal decision costs are zero. In the case where $\beta > 1$, the minimum organizational size $M_{\text{min}}$, would also be reduced if $\delta_a$ improves, while in the case where $\beta < 1$, $M_{\text{min}}$, would also be reduced if $\delta_r$ improves. Thus, the improvement in managerial expertise is compensated by appropriate adjustments in both the minimum organizational size and the marginal decision rule. In many organizations, it is generally the case that managerial expertise improves with on-the-job learning experience. In the context of the project selection model, if on-the-job experiences lead to better decision-making ability, then an organization should regularly adjust its quality of its decision-making processes, in order to ensure the decision-making process does not become overly stringent. With the possibility of improvement in managerial expertise, we should also observe that organizations will start out initially with a stringent decision-making process; this is then gradually relaxed over time, as the ability to discriminate between good and bad projects improves.

Furthermore, even though the investment environment may remain constant over time, as managerial expertise improves, the minimum organizational size to establish a business operation will also decrease, thus allowing for the entry of new firms into the industry, if it was previously unprofitable to do so. Thus, if the promotion of competition is a desired policy objective for the governments, the analysis suggests that improving the overall level of industry expertise will serve to lower the barriers to entry as the minimum firm size will be reduced. This suggests a potential linkage between an improvement in the quality of organizational decision-making and market structure. This is the subject of further research.
Again, drawing an observation from the private equity industry, the Singapore government’s Economic Development Board has recently mooted the idea to establish an Asia-Pacific Training Institute for Private Equity, with the objective of raising the level of expertise in the private equity industry, and encouraging the establishment of new venture capital firms in Singapore.

5. The Optimal Sequential Organizational Architecture

In this section, we characterize optimal sequential decision architecture when marginal decision costs are positive. Define $A_m(P(m, n))$ to be the expected project payoff if the decision is to proceed for an additional review, after $m$ reviews has been carried out, and let the constant marginal decision cost be $C$. The decision at stage $m$ is to choose the action—accept, reject, proceed for another review—that maximizes the conditional expected payoff to the organization, given the project under review. Denote

$$V_m(P(m, n)) \equiv \max \{ Z_a(P(m, n)), Z_r(P(m, n)), A_m(P(m, n)) - C \} \quad (13)$$

The value function $V_m(P(m, n))$ describes the maximum expected payoff at stage $m$, given the evaluation history. It follows then that

$$A_m(P(m, n)) \equiv E[V_{m+1}(P(m + 1, n + z_{m+1})) \mid z_{m+1} \in \{0, 1\}] \quad (14)$$
where the expectation is taken over $z_{m+1}$ with respect to the probabilities:

$$H(z_{m+1} \mid P(m, n)) = P(m, n) \Gamma(z_{m+1} \mid G) + (1 - P(m, n)) \Gamma(z_{m+1} \mid B) \quad (15)$$

where $P(0, 0) = \alpha$ and $\Gamma(1 \mid G) = g$, $\Gamma(0 \mid G) = (1 - g)$, $\Gamma(1 \mid B) = b$, $\Gamma(0 \mid B) = (1 - b)$. For the rest of our analysis, we may on occasion suppress the variable $n$, and denote $P(m, n)$ simply as $p$, to ease notation, unless otherwise indicated. The following lemmas, with proofs contained in the Appendix, are used in the proof of Proposition 4.

**Lemma 1**: $V_m(p) \geq V_{m+1}(p)$ and $A_m(p) \geq A_{m+1}(p)$ \quad \forall \ m = 1, \ldots, M-1 and p \in [0, 1]

**Lemma 2**: $V_m(p)$ and $A_m(p)$ are convex in $p$. \quad \forall \ m = 1, \ldots, M-1 and p \in [0, 1]

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7 As reported in *The Straits Times*, Singapore, June 28, 2002.
8 Formally, we shall assume the marginal decision costs are small relative to the potential payoff of selecting a good project and the minimum organizational size is feasible.
9 We have assumed that $\pi_{BR} > \pi_{GR}$ in our analysis, so that $Z(p)$ is decreasing in $p$. This assumption is not critical to the derivation of Proposition 4. If $\pi_{BR} < \pi_{GR}$, this would then imply $Z(p)$ is increasing in $p$. The critical conditions for our results are in Lemma 2, that $V_m(p)$ and $A_m(p)$ are convex in $p$, $\forall \ m = 1, \ldots, M-1$ and $p \in [0, 1]$. The assumption that $\pi_{GA} > \pi_{BR} > \pi_{GR} > \pi_{BA}$ implies $Z_c(Q') = Z(Q') < \pi_{BR}$, with $Q'$ defined in (4). Propositions 4
Utilizing Lemmas 1 and 2, we prove in the Appendix, the following Proposition, which establishes the optimal sequential decision rule:

**Proposition 4**: At each stage $m$ of the decision process, there exists two probability thresholds, an upper bound $q^a(m)$ and a lower bound $q^r(m)$, where $q^r(m) < Q_c < q^a(m)$ for all $m = 1, \ldots, M-1$. Also, $q^r(m)$ is decreasing in $m$, $q^a(m)$ is increasing in $m$ and $q^a(M) = Q_c = q^r(M)$ as $M$ defines size of the organization. If $P(m, n) > q^a(m)$, the project is accepted and the evaluation ends; if $P(m, n) < q^r(m)$, the project is rejected and the evaluation stops. If $P(m, n) \in (q^r(m), q^a(m))$, $m = 1, \ldots, M-1$, a further review should be carried out.

The intuition behind the existence of the two probability thresholds and the probability range $(q^r(m), q^a(m))$ is straightforward. Suppose after the initial review by a team of $M_{\min}$ managers, the conditional probability that the project is a good project is close to one. Then the value of further evaluation – in terms of improving the expected organizational payoff – is lower compared with the case when the conditional probability is close to $Q_c$, which defines the marginal case for considering acceptance. When a further evaluation incurs a marginal cost, the optimal decision for the organization is to weigh the net gain in expected payoff from making a more informed decision against the cost of doing so. A similar argument applies in the case when the conditional probability is close to zero. The probability thresholds – $q^r(m)$ and $q^a(m)$ – define the cases where the value of additional evaluation is equal to the marginal decision cost.

Figure 3 below provides an illustration of the general relationship between $A_m(p) - C$, $V_m(p)$, $Z_a(p)$ and $Z_r(p)$ while Figure 4 provides an illustration of the probability range $(q^r(m), q^a(m))$, $m = 1, \ldots, M$ and the decision spaces at each stage of the evaluation process.

\[^{10}\text{and 5 continue to hold if we assume } \pi_{BR} \leq \pi_{GR}. \text{ In fact, if } \pi_{BR} \leq \pi_{GR} \text{ it is straightforward to show that } V_m(p) \text{ and } A_m(p) \text{ are increasing in } p, \forall m = 1, \ldots, M-1 \text{ and } p \in [0, 1], \text{ and the results in this paper continue to hold.}\]

\[^{10}\text{In order that our problem is non-trivial, we require that } A_m(1) - C > Z_a(Q^r) = Z_r(Q^r). \text{ Otherwise, it is not worthwhile to become fully informed about the quality of the good project. Graphically (see Figure 3), it implies that } (A_m(p) - C) \text{ intersects the right axis, when } p = 1, \text{ at a level higher than } Z_a(Q^r). \text{ This is necessary; otherwise, the sequential decision rule } \{N'(m), N^0(m)\} \text{ would not exist, and the optimal decision is to always accept or always reject.}\]
Corresponding to \((q'(m), q^a(m))\) is an optimal sequential architecture \(\{N^e(m), N^a(m)\}\):

**Proposition 5**: At each stage \(m\), there exists two majority rules – an upper bound \(N^e(m)\) and a lower bound \(N^a(m)\), \(\forall m = 1, \ldots, M-1\). By definition of \(M\) as the optimal organizational size, \(N^e(M) = N^a(M) = N(M)\). If the number of approvals exceeds \(N^e(m)\), the project is accepted, and evaluation ends. If the number of approvals falls below \(N^a(m)\), the project is rejected, and evaluation ends. Otherwise, an additional evaluation is beneficial.

The existence of the sequential majority rules \(N^e(m)\) and \(N^a(m)\) follows directly from the fact that, by definition, \(P(m, N^e(m)) = q'(m)\) and \(P(m, N^a(m)) = q^a(m)\), so that

\[
N^e(m) = \text{Int} \left\{ \min \left[ \frac{1}{\ln d} \left( \ln \left( \frac{q'(m)}{1-q'(m)} \right) - \ln \left( \frac{a}{1-a} \right) + m \ln d_r \right), \ m \right] \right\}
\]

\(16a\)

\[
N^a(m) = \text{Int} \left\{ \max \left[ \frac{1}{\ln d} \left( \ln \left( \frac{q^a(m)}{1-q^a(m)} \right) - \ln \left( \frac{a}{1-a} \right) + m \ln d_r \right), \ 0 \right] \right\}
\]

\(16b\)

From \((16a)\) and \((16b)\), there exist \(M'_{min}\) and \(M^a_{min}\) such that for \(m < M'_{min}\), \(N^e(m) = m\) and for \(m < M^a_{min}\), \(N^a(m) = 0\). Routine calculation yields the following relationships:

\[
M'_{min} = \text{Int} \left\{ \frac{1}{\ln d_a} \left( \ln \left( \frac{q'(M'_{min})}{1-q'(M'_{min})} \right) - \ln \left( \frac{a}{1-a} \right) \right) \right\}
\]

\(17a\)

\[
M^a_{min} = \text{Int} \left\{ \frac{1}{\ln d_r} \left( \ln \left( \frac{a}{1-a} \right) - \ln \left( \frac{q^a(M^a_{min})}{1-q^a(M^a_{min})} \right) \right) \right\}
\]

\(17b\)

Since both \((Q^e - q'(m))\) and \((q^a(m) - Q^a)\) are decreasing in \(m\), it follows that when \(m > M'_{min}\), \((N(m) - N^e(m))\) is decreasing in \(m\); similarly, when \(m > M^a_{min}\), \((N^a(m) - N(m))\) is decreasing in \(m\). Hence, both \(N^e(m)\) and \(N^a(m)\) converge to \(N(m)\). More precisely, we are able to establish in Proposition 6 the following properties for \(\{N^e(m), N^a(m)\}\) (see the Appendix for the proof):

**Proposition 6**: For \(m < M-1\),

(a) \(N^e(m) + \frac{\ln \delta_r}{\ln \delta} < N^e(m+1) < N^e(m) + 1\);

(b) \(N^a(m) < N^a(m+1) < N^a(m) + \frac{\ln \delta_r}{\ln \delta}\)

Using the properties of \(N^e(m)\) and \(N^a(m)\) stated in Proposition 6, and beginning with the following relationships for stage \(M-1\) of the evaluation process, the explicit form of the
decision rules $N'(m)$ and $N''(m)$ can be derived recursively\(^{11}\), (the derivation is contained in the Appendix):

\[
q''(M-1) = \frac{b(\pi_{BR} - \pi_{BA}) + C}{b(\pi_{BR} - \pi_{BA}) + g(\pi_{GA} - \pi_{GR})}
\]

\[
q'''(M-1) = \frac{(1-b)(\pi_{BR} - \pi_{BA}) + C}{(1-b)(\pi_{BR} - \pi_{BA}) + (1-g)(\pi_{GA} - \pi_{GR})}
\]

\[
N'(M-1) = \text{Int}\left\{ -\frac{ln \beta_f}{ln d} + (M-1) \frac{ln d}{ln d} \right\}
\quad \text{where } \beta_f = \frac{\alpha\{g(\pi_{GA} - \pi_{GR}) - C\}}{(1-\alpha)(b(\pi_{BR} - \pi_{BA}) + C)}
\]

\[
N''(M-1) = \text{Int}\left\{ -\frac{ln \beta_a}{ln d} + (M-1) \frac{ln d}{ln d} \right\}
\quad \text{where } \beta_a = \frac{\alpha\{(1-g)(\pi_{GA} - \pi_{GR}) - C\}}{(1-\alpha)(1-b)(\pi_{BR} - \pi_{BA}) + C}
\]

Figures 5, 6 and 7 provide, respectively, illustrations of the optimal sequential decision architecture for the cases when $\beta = 1$, $\beta < 1$ and $\beta > 1$.

\[
\text{FIGURES 5, 6, 7 ABOUT HERE}
\]

In Section 3, we noted that in a neutral investment environment, i.e. when $\beta = 1$, the optimal decision architecture is potentially strictly sequential (i.e. starting the initial review with one manager), while in other cases, there is a minimum organizational size before project valuation can meaningfully improve decision-making (see Figure 1). This is true only when the marginal decision costs are zero. When we consider the benefit of a further review, as represented by $A_m(p) - C$, against the decision to accept or reject (as represented by $Z_a(p)$ and $Z_r(p)$ respectively), the minimum organizational size will be larger than one even for the case when $\beta = 1$. As illustrated in Figure 5, this is given by $M''_{min}$, so that after the initial review by the team of $M''_{min}$ managers, the organization can decide if the project should be accepted or proceed for further review. In general, the minimum organizational size is given by Min $\{M''_{min}, M''_{min}\}$. This applies also in the case for $\beta < 1$ (see Figure 6) and $\beta > 1$ (see Figure 7). The intuition, as noted before, is that when marginal decision costs are positive, the attractiveness of a naïve strategy to accept or reject all projects becomes greater; thus, the

\(^{11}\) I have explored two approximations to the optimal decision rule: a myopic one-step-look-ahead decision rule (i.e. assuming that the only decision available in the next stage of evaluation is an “accept” or “reject” decision) and a linear approximation to the decision rule $\{N''(m), N'(m)\}$, utilizing our knowledge of $N'(M-1), N''(M-1)$ and $N(M)$. The performance of both approximations, in terms of the deviation from the optimal decision architecture, depends on the particular specifications of the model.
threshold for informative evaluation, as reflected in an increase in the minimum organizational size, is raised.

The marginal decision costs

From the preceding analysis, it is clear that the optimal sequential decision architecture and the desirability of additional evaluation, as represented by the probability range \((q'(m), q''(m)), m = 1, \ldots, M\), depends on the magnitude of the marginal decision cost. While we have assumed that marginal decision costs are constant in the analysis, the results generalize to situations where the marginal decision costs are increasing with the stage of the evaluation, as would be the case where senior managers face multiple demands on their time, or when further delay in decision-making may adversely affect the organization’s chances of investing in the project. The impact of increasing marginal decision costs will make the probability range \((q'(m), q''(m))\) tighter at later stages of the evaluation, and correspondingly, the likelihood of an additional evaluation is reduced. In general, an increase in \(C\) will lead to a narrowing of the probability range \((q'(m), q''(m))\), \(\forall m = 1, \ldots, M-1\), so that the expected number of evaluations of a project will become smaller.

Conversely, if there is a reduction the marginal decision cost, the expected number of evaluations will increase, since the relative benefit of an additional evaluation increases. In the limit when the marginal decision cost \(C\) tends to zero, it is routine to verify that \(A_m(0) \rightarrow \pi_{BR}, A_m(1) \rightarrow \pi_{GA}\), so that \(q'(m) \rightarrow 0\) and \(q''(m) \rightarrow 1\). Therefore, the probability range \((q'(m), q''(m))\) converges to \((0, 1)\). Intuitively, additional reviews of the project are always desirable if they do not incur marginal decision costs to the organization. In the limit, the optimal organizational architecture converges to a fixed-size committee of size \(M\), with a simple majority decision rule of \(\hat{N}(M)\), and each project undergoes \(M\) reviews in a committee.

When the optimal sequential decision rule \(\{N'(m), N''(m)\}\) is implemented in each organization, the strategy to solve for the optimal organizational size is to compare the expected payoff of different organizations and select the organizational size that provides the highest expected terminal payoff less the expected evaluation costs. An explicit solution of the optimal organizational size \(M\) requires further specification of the model\(^{12}\).

\(^{12}\) We can derive an upper bound to the size of the management organization. Let \(B(M)\) denote the (unconditional) gross expected payoff to the organization when the optimal sequential decision architecture \(\{N'(m), N''(m)\}\) is implemented. Note that \(B(M) \leq \alpha \pi_{GA} + (1-\alpha)\pi_{BR}\), as this occurs only if the quality of each project can be ascertained perfectly. Under the optimal sequential decision rule, the expected number of evaluations for a project is
6. Concluding Comments

In this paper, we have characterized the optimal sequential decision architecture when marginal decision costs are positive. We also note that the quality of the investment environment and the level of managerial expertise entail a minimum organizational size, in order that project evaluation is informative and is not dominated by a simple strategy of always accepting projects or always rejecting projects. In our analysis, we have not considered several issues that are clearly important in understanding the optimal design of organizations. For instance, we have not considered the possibility of managers investing in acquiring skills or a better understanding of the investment environment, that could aid in improving their expertise in evaluating and choosing projects. This issue is important when the size of the organization is constrained, as the economic organization will direct managers to optimally adjust their decision criteria in response to changes in the investment environment. Ben-Yashar and Nitzan (1998) and Koh (1992b, 1994a) have studied this aspect of collective decision-making in organizations.

While our analysis considered the case that evaluation at each stage is undertaken by one manager, we can generalize the results to situations where the decision at each stage is undertaken by a committee. This has the effect of quickening the decision process, reduce the risk that other rival firms will adopt the project earlier – which may adversely impact its chances of success should it decide to pursue the same investment opportunity, as is the case in most venture capital investments where the first-mover advantage may be significant.

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Assuming other costs of the organization are $K$, the optimal organizational size $M^*$ satisfies $M^* < \{B(M) - K\}/C < \{(\alpha \pi_{GA} + (1-\alpha)\pi_{BR}) - K\}/C$. 
Appendix

Characterization of $\gamma(m)$, for the illustration in Figure 1:

From (5), we note the following:

- If $\beta > 1$ and $m \leq \frac{\ln \beta}{\ln \delta_r}$,
  $\gamma(m) < 0$.
- If $\beta = 1$ at $m = 0$, $\gamma(m) = 0$.
- If $\beta > 1$ at $m = \frac{\ln \beta}{\ln \delta_r}$,
  $\gamma(m) > 0$.

For $m > 0$,

- If $\beta \geq 1$, $\forall m$; $\beta < 1$ and $m > -a\frac{\ln \beta}{\ln \delta_a}$,
  $\gamma(m) < m$.
- If $\beta < 1$ and $m = -a\frac{\ln \beta}{\ln \delta_a}$,
  $\gamma(m) = m$.
- If $\beta < 1$ and $m < -a\frac{\ln \beta}{\ln \delta_a}$,
  $\gamma(m) > m$.

Proof of Lemma 1: We first show that if $V_m(p) \geq V_{m+1}(p)$ is true for some $m$, then $V_{m-1}(p) \geq V_m(p)$. Suppose $V_m(p) \geq V_{m+1}(p)$, we have $A_{m-1}(p_{m-1}) = \mathbb{E}[V_m(p_m)] \geq \mathbb{E}[V_{m+1}(p_m)] = A_m(p_{m-1})$. Therefore,

- $V_{m-1}(p_{m-1}) = \max \{ Z_a(p_{m-1}), Z_r(p_{m-1}), A_{m-1}(p_{m-1}) - C \} \geq \max \{ Z_a(p_{m-1}), Z_r(p_{m-1}), A_m(p_{m-1}) - C \} = V_m(p_{m-1})$.

Next, we show that $V_{M-1}(p) \geq V_M(p)$. This is obvious, by the definition of the fact that $M$ denotes the maximum number of evaluations,

- $V_{M-1}(p_{M-1}) = \max \{ Z_a(p_{M-1}), Z_r(p_{M-1}), A_{M-1}(p_{M-1}) - C \} \geq \max \{ Z_a(p_M), Z_r(p_M) \} = V_M(p_{M-1})$. 


Proof of Lemma 2:

First, it is obvious that $V_M(p)$ is convex in $p$ since it is the maximum of two linear functions, $Z_a(p)$ and $Z_r(p)$. Assume there exists some $m$ such that $V_{m+1}(p)$ is convex in $p$. First, note that

$$A_m(p) = (pg + (1-p)b)V_{m+1}\left(\frac{pg}{pg + (1-p)b}\right)$$

$$+ (p(1-g) + (1-p)(1-b))V_{m+1}\left(\frac{p(1-g)}{p(1-g) + (1-p)(1-b)}\right)$$

We can rewrite $A_m(p) = U(1|p) + U(0|p)$ where

$$U(z_{m+1}|p) \equiv [p\Gamma(z_{m+1}|G) + (1-p)\Gamma(z_{m+1}|B)]V_{m+1}\left(\frac{p\Gamma(z_{m+1}|G)}{p\Gamma(z_{m+1}|G) + (1-p)\Gamma(z_{m+1}|B)}\right)$$

and $\Gamma(1|G) = g$, $\Gamma(0|G) = (1-g)$, $\Gamma(1|B) = b$, $\Gamma(0|B) = (1-b)$. To show the convexity of $A_m(p)$ in $p$, $\forall m = 1, \ldots, M-1$ and $p \in [0, 1]$, it is sufficient to show that $A_m(p)$ is convex in $p$ in the above expression. Indeed, assume $V_{m+1}(p)$ is convex in $p$ for some $m$. To demonstrate the convexity of $U(z_m|p)$ in $p$ and therefore the convexity of $A_m(p)$ in $p$, we must show that for every $\lambda \in [0, 1]$, and $p_1$ and $p_2 \in [0, 1]$, we must have

$$\lambda U(z_{m+1}|p_1) + (1-\lambda) U(z_{m+1}|p_2) \geq U(z_{m+1}|\lambda p_1 + (1-\lambda) p_2)$$

This expression can be rewritten, suppressing the subscript for $z$, to be

$$\frac{\lambda H(z|p_1)}{\lambda H(z|p_1) + (1-\lambda) H(z|p_2)} V_{m+1}\left(\frac{p_1\Gamma(z|G)}{H(z|p_1)}\right) + \frac{(1-\lambda) H(z|p_2)}{\lambda H(z|p_1) + (1-\lambda) H(z|p_2)} V_{m+1}\left(\frac{p_2\Gamma(z|G)}{H(z|p_2)}\right)$$

$$\geq V_{m+1}\left(\frac{(\lambda p_1 + (1-\lambda) p_2)\Gamma(z|G)}{\lambda H(z|p_1) + (1-\lambda) H(z|p_2)}\right)$$

where $H(z|p)$ is defined in (15). The relationship as derived above is implied by our assumption that $V_{m+1}(p)$ is convex in $p$. If $A_m(p)$ is convex in $p$, it follows that since $V_m(p)$ is the maximum of three convex functions, it is also convex in $p$. 

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Proof of Proposition 4:

First, we note that as \( p \to 1 \), \( A_m(p) \to Z_a(1) = \pi_{GA} \); similarly, as \( p \to 0 \), \( A_m(p) \to Z_a(0) = \pi_{BR} \), \( \forall m = 1, \ldots, M \). This is clearly seen from the formula given for \( A_m(p) \) in the proof of Lemma 2. In Footnote 11, we noted that one of the conditions we need to assume, in order that our problem is non-trivial, is that \( A_m(1) - C > Z_a(Q^c) = Z_r(Q^c) \). Otherwise, it is not worthwhile to become informed about the quality of a good project. Furthermore, we have \( Z_a(Q^c) = Z_r(Q^c) < A_m(0) \), by our assumption that \( \pi_{BR} \geq \pi_{GR} \). Figure 3 illustrates the relationship. Since \( M \) denotes the organizational size, no further reviews should be carried out if a project reaches the \( M \)th stage, and a decision to accept or reject needs to be taken. By its definition, we have

\[
A_M(Q^c) - C = Z_a(Q^c) = Z_r(Q^c)
\]

so that \( V_M(Q^c) = \max\{Z_a(Q^c), Z_r(Q^c)\} \).

Next, we note from Lemma 2, that \( A_{m+1}(p) \geq A_m(p) \), so that for stage \( M-1 \), we would have

\[
A_{M-1}(Q^c) - C > Z_a(Q^c) = Z_r(Q^c)
\]

Since \( Z_a(p) \) and \( Z_r(p) \) are linear, and \( A_m(p) \) is convex in \( p \), with \( A_m(1) - C > Z_a(Q^c) = Z_r(Q^c) \) and \( A_m(0) > Z_a(Q^c) = Z_r(Q^c) \), it is straightforward to see that the function \( \{A_{M-1}(p) - C\} \) intersects the function \( \max\{Z_a(p), Z_r(p)\} \) at two points, \( (q^a(M-1), Z_a(q^a(M-1))) \) and \( (q^r(M-1), Z_r(q^r(M-1))) \) so that

\[
A_{M-1}(q^a(M-1)) - C = Z_a(q^a(M-1))
\]

\[
A_{M-1}(q^r(M-1)) - C = Z_r(q^r(M-1))
\]

and \( q^r(M-1) < Q^c < q^a(M-1) \). Thus, if \( P(M-1, n) \in (q^r(M-1), q^a(M-1)) \), requesting for another review of the project is the correct decision.

Next, utilizing Lemma 2, it follows that the probability range \( (q^r(m), q^a(m)) \) narrows as \( m \) increases and converges to zero at \( m = M \), since by definition, \( M \) denotes the final stage of evaluation when it is reached. Since \( A_{m-1}(p) \geq A_m(p) \) for \( p \neq 0 \) or 1, it is obvious that \( q^a(m) \) is decreasing in \( p \) and \( q^r(m) \) is increasing in \( p \).
Proof of Proposition 6: To prove that for \( m = 1, \ldots, M-1 \), (a) \( N'(m) + \frac{\ln \delta_r}{\ln \delta} < N'(m+1) < N'(m) + 1 \). First, we note from Proposition 4 that \( q'(m) \) is increasing, so that \( q'(m) < q'(m+1) \); this implies \( P(m, N'(m)) < P(m+1, N'(m+1)) \). Using the definition of \( P(m, n) \) in (2), it is routine to show that \( N'(m) + \frac{\ln \delta_r}{\ln \delta} < N'(m+1) \). Next, to show that \( N'(m+1) < N'(m) + 1 \), suppose that \( n_{m+1} \leq N'(m+1) \). This implies that \( V_{m+1}(P(m+1, n_{m+1})) = Z_r(P(m+1, n_{m+1})) \) and \( V_{m+1}(P(m+1, n_m)) = Z_r(P(m+1, n_m)) \). This in turn implies, using (14) and (15) to take conditional expectation, that \( A_m(P(m, n_m)) = Z_r(P(m, n_m)) \). Therefore, we have \( V_m(P(m, n_m)) = Z_r(P(m, n_m)) \). Since this is true for \( n_m < N'(m) \), it follows from our assumption that \( n_{m+1} \leq N'(m+1) \) that \( n_{m+1} \leq N'(m+1) < N'(m) + 1 \).

Next, to prove that for \( m = 1, \ldots, M-1 \), (b) \( N'(m) < N'(m+1) < N'(m) + \frac{\ln \delta_r}{\ln \delta} \). Again, we note from Proposition 4 that \( q^*(m) \) is decreasing, so that \( q^*(m) > q^*(m+1) \); this implies \( P(m, N''(m)) > P(m+1, N''(m+1)) \). Similarly, using the definition of \( P(m, n) \) in (2), it is routine to show that \( N''(m+1) < N''(m) + \frac{\ln \delta_r}{\ln \delta} \). Next, to show that \( N''(m) < N''(m+1) \), suppose that \( n_m \geq N''(m+1) \). This implies that \( V_{m+1}(P(m+1, n_{m+1})) = Z_r(P(m+1, n_{m+1})) \) and \( V_{m+1}(P(m+1, n_m)) = Z_r(P(m+1, n_m)) \). This in turn implies, using (14) and (15) to take conditional expectation, \( A_m(P(m, n_m)) = Z_r(P(m, n_m)) \). Therefore, we have \( V_m(P(m, n_m)) = Z_r(P(m, n_m)) \). Since this is true for \( n_m > N''(m) \), it follows from our assumption that \( n_m \geq N''(m+1) \) that \( n_m \geq N''(m+1) > N''(m) \).

Derivation of the results in (18) to (21): Consider the case for \( q'(M-1) \). By definition, \( q'(M-1) = P(M-1, N(M-1)) \). Using Proposition 6a, and noting that \( N'(M) = N''(M) \), it follows that \( V_M(P(M, N'(M-1)+1)) = Z_r(P(M, N'(M-1)+1)) \) and \( V_M(P(M, N'(M-1)-1)) = Z_r(P(M, N'(M-1)-1)) \). Next, using (14) and (15), and writing \( q'(M-1) \) as \( q'_{M-1} \) for short,

\[
A_{M-1}(q'_{M-1}) = q'_{M-1} g \pi_{GA} + (1-q'_{M-1} b) \pi_{BA} + q'_{M-1} (1-g) \pi_{GR} + (1-q'_{M-1})(1-b) \pi_{BR}
\]

Noting that \( A_{M-1}(q'(M-1)) - C \geq Z_r(q'(M-1)) \), we derive the results in (18) and (19).

The derivation for (20) and (21) is similarly carried out.
References


The optimal decision rule for an accept-reject decision

\[ N(m) = \text{Int}( \text{Min} \{ m, \text{Max}\{0, \gamma(m)\} \} ) \]

\[ M_{\beta \leq 1} = \text{Int} \left( -\frac{\ln \beta}{\ln \delta} \right) \]

\[ M_{\beta > 1} = \text{Int} \left( \frac{\ln \beta}{\ln \delta} \right) \]

Figure 1:
Figure 2:

The conditions for $g$ and $b$ under which the marginal decision rule
is greater or less than 0.5

$$g = b$$

$$g = (1 - b)$$

$$\frac{d\gamma(m)}{dm} > 0.5$$

$$\frac{d\gamma(m)}{dm} < 0.5$$
Figure 3:

An illustration of the general relationship between

\[ A_m(p), V_m(p), Z_a(p) \text{ and } Z_r(p) \]

\[ V_m(p) \equiv \text{Max} \{ Z_a(p), Z_r(p), A_m(p) - C \} \]
Figure 4:

An illustration of the relationship between $Q^c$ and the probability range $(q'(m), q''(m))$
Figure 5:

An illustration of the optimal sequential decision architecture \( \{N'(m) N''(m)\} \) when \( \beta = 1 \)
Figure 6:

An illustration of the optimal sequential decision architecture \( \{N'(m) \ N''(m)\} \)

when \( \beta < 1 \)
Figure 7:

An illustration of the optimal sequential decision architecture \( \{N'(m) N''(m)\} \)
when \( \beta > 1 \)